Fri: HW by 5pm

Ex 19, 22, 27, 31, 32, 34, 38, 39
Yes!

Mon: ~~HW~~ Warm up 3 D2L

Thurs: Seminar Wubben 264 12:30 - 1:30

Electric field produced by a continuous charge distribution

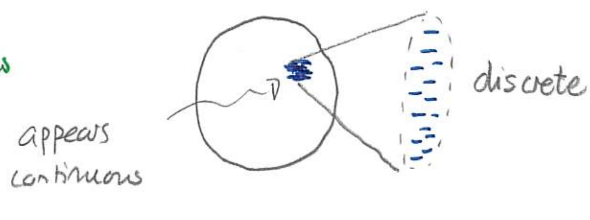
We have a basic rule to calculate the electric field produced by a single point charge and this can be used to calculate the electric field produced by multiple point charges, provided they are stationary. What if there are very many source point charges?

DEMO: Large capacitor
- charge one plate

Quiz!

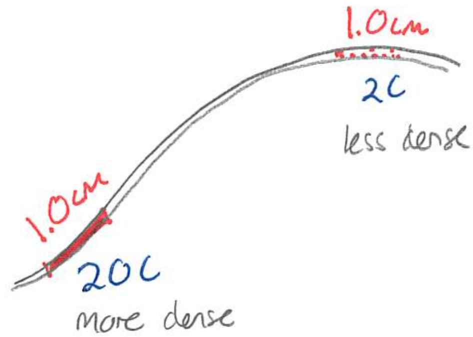
There are too many individual point charges to use the basic electric field rule and to add the results effectively. We can simplify by assuming that the charge is continuously distributed.

DEMO: Falstad - Two charge planes



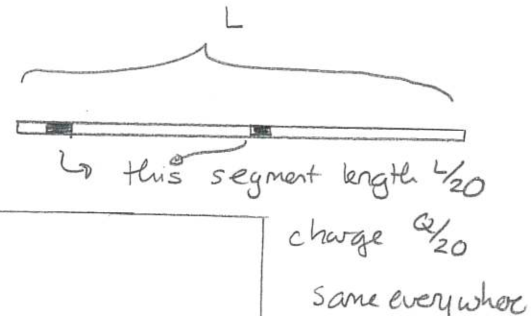
We need methods for describing:
* how the charge is distributed
* how to calculate the field from the charge distribution

One dimensional charge distribution



The simplest situation is a one dimensional wire with negligible cross-section. We can describe how much charge there is per unit length and this could vary.

The simplest version of this is a straight wire with a uniform charge distribution. Let L be the total length of the wire and Q the total charge



We define

The linear charge density

$$\lambda = Q/L$$

where Q = total charge

L = total length

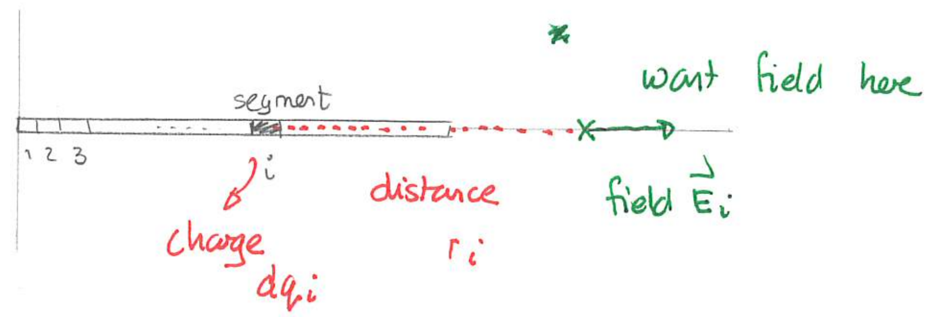
only for uniformly distributed charge

units: C/m

Quiz 2

Quiz 3

The strategy for determining the electric field is:



Break the wire into many small segments

↳ Treat each segment as a point charge

* use Coulomb's law to determine the contribution from the field at segment i ⇝ call this $\vec{E}_i$

↳ Add contributions from all segments

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \dots$$

$$= \sum_{\text{all segments}} \vec{E}_i$$

Approximation

↳ Improve approximation by

* increasing the number of segments
* decreasing the segment size.

37 Field produced by a uniformly charged rod

A uniformly charged rod is oriented as illustrated. The length of the rod is L and the total charge (positive) is Q. The aim of this is to determine the electric field at P. *(132S22 Class)*

a) As an example, suppose that the length of the rod is 2.0 m and the total charge is 60 C. Determine the charge density and use it to determine the charge in any segment of the rod with length 0.010 m. Repeat this for a segment with length 0.0050 m.

b) Consider a segment from $x \to x + \mathrm{d}x$ where x is some point in the rod. Write expressions for the length of the segment and the charge it contains (in terms of $\mathrm{d}x$ and λ).

c) Indicate the field produced by this segment and determine an expression for its components (in terms of $x_p, x, \mathrm{d}x$ and λ).

d) Write an expression (eventually in terms of calculus) for the components of the net electric field produced by all segments.

Answer: a) $\lambda = \dfrac{Q}{L} = \dfrac{60C}{2.0m} = 30C/m$

In segment length Δx, charge is Δq

$$\frac{\Delta q}{\Delta x} = \lambda \Rightarrow \Delta q = \lambda \Delta x$$

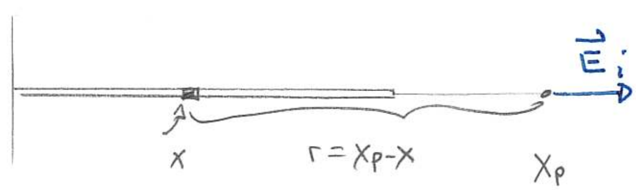
For $0.010m$ $\quad \Delta q = 30C/m \times 0.010m = 0.30C$

For $0.0050m$ $\quad \Delta q = 30C/m \times 0.0050m = 0.15C$

b) length is $\quad x + dx - x = dx$

charge is $\quad dq = \lambda dx$.

c) $\vec{E}_i = \dfrac{1}{4\pi\epsilon_0} \dfrac{dq}{r^2} \hat{i}$



$$\vec{E}_i = \frac{1}{4\pi\epsilon_0} \frac{\lambda dx}{(x_p - x)^2} \hat{i} \Rightarrow E_{ix} = \frac{1}{4\pi\epsilon_0} \frac{\lambda}{(x_p - x)^2} dx$$

$$E_{iy} = 0$$

d) $E_x = \sum_{\text{all segments}} E_{ix} \rightsquigarrow E_x = \int_{x=0}^{L} \frac{1}{4\pi\epsilon_0} \frac{\lambda}{(x_p - x)^2} dx$

$$\boxed{E_x = \frac{\lambda}{4\pi\epsilon_0} \int_0^L \frac{dx}{(x_p - x)^2}}$$

We do the integral with a "u-substitution."

$u = x_p - x$

$du = -dx$

$x = 0 \Rightarrow u = x_p$

$x = L \Rightarrow u = x_p - L$

$$E_x = \frac{\lambda}{4\pi\epsilon_0} \int_{x_p}^{x_p - L} \frac{-du}{u^2} = \frac{\lambda}{4\pi\epsilon_0} \frac{1}{u} \Big|_{x_p}^{x_p - L}$$

$$\Rightarrow E_x = \frac{\lambda}{4\pi\epsilon_0} \left[\frac{1}{x_p - L} - \frac{1}{x_p} \right] = \frac{\lambda}{4\pi\epsilon_0} \frac{x_p - (x_p - L)}{(x_p - L) x_p}$$

$$\Rightarrow E_x = \frac{\lambda}{4\pi\epsilon_0} \frac{L}{x_p(x_p - L)}$$

Then $\lambda = Q/L$ gives $Q = \lambda L$

$$E_x = \frac{1}{4\pi\epsilon_0} \frac{Q}{x_p(x_p - L)} \Rightarrow$$

on the axis of the rod, right of end $\quad \vec{E} = \frac{1}{4\pi\epsilon_0} \frac{Q}{x_p(x_p - L)} \hat{i}$

How can we check this?

Check: Field point very far from end of rod $\Rightarrow$ the rod will appear as a point charge at distance $\approx x_p$

$\hookrightarrow$ charge Q

Then $x_p \gg L \Rightarrow x_p - L \approx x_p$

$$\Rightarrow \vec{E} \approx \frac{1}{4\pi\epsilon_0} \frac{Q}{x_p^2} \hat{i}$$

This is exactly the field produced by a point source charge Q at distance x_p from the charge.

We can reexpress the result using distance d from the end of the rod. Then $x_p = L + d$ gives

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{Q}{(L+d)\, d} \hat{i}$$

↳ only for <u>uniformly</u> charged rod

only for point along axis of rod.