Electromagnetic Theory II: Homework 10

Due: 4 March 2025

1 Standing waves

Consider two waves on the same string with exactly the same wavenumber and amplitude but which travel in opposite directions. These are described by:

$$f_1(z,t) = A\cos(kz - \omega t)$$
$$f_2(z,t) = A\cos(kz + \omega t)$$

where $A > 0$ is real.

a) Using the complex exponential representation, determine an expression for the superposition in terms of a *single product of sines and cosines.*

b) Does the overall pattern propagate along the z axis? Explain your answer.

2 Sinusoidal waves and energy transport

A sinusoidal traveling wave on a string has frequency ω, wavenumber k and amplitude A. In each of the following, briefly explain your answer.

a) Suppose that the amplitude is increased to $3A$ without changing the other parameters. Which of the following is true regarding in rate at which energy is transported?

   i) It remains unchanged.
   ii) It increases to a factor of 3 of what it had been.
   iii) It increases to a factor of 9 of what it had been.

b) Suppose that the tension in the string is adjusted so that the wave speed increases to three times what it had been previously. The frequency and the amplitude remain unchanged. Which of the following is true regarding in rate at which energy is transported?

   i) It remains unchanged.
   ii) It increases to a factor of 3 of what it had been.
   iii) It increases to a factor of 9 of what it had been.

3 Energy transport on a string

The displacement of an infinitely long string with mass per unit length μ and tension T has the form

$$f(z,t) = Be^{-(z-vt)^2/a^2}$$

where $B > 0$ and $a > 0$ are constants, each with units of meters.

a) Determine the total energy stored in the string.

b) Determine the rate at which energy passes from left to right at $z = 0$. At what time does the peak energy flow rate occur at this point?

c) Consider another point, $z_0 > 0$. At what time does the peak energy flow rate occur at this point? Is the peak flow rate the same as at $z = 0$?

4 Energy of a normal mode of a string with both ends fixed

Consider a string spanning the range $0 \leqslant x \leqslant L$ with both ends fixed. The standing waves on this string, or normal modes, are of the form

$$y_n(x,t) = A_n \sin(k_n x)\cos(\omega_n t + \phi).$$

where A_n is a constant, $k_n = n\pi/L$, $\omega = vk_n$ with v being the wavespeed and $n = 1, 2, \ldots$

a) Determine an expression for the energy of the n^{th} normal mode.

b) Rewrite this in terms of the total mass of the string.

It is possible to create electromagnetic standing waves. Using the analogous quantity for the energy of these is the starting point for a quantum theory description of light.