Tues: Discussion / quiz

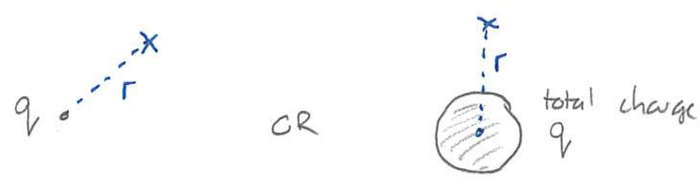
Ex 40, 41, 42ac, 44, 45, 47, 49

Weds: Usual class

Electric fields

One can calculate electric fields produced by certain continuous charge arrangements. Examples include

Point source or spherically symmetric distribution (outside of all charges)



magnitude

$$E = \frac{1}{4\pi\epsilon_0}\frac{|q|}{r^2}$$

Infinite uniformly charged sheet magnitude

$$E = \frac{|\eta|}{2\epsilon_0}$$

where η = surface charge density

Note that

The electric field depends strongly on the arrangement of source charges and can vary significantly from one distribution to another.

DEMO: Falstad
- quadrupole charge
- floating cylinder + charge.

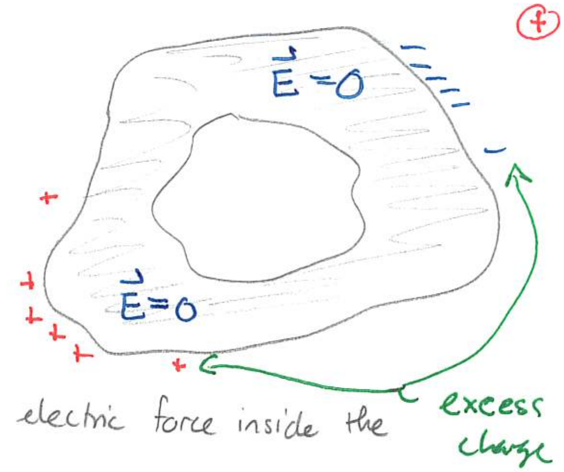
In all cases the force exerted by the electric field on a probe charge is

$$\vec{F} = q_{probe}\vec{E}$$

where q_{probe} = probe charge and $\vec{E}$ is the electric field at the probe location.

DEMO: PhET Electric Field Hockey

- show fields



Fields in conductors

In an ideal perfect conductor

* charge flows freely without resistance
* there is always enough charge to flow

In electrostatic equilibrium, there cannot be a net electric force inside the conductor (otherwise charges would accelerate). Thus

> In electrostatic equilibrium the electric field inside a conductor is zero, $\vec{E} = 0$.

Then Gauss' Law predicts

> The charge density everywhere inside a conductor is zero. Any excess charge must reside on the surface.

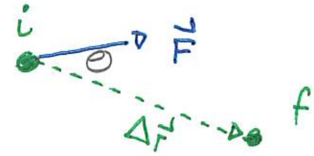
Then one can show that within a cavity inside the conductor there is no electric field (provided there is no charge within the cavity).

~~Warm Up!~~ Warm Up!

Work + energy overview

Classical physics allows one to assess the motion of an object via forces and acceleration. An alternative method uses work and energy. The scheme is

Basic definition of work for object moving in a straight line and constant force



Work done by force is

$$W = \vec{F}\cdot\Delta\vec{r} = F\Delta r\cos\theta$$

For general forces

$$W = \int \vec{F}\cdot d\vec{r}$$

along trajectory

Kinetic energy

$$K = \frac{1}{2}mv^2$$

Work - kinetic energy theorem

$$W_{net} = \Delta K = K_f - K_i$$

where W_{net} = work done by all forces

Quiz 1 80% - 100%

Quiz 2 30% - 40%

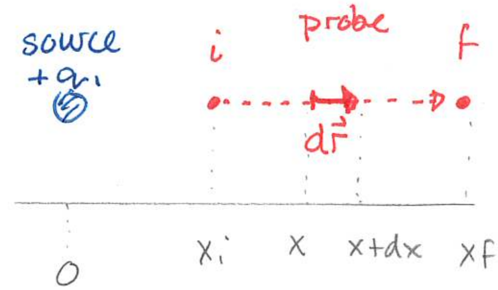
In some situations, the force is conservative and the work is independent of the path between initial and final points. Then there exists a potential energy U_{force} such that

$$W_{force} = -\Delta U_{force}$$

and this gives energy conservation

> If the non-conservative forces do zero work then
>
> $$\Delta E_{mech} = \Delta K + \underbrace{\Delta U_{force1} + \Delta U_{force2} + \cdots}_{\text{for conservative forces}} = 0$$
>
> and mechanical energy is constant

Electric potential energy



Is the electrostatic force conservative?

Consider a probe charge moving directly away from a point source. Then we break the trajectory into infinitesimal pieces and form

$$W = \int \vec{F} \cdot d\vec{r}$$

Here $d\vec{r} = dx\,\hat{\imath}$ and $\vec{F} = k\dfrac{q_1 q_2}{x^2}\hat{\imath}$. Thus

$$W = \int_{x_i}^{x_f} k\frac{q_1q_2}{x^2}\hat{\imath} \cdot dx\,\hat{\imath} = \int_{x_i}^{x_f} k\frac{q_1q_2}{x^2}\,dx$$

$$= -kq_1q_2\,\frac{1}{x}\Bigg|_{x_i}^{x_f}$$

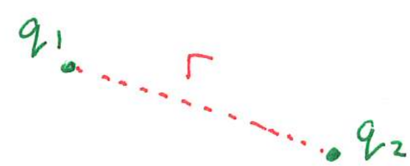
$$W = -\left[\frac{kq_1q_2}{x_f} - \frac{kq_1q_2}{x_i}\right]$$

So we define

The electric potential energy for two point charges is

$$U_{elec} = k\frac{q_1q_2}{r}$$

Units: J



where r is the distance between the charges

Then the derivative gives that the electrostatic force is conservative and

$$W_{elec} = -\Delta U_{elec}$$

If the only forces are electric then $\Delta K + \Delta U_{elec} = 0$

Warm up 2

53 Proton and a fixed point charge

A +40.0 nC particle is held fixed. A proton is released from rest 0.025 m from the fixed charge. The aim is to determine the speed of the proton when it is 0.050 m from the fixed charge. *(132 Fall 2026)*

a) Suppose that you tried to use Coulomb's law to determine the speed. What mathematical difficulties would arise?

b) Use energy conservation to determine the velocity of the proton when it is 0.050 m from the fixed charge.

c) Determine the maximum speed attained by the proton.

$r_i = 0.025\,\text{m}$ $\quad r_f = ?$

$v_i = 0\,\text{m/s}$ $\quad v_f = ?$

Answer: a) $F = k\dfrac{q_1 q_2}{r^2}$ would change as the particle moves $\Rightarrow$ acceleration not constant $\Rightarrow$ cannot use constant acceleration kinematics.

b) $\Delta E_{mech} = 0 \Rightarrow K_f + U_{elec\,f} = K_i + U_{elec\,i}$

$$\Rightarrow \frac{1}{2}mv_f^2 + k\frac{q_1 q_2}{r_f} = \cancel{\frac{1}{2}mv_i^2} + k\frac{q_1 q_2}{r_i}$$

$$\Rightarrow \frac{1}{2}mv_f^2 = k\frac{q_1 q_2}{r_i} - k\frac{q_1 q_2}{r_f} = kq_1q_2\left[\frac{1}{r_i} - \frac{1}{r_f}\right]$$

So
$$v_f^2 = \frac{2kq_1q_2}{m}\left[\frac{1}{r_i} - \frac{1}{r_f}\right]$$

$$= \frac{2 \times 8.99\times 10^{9}\,\text{N m}^2/\text{C}^2 \times 40\times 10^{-9}\,\text{C} \times 1.6\times 10^{-19}\,\text{C}}{1.67\times 10^{-27}\,\text{kg}}\left[\frac{1}{0.025\,\text{m}} - \frac{1}{0.050\,\text{m}}\right]$$

$$= 1.38\times 10^{12}\,\text{m}^2/\text{s}^2$$

$$\Rightarrow v_f = 1.2\times 10^{6}\,\text{m/s}$$

c) Same as b) except $r_f = \infty$

$$V_f^2 = \frac{2kq_1q_2}{m}\left[\frac{1}{r_i} - \frac{1}{\infty}\right]$$

$$= \frac{2kq_1q_2}{m}\frac{1}{r_i}$$

$$= \frac{2 \times 8.99\times10^{9}\,Nm^2/C^2 \times 40\times10^{-9}C \times 1.6\times10^{-19}C}{1.67\times10^{-27}\,kg}\ \frac{1}{0.025m}$$

$$= 2.76 \times 10^{12}\,m^2/s^2$$

$$V_f = 1.7 \times 10^{6}\,m/s$$