

Electromagnetism and Optics: Final Exam

18 May 2022

Name: Solution

Total: /150

Instructions

- There are 18 questions on 12 pages.
- Show your reasoning and calculations and always explain your answers.

Physical constants and useful formulae

$$\begin{aligned}
 e &= 1.61 \times 10^{-19} \text{ C} & q_{\text{electron}} &= -e & q_{\text{proton}} &= +e & \text{area sphere} &= 4\pi r^2 \\
 m_{\text{electron}} &= 9.11 \times 10^{-31} \text{ kg} & m_{\text{proton}} &= 1.67 \times 10^{-27} \text{ kg} & c &= 3.0 \times 10^8 \text{ m/s} \\
 k &= 9.0 \times 10^9 \text{ Nm}^2/\text{C}^2 & \epsilon_0 &= 8.85 \times 10^{-12} \text{ C}^2/\text{Nm}^2 & \mu_0 &= 4\pi \times 10^{-7} \text{ Tm/A}
 \end{aligned}$$

✓ Question 1

In several situations charged particles (labeled A, B, ...) are placed at the same location near to a collection of fixed source charges whose arrangement is always the same. Suppose that in the first situation particle A has charge 3.0 nC and the net force on it has magnitude 0.0050 N. In the second situation A is replaced at the same location with B, whose charge is 12.0 nC. Is it possible to determine the magnitude of the force on charge B or not? If so, explain how. If not, explain why not.

YES. In general $\vec{F} = q\vec{E}$ and the electric field will be the same for all charges. So

Cannot assume
source is point
charge
(-2)

$$\begin{aligned}
 \vec{F}_B &= q_B \vec{E} \\
 \text{and } \vec{F}_A &= q_A \vec{E} \Rightarrow \vec{E} = \vec{F}_A / q_A
 \end{aligned}$$

Use $\vec{E}$ field
essential. No use
of flux... (-2)

$$\begin{aligned}
 \Rightarrow \vec{F}_B &= \frac{q_B}{q_A} \vec{F}_A \\
 &= \frac{12 \text{ nC}}{3 \text{ nC}} \vec{F}_A \Rightarrow \vec{F}_B = 4 \vec{F}_A
 \end{aligned}$$

/8

Question 2

Two charged particles are held fixed as illustrated. The grid units are each 1.00 cm.

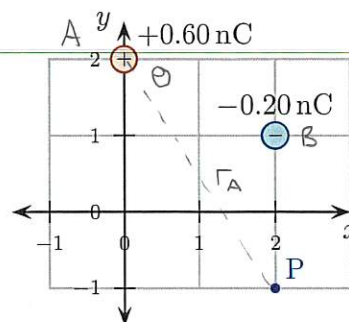
- a) Determine the net electric field produced by the charge arrangement at the point P.

$$\vec{E} = \vec{E}_A + \vec{E}_B \quad \text{Add vectors}$$

Magnitudes:

$$E_A = k \frac{|q_A|}{r_A^2} = 9.0 \times 10^9 \frac{\text{Nm}^2}{\text{C}^2} \frac{0.60 \times 10^{-9} \text{C}}{(0.036 \text{m})^2} = 4200 \text{N/C}$$

$$E_B = k \frac{|q_B|}{r_B^2} = 9.0 \times 10^9 \frac{\text{Nm}^2}{\text{C}^2} \frac{0.20 \times 10^{-9} \text{C}}{(0.02 \text{m})^2} = 4500 \text{N/C}$$



At P

$$\tan \theta = \frac{3}{2}$$

$$\Rightarrow \theta = 56.3^\circ$$

$$r_A = \sqrt{(0.02 \text{m})^2 + (0.03 \text{m})^2}$$

$$= 0.036 \text{m}$$

Components: $E_{Bx} = 0$ $E_{By} = 4500 \text{N/C}$

$$E_{Ax} = E_A \cos \theta = 4200 \text{N/C} \cos 56.3^\circ = 2300 \text{N/C}$$

$$E_{Ay} = -E_A \sin \theta = 4200 \text{N/C} \sin 56.3^\circ = -3500 \text{N/C}$$

Net:

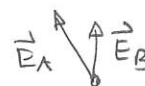
$$\vec{E} = 2300 \text{N/C} \hat{i} - 3500 \text{N/C} \hat{j} + 4500 \text{N/C} \hat{j}$$

$$= 2300 \text{N/C} \hat{i} + 1000 \text{N/C} \hat{j}$$

- b) Suppose that the $+0.60 \text{nC}$ charge were replaced by a -0.60nC charge at the same location. Which of the following (choose one) best describes the effect of this change on the magnitude of the net electric field at P?

- i) The magnitude does not change.
 ii) The magnitude increases.
 iii) The magnitude decreases.

The vectors will be

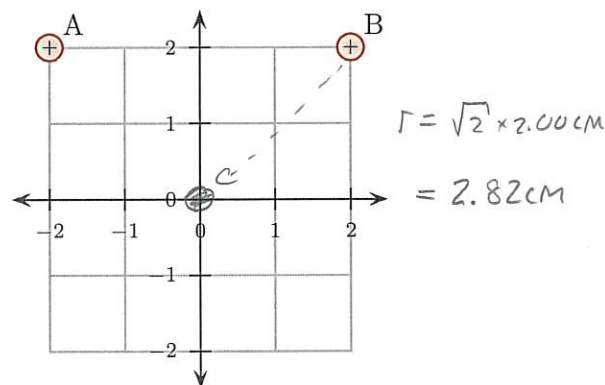


less cancellation...

/12

Question 3

Two charged particles, labeled A and B and each with charge $+5.00 \text{ nC}$ are held fixed at the illustrated locations. The grid units are each 1.00 cm . A third particle, C, whose charge is $+3.00 \text{ mC}$ and whose mass is 10.0 g is released from rest at the origin.



- a) Explain why it would be extremely difficult or even impossible in practice to use Newton's second law to determine the velocity of particle C at any time after it has been released.

$$\vec{F} = m\vec{a} \Rightarrow \vec{a} = \vec{F}/m.$$

We can determine $\vec{F}$ but this will vary with time and result in a time varying acceleration. We could not use a simple strategy like constant acceleration kinematics.

- b) Determine the velocity of this particle when it is infinitely far from both A and B.

$$\Delta K + \Delta U_{\text{elec}} = 0 \Rightarrow \Delta K + q_c \Delta V = 0$$

$$\Rightarrow K_f - K_i + q_c [V_f - V_i] = 0 \quad \begin{matrix} \text{Potential} \\ \text{produced by} \\ A, B. \end{matrix}$$

$$V = k \frac{q_A}{r_A} + k \frac{q_B}{r_B} = 2k \frac{q_A}{r_A} \quad \text{since } r_A = r_B \text{ and } q_A = q_B.$$

$$\text{Initially } v_i = 0 \text{ m/s} \quad \Rightarrow \quad \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2 + q_c \left[2k \frac{q_A}{r_{Af}} - 2k \frac{q_A}{r_{Ai}} \right] = 0$$

$$r_{Ai} = 0.028 \text{ m}$$

Finally

$$r_{Af} = \infty$$

$$\Rightarrow v_f^2 = \frac{4kq_A q_c}{r_{Ai} m}$$

$$\Rightarrow v_f = \sqrt{\frac{4 \times 9 \times 10^9 \text{ Nm}^2/\text{C}^2 \times 5 \times 10^{-9} \text{ C} \times 3 \times 10^{-3} \text{ C}}{0.0282 \times 0.010 \text{ kg}}}$$

$$\Rightarrow v_f = 44 \text{ m/s}$$

Question 4

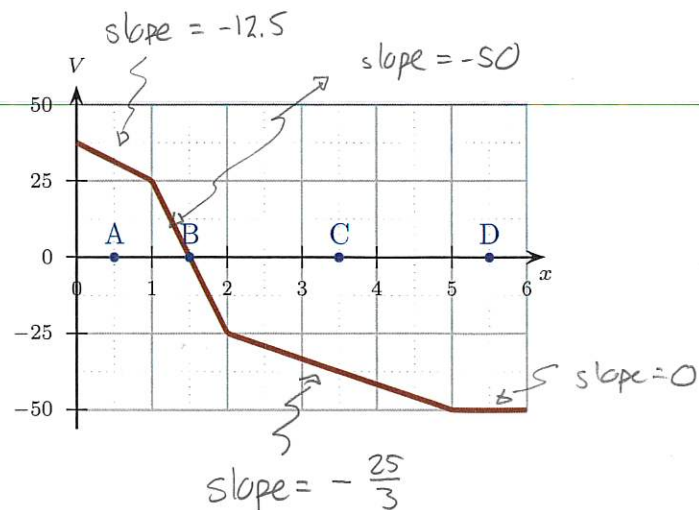
Various hidden sources produce an electric potential, V , whose plot versus horizontal location is illustrated. Consider the electric field at each of the indicated locations: A, B, C, and D. Rank the electric fields in order of increasing magnitude (indicate which are largest and smallest).

$$E = -\frac{dV}{dx} = -\text{slope}$$

Using slopes:

smallest largest

$$E_D < E_C < E_A < E_B$$



/5

Question 5

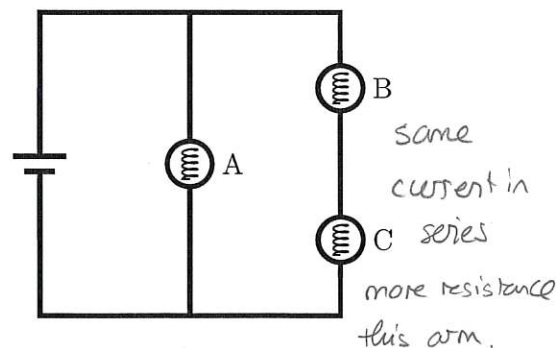
Three identical bulbs are connected in the illustrated circuit. Which (choose one) of the following best represents the rank of the currents through the bulbs?

i) $I_A > I_B > I_C$

ii) $I_A = I_B > I_C$

iii) $I_B = I_C > I_A$

iv) $I_A > I_B = I_C$



Assuming that the bulbs are each resistors and obey Ohm's law, which of the following (choose one) best represents the relationship between the power produced by the bulbs?

i) Power produced by A is same as power produced by each of B and C.

ii) Power produced by A is same as total power produced by B and C together.

iii) Power produced by A is half of the total power produced by B and C together.

iv) Power produced by A is twice the total power produced by B and C together.

$$P = I^2 R$$

$$I_A = 2 I_B = 2 I_C$$

/8

$$\Rightarrow P_A = 4 P_B = 2 (P_B + P_C)$$

$$P_B = P_C$$

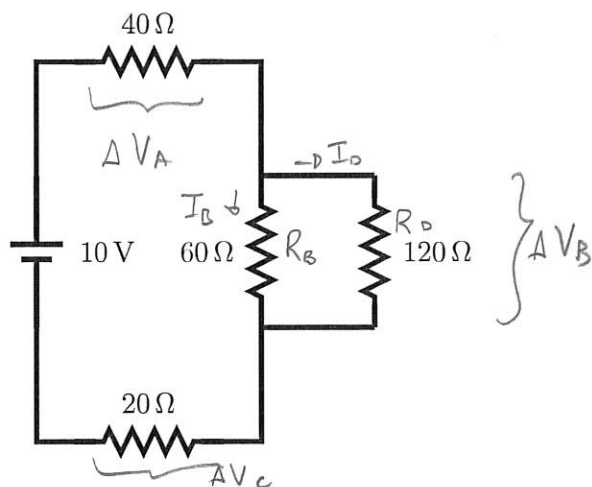
✓ Question 6

Several resistors are connected in the illustrated circuit.

- a) Determine the current through the 20Ω resistor.

$$\Delta V_{\text{battery}} = I R_{\text{eq}} \quad (+1)$$

gives current through battery
= current through 20Ω .



For R_{eq} the $60/120$ parallel combination has equivalent resistance

$$\frac{1}{R_{\text{eq1}}} = \frac{1}{60} + \frac{1}{120} = \frac{3}{120} \Rightarrow R_{\text{eq1}} = 40\Omega \quad (+2)$$

Then the entire circuit has equivalent resistance $40\Omega + 40\Omega + 20\Omega = 100\Omega$

$$\Rightarrow 10V = I 100\Omega \Rightarrow I = 0.10A \quad (+2)$$

- b) Determine the current through the 60Ω resistor.

We need voltage across the $60/120$ combination. Using the symbols in the diagram

$$\Delta V_{\text{battery}} = \Delta V_A + \Delta V_B + \Delta V_C \Rightarrow 10V = \Delta V_A + \Delta V_B + \Delta V_C$$

$$\Delta V_A = 0.10A \times 40\Omega = 4.0V$$

$$\Delta V_C = 0.10A \times 20\Omega = 2.0V$$

$$\Rightarrow \Delta V_B = 10V - 4.0V - 2.0V = 4.0V \quad (+3)$$

$$\text{Then } \Delta V_B = I_B R_B$$

$$4.0V = I_B 60\Omega \Rightarrow I_B = 0.067A$$

✓ Question 7

Two capacitors, one with capacitance $2C$ and the other $6C$ (here C is a constant with units of capacitance) are connected in series to a battery that provides potential difference ΔV . Determine an expression for the magnitude of the charge supplied by the battery in terms of ΔV and C .

$$Q = C_{eq} \Delta V$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{2C} + \frac{1}{6C} = \frac{3+1}{6C} = \frac{4}{6C}$$

$$\Rightarrow C_{eq} = \frac{3}{2} C$$

$$\Rightarrow Q = \frac{3}{2} C \Delta V$$

/6

✓ Question 8

An infinitely long wire is placed in the illustrated uniform magnetic field, $\vec{B}_{\text{other}}$ (created by something else). The wire carries a current into the page. Consider the net magnetic field produced at each illustrated point. Which of the following (choose one) ranks the magnitudes of the magnetic fields at each point?

- i) $B_A = B_B = B_C$
- ii) $B_A = B_C > B_B$
- iii) $B_A = B_C < B_B$
- iv) $B_A > B_B > B_C$
- v) $B_A < B_B < B_C$

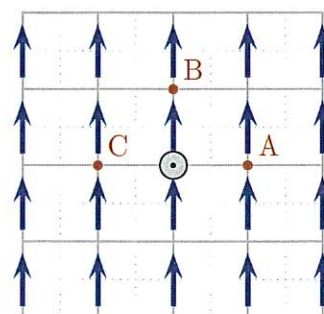
For the wire
← B_{wire}

↓ B_{wire} ⊙ ↑ B_{wire}

At A: ↑ B_{other} ↑ B_{wire}

B: ← B_{wire} ↑ B_{other}

6



$\vec{B}_{\text{other}}$

C: ↓ B_{other} ↓ B_{wire}

/5

✓ Question 9

Two infinitely long wires point perpendicular to the page.
Determine the net magnetic field at point A.

At A

$$\vec{B} = \vec{B}_\alpha + \vec{B}_\beta \quad (\text{Add vectors})$$

For a wire the magnitude is

$$B = \frac{\mu_0 I}{2\pi r} \quad (+1)$$

and direction is given by r.h. rule

Magnitudes:

$$B_\alpha = \frac{4\pi \times 10^{-7} \text{ Tm/A} \times 12.0 \text{ A}}{2\pi \times 0.082 \text{ m}}$$

$$= 2.9 \times 10^{-5} \text{ T} \quad (+1)$$

$$B_\beta = \frac{4\pi \times 10^{-7} \text{ Tm/A} \times 3.0 \text{ A}}{2\pi \times 0.02 \text{ m}} = 3.0 \times 10^{-5} \text{ T} \quad (+1)$$

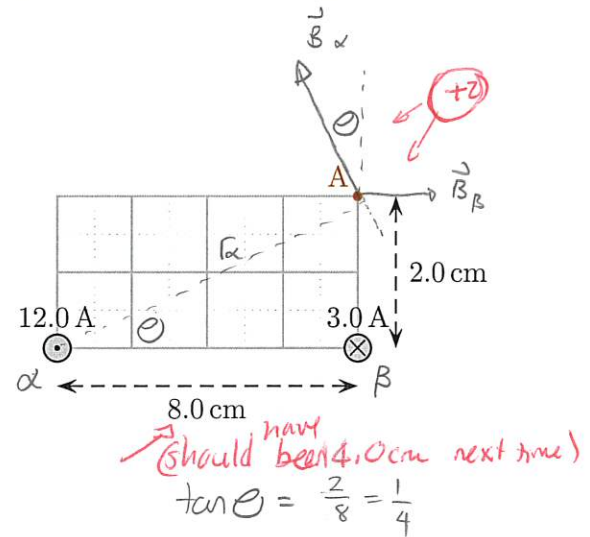
Components:

$$\left. \begin{aligned} B_{\alpha x} &= -B_\alpha \sin \theta = -2.9 \times 10^{-5} \text{ T} \sin 14.0^\circ = -0.70 \times 10^{-5} \text{ T} \\ B_{\alpha y} &= B_\alpha \cos \theta = 2.9 \times 10^{-5} \text{ T} \cos 14.0^\circ = 2.8 \times 10^{-5} \text{ T} \end{aligned} \right\} (+3)$$

$$B_{\beta x} = 3.0 \times 10^{-5} \text{ T}$$

$$B_{\beta y} = 0$$

$$\begin{aligned} \vec{B} &= (-0.70 + 3.0) \times 10^{-5} \text{ T } \hat{i} + 2.8 \times 10^{-5} \text{ T } \hat{j} \\ &= 2.3 \times 10^{-5} \text{ T } \hat{i} + 2.8 \times 10^{-5} \text{ T } \hat{j} \end{aligned} \quad (+2)$$



$$\Rightarrow \theta = 14.0^\circ \quad (+1)$$

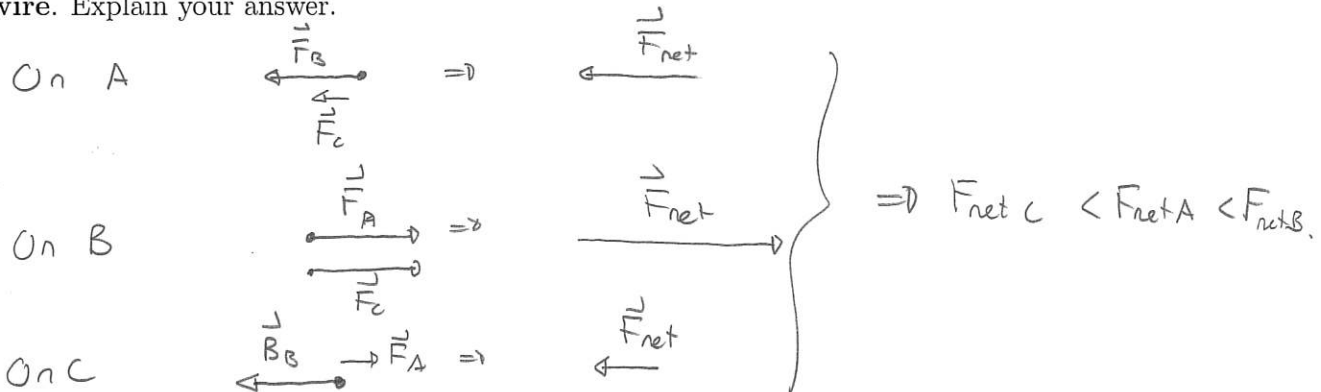
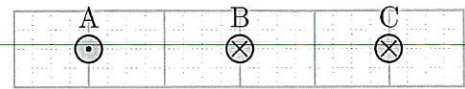
$$r_\alpha = \sqrt{(0.02 \text{ m})^2 + (0.08 \text{ m})^2}$$

$$= 0.082 \text{ m} \quad (+1)$$

/12

Question 10

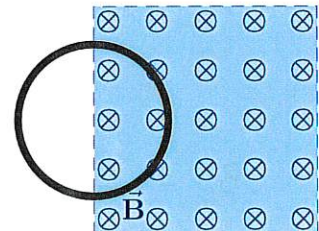
Three very long wires, each perpendicular to the page, carry currents of identical magnitudes. The distances between adjacent wires are equal. The directions of the currents are as indicated. Rank the wires in order of increasing **magnitudes of the net force exerted on the wire**. Explain your answer.



/8

Question 11

A loop with radius 0.2m and resistance 5Ω is placed so that half of it overlaps a region in which there is a uniform magnetic field pointing ~~out~~ ^{into} of the page. The magnetic field decreases from 0.4T to 0T at a steady rate over 4s . Determine the magnitude and direction of the current in the loop.



$$\mathcal{E} = IR \Rightarrow I = \mathcal{E}/R$$

$$\mathcal{E} = \left| -\frac{\Delta\Phi}{\Delta t} \right| \quad \text{and} \quad \Phi = \overset{\text{shaded half}}{AB} = \frac{1}{2} \pi r^2 B$$

$$\Rightarrow \mathcal{E} = \frac{1}{2} \pi r^2 \left| \frac{\Delta B}{\Delta t} \right| = \frac{\pi \times (0.20\text{m})^2}{2} \frac{0.4\text{T}}{4\text{s}} = 0.0063\text{V}$$

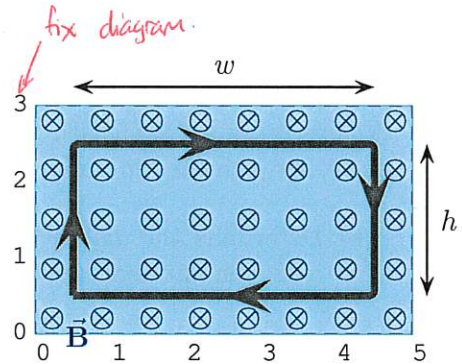
$$I = \frac{0.0063\text{V}}{5.0\Omega} = 0.0013\text{A}$$

/8

2 [Direction: $\rightarrow$ induced field opposes change $\Rightarrow$ into page.
 $\Rightarrow$ current is clockwise.

Question 12

A rectangular loop is placed in a magnetic field as illustrated. The loop has height h , length w and carries current I . The magnetic field is uniform, points into the page and has magnitude B .



- a) Determine expressions for the forces on **each section** of the loop.

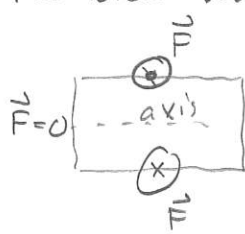
$$\vec{F} = I \vec{l} \times \vec{B}$$

directions	2	Top: $\vec{B} \otimes \rightarrow \vec{l}$	$\uparrow \vec{F}$	$F = IwB \Rightarrow \vec{F} = IwB \hat{j}$
magnitudes	3	Right: $\vec{B} \otimes \downarrow \vec{l}$	$\rightarrow \vec{F}$	$F = IhB \Rightarrow \vec{F} = IhB \hat{i}$
		Bottom: $\vec{l} \otimes \leftarrow \vec{B}$	$\downarrow \vec{F}$	$F = IwB \Rightarrow \vec{F} = -IwB \hat{j}$
		Left: $\uparrow \vec{l} \otimes \vec{B}$	$\leftarrow \vec{F}$	$F = IhB \Rightarrow \vec{F} = -IhB \hat{i}$

- b) Suppose that the loop was held at rest in this position. Explain how it would begin to move just after it is released.

3 [The net force is zero. $\Rightarrow$ no translation motion
 Each force produces zero torque $\Rightarrow$ no rotational "
 $\Rightarrow$ stays at rest.

- c) Suppose the following single change was made to the situation: the magnetic field direction is up the page ($\uparrow$). The loop is again released from rest. Explain how it would begin to move just after it is released.

3 [The forces would be $\sim$ no net force $\Rightarrow$ no translational motion
 $\vec{F}=0$  $\vec{F}=0 \sim$ net torque $\neq 0 \Rightarrow$ rotates about axis
 top out of page...
 bottom into page.

/12

Question 13

A laser produces light with wavelength 532 nm. Suppose that this was the result of a standing wave between two mirrors.

- a) Determine the distance between the mirrors such that this produces a standing wave with one antinode ($n = 1$).

$$\lambda_n = \frac{2L}{n}$$

$$n=1 \quad \lambda_n = 2L \quad \Rightarrow L = \frac{\lambda}{2} = \frac{532 \text{ nm}}{2} = 266 \text{ nm}$$

- b) Suppose that the distance between the mirrors is actually 8.0 cm. Determine the approximate number of antinodes in the standing wave between the mirrors.

$$n = \frac{2L}{\lambda_n} = \frac{2 \times 0.080 \text{ m}}{532 \times 10^{-9} \text{ m}} = 300752 \text{ nodes}$$

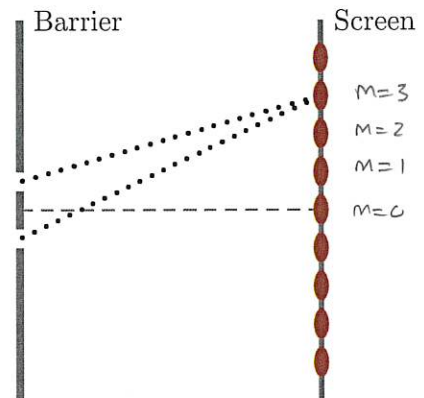
/6

Question 14

Light with wavelength $590 \times 10^{-9} \text{ m}$ is incident upon a double slit and produces bright fringes on a screen as indicated.

- a) Which of the following (choose one) would occur if the separation between the slits were increased to double of what it originally was?

- i) Nothing would change.
- ii) The bright fringes would be in the same places but would become dimmer.
- iii) The bright fringes would be further apart.
- iv) The bright fringes would be closer together.



- b) The dotted lines indicate the paths taken by light to travel from each of the slits to a particular bright fringe. The light from the lower slit has traveled a further distance than that from the upper slit. By how much further has it traveled?

$$d \sin \theta_m = m\lambda$$

$$\sin \theta_m = \frac{m\lambda}{d}$$

$$d \uparrow \Rightarrow$$

$$\sin \theta_m \downarrow$$

$$\Rightarrow \theta_m \downarrow$$

$$\Delta r = m\lambda$$

$$= 3\lambda = 3 \times 590 \times 10^{-9} \text{ m}$$

$$= 1.77 \times 10^{-6} \text{ m}$$

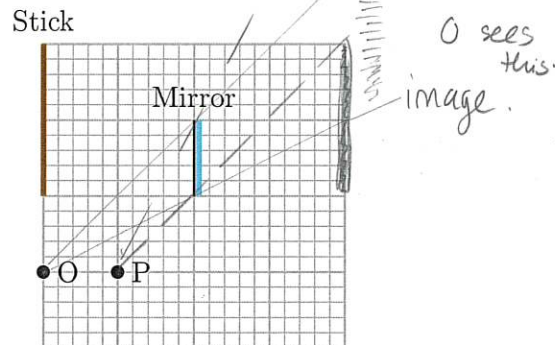
/8

Question 15

A stick with length 1.0m is held parallel to a flat mirror as illustrated. A small observer is initially located at the point O.

- a) What length or fraction of the image will the observer at O be able to see? Explain your answer.

Diagram shows only top half



- b) The observer moves from point O to point P. As the observer moves does the fraction of the image visible change or not? Explain your answer.

It becomes smaller. At P none of the image can be moved.

/8

Question 16

Light, initially in air (index of refraction 1.00), enters a rectangular block of glass (index of refraction 1.50) from the left as illustrated. The glass block is surrounded by air.

- a) Determine θ_2 and θ_3 .

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

$$1.00 \sin 45^\circ = 1.50 \sin \theta_2$$

$$\Rightarrow \sin \theta_2 = \frac{\sin 45^\circ}{1.5} = 0.47 \Rightarrow \theta_2 = 28.1^\circ$$

By geometry
 $\theta_2 + \theta_3 = 90^\circ$
 $\Rightarrow \theta_3 = 61.9^\circ$

- b) Will any light pass through the surface of the glass at the bottom of the diagram? Explain your answer.

θ_3 must be less than the critical angle.

$$\theta_c = \sin^{-1}\left(\frac{1}{1.5}\right) = 41.8^\circ$$

It is larger $\Rightarrow$ total internal reflection

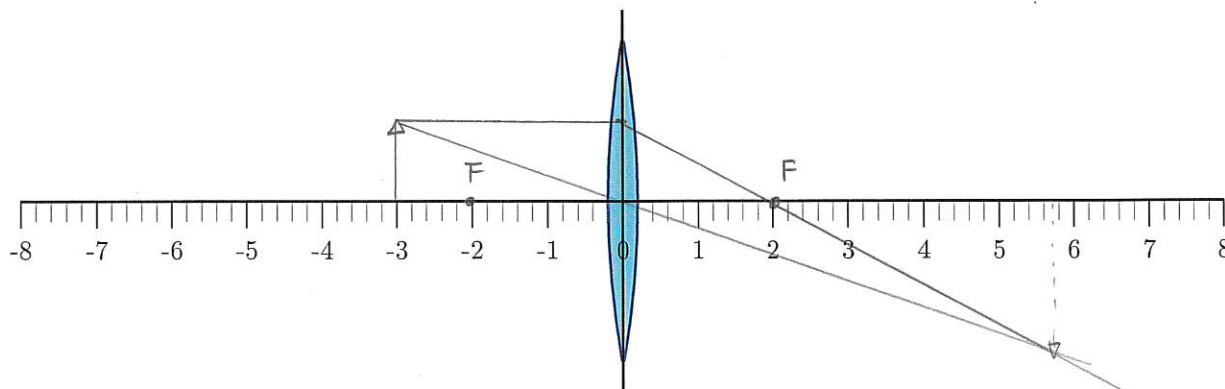
11

$\Rightarrow$ none emerges.

/6

Question 17

A convex lens has focal length f . An arrow is placed a distance $\frac{3}{2}f$ left of the lens.



a) Determine an expression for the location of the image in terms of f .

6

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \Rightarrow \frac{1}{3f/2} + \frac{1}{s'} = \frac{1}{f} \Rightarrow \frac{1}{s'} = \frac{1}{f} - \frac{2}{3f} \Rightarrow \frac{3-2}{3f} = \frac{1}{3f}$$

$$\Rightarrow s' = 3f$$

5 5 b) Determine the (approximate) location and height of the image produced by the lens using accurate ray tracing for the case where the focal length of the lens is $+2\text{ cm}$.

4 4

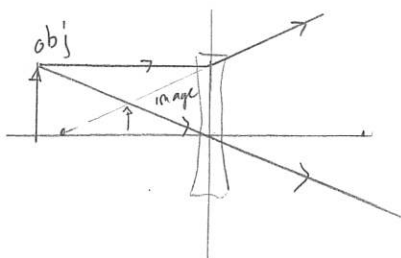
$$\Rightarrow s = 3\text{ cm}$$

/11

Question 18

An object is placed a distance s to the left of a concave (diverging) lens. Which of the following (choose one) is true regarding the magnification produced by the lens?

- i) The magnification is always positive and is unaffected by s .
- ii) The magnification is always positive and its magnitude increases as s decreases.
- iii) The magnification is always positive and its magnitude decreases as s decreases.
- iv) The magnification is always negative and is unaffected by s .
- v) The magnification is always negative and its magnitude increases as s decreases.
- vi) The magnification is always negative and its magnitude decreases as s decreases.



Always upright $\Rightarrow m > 0$

As s decreases image becomes larger.

/5