

Fri: HW 2 5pm

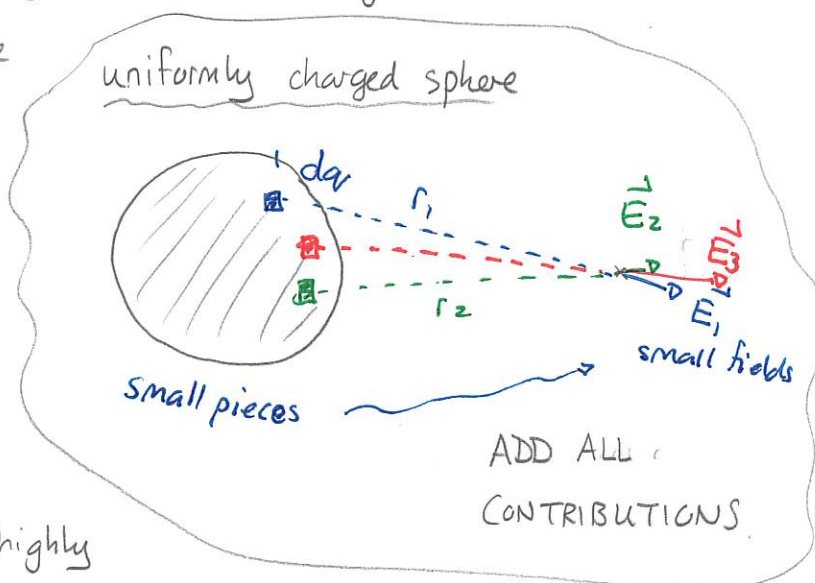
Mon: Warm Up 3 D2L 10am

Tues: Discussion/quiz EX 40, 41, 42ac, 44, 45, 47, 49

Electric field produced by a continuous charge distribution

It is often convenient to approximate a charge distribution as continuous even though it consists of many individual charges. We can determine the field by decomposing the distribution into many small pieces, using the point charge field for each piece and then adding the results. This usually produces a multivariable integral and sometimes this can be evaluated.

An alternative strategy uses a mathematical rule called Gauss' Law which involves vector calculus. A simple example applicable for a highly symmetrical situation is:



Consider charge that is uniformly distributed on a sphere of radius R



Volume charge density $\rho = \frac{Q}{V}$

Construct another sphere of radius r centered at the origin



Then on the sphere of radius r

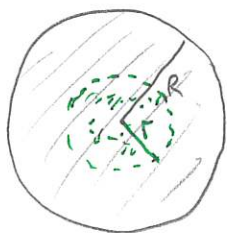
$$E \times \text{Area} = \frac{1}{\epsilon_0} q_{\text{enc}}$$

$$E 4\pi r^2 = \frac{1}{\epsilon_0} q_{\text{enc}}$$

where q_{enc} is the total charge enclosed in the constructed sphere

Quiz 1 30% - 70%

This applies whether the Gaussian sphere is inside or outside the physical sphere. For example, for a solid uniformly charged sphere



$$E 4\pi r^2 = \frac{1}{\epsilon_0} q_{enc} = \frac{1}{\epsilon_0} \rho \times \text{Volume Gaussian sphere}$$

$$= \frac{1}{\epsilon_0} \rho \frac{4}{3} \pi r^3$$

$$= \frac{1}{\epsilon_0} \frac{Q \cancel{\frac{3}{4\pi R^3}} \frac{4}{3} \pi r^3}{\cancel{\frac{3}{3}}} \Rightarrow \frac{1}{\epsilon_0} \frac{Q r^3}{R^3} = E 4\pi r^2$$

$\Rightarrow$ At radius r inside sphere

$$E = \frac{1}{4\pi\epsilon_0} \frac{Q r}{R^3}$$

This scheme applies to any spherically symmetric charge distribution.

Quiz 2 Only once.

Infinite uniformly charge flat sheet

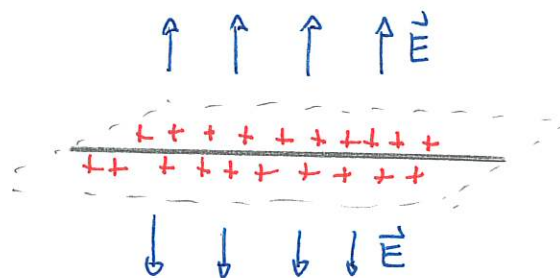
Consider a flat, infinite sheet. The surface charge density

η = charge per unit area

Symmetry dictates that the electric field

points perpendicular to the sheet - away if η is positive

- towards if η is negative



Gauss' Law eventually gives

For an infinite uniformly charged flat sheet the electric field is uniform on either side with

1) magnitude $E = \eta / 2\epsilon_0$

2) direction = $\begin{cases} \text{away from sheet if positively charged} \\ \text{toward sheet if negatively charged.} \end{cases}$

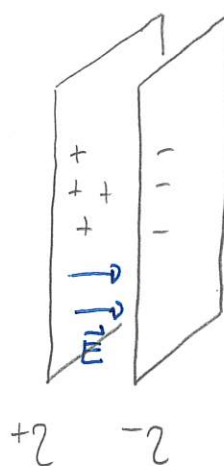
Quiz 3 20% - 60%

Parallel Plate Capacitor

A parallel plate capacitor consists of two sheets with equal area which are parallel. The sheets are configured to hold exactly opposite charges.

Then the electric field at any point is

$$\vec{E} = \vec{E}_{\text{from + plate}} + \vec{E}_{\text{from - plate}}$$



If the plates are infinite then

$$\vec{E} = \begin{cases} \sigma/\epsilon_0 & \text{inside from positive to negative} \\ 0 & \text{outside} \end{cases}$$

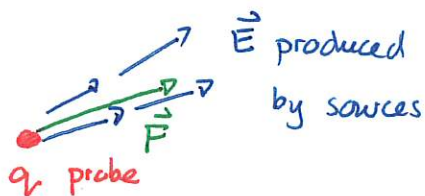
These plates can produce a uniform electric field.

Motion of charges in fields

Consider a probe charge placed in an electric field produced by (other) sources. How will the probe move?

DEMO: PHET Field Hockey

Newton's mechanics gives



Determine electric field produced by sources

Force exerted by field (i.e. sources) on probe

$$\vec{F}_{\text{elec}} = q_{\text{probe}} \vec{E}$$

where $\vec{E}$ is field vector at probe location

↳ Newton's 2nd Law

$$\vec{F}_{\text{net}} = m\vec{a}$$

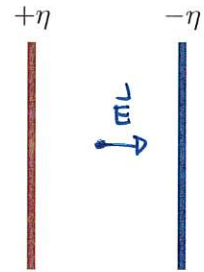
and if no other forces $\vec{F}_{\text{elec}} = m\vec{a}$

↳ Get acceleration

→ Kinematics describes motion

46 Proton between parallel capacitor plates

A parallel plate capacitor has plates separated by 0.0020 m. The plates are uniformly charged as illustrated with magnitude $\eta = 6.0 \times 10^{-12} \text{ C/m}^2$. They are large enough that the field between the plates can be regarded as being produced by infinitely large plates. A proton is released from rest midway between the plates. Ignore gravity in this problem. (132S22 Class)



- Describe the trajectory of the proton.
- Determine how long it takes for the proton to hit a plate after release.

a) $\vec{E}$ is $\rightarrow$ and uniform $\vec{F} = q \vec{E} \Rightarrow \vec{F}$ is right
 $q_{\text{proton}} > 0 \Rightarrow$ proton accelerates to right

The proton moves with constantly increasing speed right

b) Newton's 2nd Law $\rightarrow$ acceleration $\rightarrow$ kinematic equations

Newton's 2nd Law



$$\vec{F}_{\text{net}} = m_{\text{proton}} \vec{a} \Rightarrow \vec{F}_{\text{elec}} = m_{\text{proton}} \vec{a}$$

Then $\vec{F}_{\text{elec}} = q_{\text{proton}} \vec{E}_{\text{plates}} \Rightarrow q_{\text{proton}} \vec{E}_{\text{plates}} = m_{\text{proton}} \vec{a}$

The electric field is $\vec{E} = \frac{\eta}{\epsilon_0} \hat{i}$

Thus $q_{\text{proton}} \frac{\eta}{\epsilon_0} \hat{i} = m_{\text{proton}} \vec{a}$

$$\Rightarrow \vec{a} = \frac{q_{\text{proton}} \eta}{m_{\text{proton}} \epsilon_0} \hat{i}$$

So $a_x = \frac{q_{\text{proton}} \eta}{m_{\text{proton}} \epsilon_0} = \frac{1.60 \times 10^{-19} \text{ C} \times 6.0 \times 10^{-12} \text{ C/m}^2}{1.67 \times 10^{-27} \text{ kg} \times 8.85 \times 10^{-12} \text{ C}^2/\text{Nm}^2} = 6.5 \times 10^7 \text{ m/s}^2$

Kinematics

initial	final
0	0
$x_i = 0.0\text{m}$	$x_f = 0.0010\text{m}$
$v_{ix} = 0\text{m/s}$	$v_{fx} = ?$
$a_x = 6.5 \times 10^7 \text{m/s}^2$	

Then

$$x_f = \cancel{x_i} + \cancel{v_{ix}} \Delta t + \frac{1}{2} a_x \Delta t^2$$

$$x_f = \frac{1}{2} a_x \Delta t^2$$

$$\Delta t = \sqrt{\frac{2x_f}{a_x}} = \sqrt{\frac{2 \times 0.0010\text{m}}{6.5 \times 10^7 \text{m/s}^2}} =$$

$$\Delta t = 5.5 \times 10^{-6} \text{s} \quad \square$$

Quiz 4