

Electromagnetic Theory I: Homework 3

Due: 28 August 2026

This assignment will be graded immediately after the due date. If you get all problems correct, then you will receive 100%. If you have made any errors, then I will deduct 10%, point the errors out and you must submit a corrected assignment by 4 September 2026. If there are still errors, then I will deduct another 10% and you must submit the corrected assignment by 11 September 2026. This will continue until you **have solved every problem correctly**.

1 Divergence of a vector field

Consider vector fields in a two dimensional plane. These have the form:

$$\mathbf{v} = v_x(x, y)\hat{\mathbf{x}} + v_y(x, y)\hat{\mathbf{y}}$$

where $v_x(x, y)$ and $v_y(x, y)$ are two functions of x and y . Construct a vector field by choosing two such functions, which must satisfy the following: they were not used previously in class, they are not just constant functions and they are relatively simple (don't try complicated expressions with trigonometric and other special functions).

- Sketch several vectors for the vector field that you chose. Based on the appearance of the vector field, will the divergence be zero or not? Explain your answer.
- Compute the divergence of the vector field. Was your prediction correct?

2 Divergence and curl of a vector field with three components, 2

Determine the divergence and curl of

$$\mathbf{v} = xy\hat{\mathbf{x}} + yz\hat{\mathbf{y}} + xz\hat{\mathbf{z}}.$$

3 Radial vector field

Let

$$\mathbf{v} = \frac{\hat{\mathbf{r}}}{r^n}$$

where $\mathbf{r} = x\hat{\mathbf{x}} + y\hat{\mathbf{y}} + z\hat{\mathbf{z}}$ and n is an integer.

- Sketch this vector field. Use the sketch describe whether you expect $\nabla \cdot \mathbf{v}$ to be positive, negative or zero. Use the sketch to describe whether you expect $\nabla \times \mathbf{v}$ to be zero or not.
- Show that

$$\nabla \cdot \mathbf{v} = \frac{2 - n}{r^{n+1}}$$

For which values of n is this positive, negative or zero? Do the results result agree your predictions? *Hint: first rewrite $\mathbf{v}$ in terms of $\mathbf{r}$.*

c) Determine $\nabla \times \mathbf{v}$. Does the result agree with your prediction?

4 Differentiating products, 4

Consider

$$\begin{aligned}\mathbf{A} &= \hat{\mathbf{x}} + \hat{\mathbf{y}} \quad \text{and} \\ \mathbf{B} &= y\hat{\mathbf{x}} - x\hat{\mathbf{y}}.\end{aligned}$$

Show, by direct substitution into either side that

$$\nabla(\mathbf{A} \cdot \mathbf{B}) = \mathbf{A} \times (\nabla \times \mathbf{B}) + \mathbf{B} \times (\nabla \times \mathbf{A}) + (\mathbf{A} \cdot \nabla) \mathbf{B} + (\mathbf{B} \cdot \nabla) \mathbf{A}.$$

for these vector fields.

5 Gradient and vector fields, 1

Consider the vector fields

$$\begin{aligned}\mathbf{A} &= x\hat{\mathbf{y}} \\ \mathbf{B} &= y\hat{\mathbf{y}}\end{aligned}$$

- a) Based on sketches of these vectors fields would you say that either is the gradient of a function? That is, is there some function f so that $\mathbf{A} = \nabla(f)$ and similarly for $\mathbf{B}$. Explain your answer.
- b) How could you check precisely if either vector could be the gradient of some function? Apply your check to $\mathbf{A}$ and $\mathbf{B}$.