Electromagnetic Theory I: Class Exam I

2 October 2025

Name: _____ **Total:** **/50**

Instructions

- There are 6 questions on 8 pages.
- Show your reasoning and calculations and always explain your answers.

Physical constants and useful formulae

Permittivity of free space	$\epsilon_0 = 8.85 \times 10^{-12}\,\mathrm{C}^2/\mathrm{Nm}^2$
Charge of an electron	$e = -1.60 \times 10^{-19}\,\mathrm{C}$

Integrals

$$\int \sin(ax)\sin(bx)\,\mathrm{d}x = \frac{\sin((a-b)x)}{2(a-b)} - \frac{\sin((a+b)x)}{2(a+b)} \quad \text{if } a \neq b$$

$$\int \cos(ax)\cos(bx)\,\mathrm{d}x = \frac{\sin((a-b)x)}{2(a-b)} + \frac{\sin((a+b)x)}{2(a+b)} \quad \text{if } a \neq b$$

$$\int \sin(ax)\cos(ax)\,\mathrm{d}x = \frac{1}{2a}\sin^2(ax)$$

$$\int \sin^2(ax)\,\mathrm{d}x = \frac{x}{2} - \frac{\sin(2ax)}{4a}$$

$$\int \cos^2(ax)\,\mathrm{d}x = \frac{x}{2} + \frac{\sin(2ax)}{4a}$$

$$\int x\sin^2(ax)\,\mathrm{d}x = \frac{x^2}{4} - \frac{x\sin(2ax)}{4a} - \frac{\cos(2ax)}{8a^2}$$

$$\int x^2\sin^2(ax)\,\mathrm{d}x = \frac{x^3}{6} - \frac{x^2}{4a}\sin(2ax) - \frac{x}{4a^2}\cos(2ax) + \frac{1}{8a^3}\sin(2ax)$$

Question 1

A solid sphere with radius R carries charge with density, in spherical coordinates,

$$\rho(\mathbf{r'}) = \frac{\beta}{r'^2}$$

where $\beta > 0$ is a constant with units of C/m.

a) Determine an expression for the total charge in the sphere, Q, in terms of β, R and constants.

b) Determine the electric field at all points inside and outside the sphere.

Question 1 continued ...

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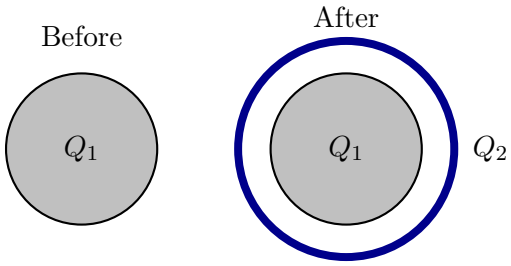
Question 2

A solid sphere is uniformly charged with a total charge of Q_1. This produces an electric field everywhere. Subsequently a hollow spherical shell with uniformly distributed charge Q_2 is situated beyond the solid sphere without changing the distribution within the solid sphere.



a) Does the additional charge Q_2 alter the field *inside the solid sphere?* Explain your answer.

b) Does the additional charge Q_2 alter the field *between the spheres?* Explain your answer.

c) Does the additional charge Q_2 alter the field *outside the hollow sphere?* Explain your answer.

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Question 3

An infinitely long solid cylinder with radius R carries charge whose density is given, in cylindrical coordinates, by

$$\rho(s', \phi', z') = \alpha s'$$

and α is a constant with units of $\mathrm{C/m^4}$. Determine the electric field at all points *inside and outside* the cylinder.

Question 3 continued ...

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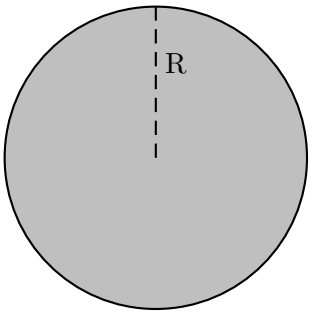
Question 4

A solid sphere with radius R contains charge Q; this is uniformly distributed. The field produced by the shell is (in spherical coordinates)



$$\mathbf{E} = \begin{cases} \dfrac{Q}{4\pi\epsilon_0}\,\dfrac{r}{R^3}\,\hat{\mathbf{r}} & \text{if } r < R \\ \dfrac{Q}{4\pi\epsilon_0}\,\dfrac{1}{r^2}\,\hat{\mathbf{r}} & \text{if } r > R \end{cases}$$

Determine the electrostatic potential at *all locations inside the sphere* assuming that $V = V_0$ at any point on the surface of the sphere.

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Question 5

Someone proposes to create a "quadratic electric" field using stationary charges so that

$$\mathbf{E} = \alpha \left(x^2 \hat{\mathbf{x}} + y^2 \hat{\mathbf{y}} \right)$$

where $\alpha > 0$ is a constant.

a) Is this a possible electrostatic field? Explain your answer.

b) What charge distribution might give rise to this electric field?

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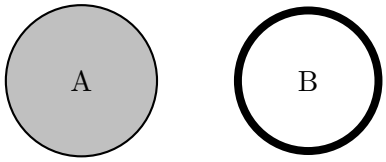
Question 6

Consider two spheres, each with the same radius. The same charge is placed on each sphere. However, in sphere A, it is uniformly distributed throughout the sphere, while in sphere B it is uniformly distributed on the surface of the sphere. Consider the work done to assemble each charge distribution. Is the work done to assemble A the same as, larger than or smaller than that for B? Explain your answer. *Note: the diagram illustrates these near to each other; they are supposed to be considered separately.*



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