Electromagnetic Theory II: Homework 11

Due: 7 March 2025

1 Reflection and transmission of waves

Consider waves traveling in one dimension along a medium with a single discontinuity; to the left the wave speed is v_1 and to the right v_2. Consider a wave that is incident from the left.

a) Express the amplitudes of the reflected and transmitted waves in terms of the amplitude of the incident waves and the velocities of the waves (rather than wavenumbers).

b) Suppose that the wave speed to the left is 100 m/s and to the right 120 m/s. Determine the ratio of the amplitude of the reflected wave to the incident wave. Repeat this for the transmitted wave.

c) Suppose that the wave speed to the left is 100 m/s and to the right 80 m/s. Determine the ratio of the amplitude of the reflected wave to the incident wave. Repeat this for the transmitted wave.

d) Determine the ratio of the wave speeds so that the amplitude of the reflected wave is half that of the incident wave.

2 Energy flow and reflection and transmission

Two strings meet at a point. To the left of this point wavespeed is v_1 and to the right it is v_2. Suppose that a wave with angular frequency ω is incident from the left.

a) Show that the rate at which energy is reflected is always less than the rate at which energy is incident.

b) Show that the rate at which energy is transmitted is always less than the rate at which energy is incident.

c) Show that the rate at which energy is incident on the junction between the strings equals the rate at which it leaves the junction.

3 Reflection at a boundary

A wave travels along a string that has three segments with boundaries at z_0 and $z_1 > z_0$. In the region between the boundaries the wavespeed is v_2. In the regions on either side it is v_1. Consider waves that are incident from the left of z_0. First suppose that $v_1 > v_2$.

a) The incident wave is reflected and transmitted at z_0. The subsequent transmitted wave is again reflected and transmitted at z_1. Describe whether each of the transmitted and reflected waves at both boundaries are upright or inverted compared to the incident wave.

b) Repeat this for the case where $v_1 < v_2$.

4 Polarization

Consider complex representations of vector waves,

$$\tilde{\mathbf{f}} = \tilde{\mathbf{A}} e^{i(kz-\omega t)}$$

where

$$\tilde{\mathbf{A}} = \tilde{A}_x \hat{\mathbf{x}} + \tilde{A}_y \hat{\mathbf{y}}$$

and $\tilde{A}_x = A_x e^{i\delta_x}$, $\tilde{A}_y = A_y e^{i\delta_y}$ with A_x, A_y and δ_x, δ_y all real.

a) Suppose that $\tilde{A}_x = Ae^{i\pi/2}$ and $\tilde{A}_y = A$. Determine a real expression for the wave and describe how the vector evolves at $z = 0$.

b) Suppose that $\tilde{A}_x = A$ and $\tilde{A}_y = Ae^{i\pi/2}$. Determine a real expression for the wave and describe how the vector evolves at $z = 0$.

c) Suppose that $\tilde{A}_x = Ae^{i\delta}$ and $\tilde{A}_y = Be^{i\delta}$ where the two phases are identical. Describe the polarization of this wave.

5 Superpositions of polarized waves

Consider two waves with the same frequency and wavenumber that travel along the $+z$ axis. They have the same amplitude. They combine and form a superposition.

a) One of the waves is polarized along $(\hat{\mathbf{x}}+\hat{\mathbf{y}})/\sqrt{2}$. The other is polarized along $(\hat{\mathbf{x}}-\hat{\mathbf{y}})/\sqrt{2}$. If the overall phase of one wave is the same as the other, what is the polarization of the superposition? Explain your answer.

b) One of the waves is polarized along $(\hat{\mathbf{x}}+\hat{\mathbf{y}})/\sqrt{2}$. The other is polarized along $(\hat{\mathbf{x}}-\hat{\mathbf{y}})/\sqrt{2}$. If the overall phase of one wave is shifted by π from the other, what is the polarization of the superposition? Explain your answer.

c) Is it possible that linearly polarized waves could be regarded as a superposition of circularly polarized waves? Explain you answer.