# Electromagnetic Theory II: Homework 7

Due: 14 February 2025

## 1 Boundary conditions: electric fields

Two conducting spherical shells are concentric. The inner shell has radius $a$ and free surface charge $Q > 0$ that is uniformly distributed. The outer shell has radius $b$ and free charge surface $-Q$, also distributed uniformly. The region between the shells is filled with a linear dielectric with dielectric constant $\epsilon_r$.

a) Determine the electric field everywhere.

b) Verify that the *electric field* boundary conditions are satisfied at each spherical shell.

## 2 Boundary conditions: magnetic field

An infinitely long hollow cylinder has radius $R$. The cylinder carries a uniform surface current, in cylindrical coordinates,

$$\mathbf{K} = K\hat{\boldsymbol{\phi}}$$

where $K > 0$ is constant.

a) Determine the magnetic field at all locations.

b) Verify that the *magnetic field* boundary conditions are satisfied at the cylinder boundary.

## 3 Perfectly conducting cylinder

Inside a perfect conductor the electric field is zero.

a) Using one of Maxwell's equations, show that the magnetic field inside the conductor does not vary with time.

b) A superconductor is a perfect conductor and the magnetic field inside is zero. Using one of Maxwell's equations, show that the current in a superconductor only flows along the surface.

c) Consider a very long cylindrical superconductor placed in a magnetic field produced by an external source. The field is uniform and is directed along the axis of the superconductor. Determine the surface current density on the superconductor and show that it agrees with the magnetic field boundary conditions.