

Electromagnetic Theory II: Homework 6

Due: 11 February 2025

1 Cylindrical magnetic material

An infinitely long cylindrical material has radius R . A free current flows along the length of the cylinder. The free current density given in cylindrical coordinates, is

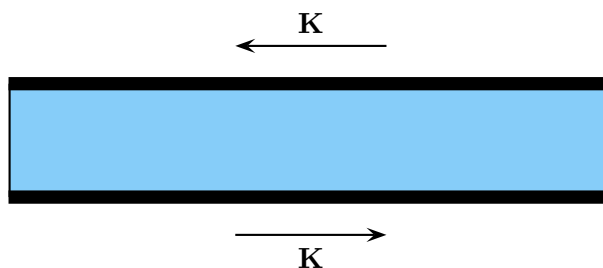
$$\mathbf{J}(\mathbf{r}') = \alpha s' \hat{\mathbf{z}}$$

where $\alpha > 0$ is a constant. The material has susceptibility χ_m .

- a) Let I_{free} be the total free current flowing along the cylinder. Determine an expression for α in terms of I_{free} .
- b) Determine the magnetic field everywhere in terms of R , χ_m and I_{free} .
- c) Determine the bound surface and volume current densities in the material. Sketch these.
- d) Do the bound currents have any affect on the field outside the material? Explain your answer.

2 Slab of linear magnetic material

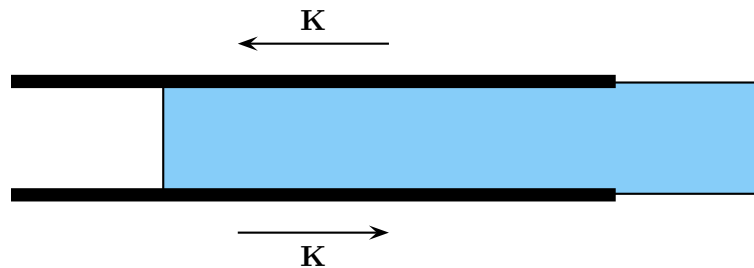
Two infinite sheets of material carry surface currents with the same surface current density with magnitude K but in opposite directions. An infinite slab of linear magnetic material with permeability μ is placed between these. The situation, as viewed from the side, is as illustrated.



- a) Suppose that the material were absent. Determine the direction of the magnetic field between the sheets.
- b) Suppose that the magnetic material is introduced without changing the free currents. Describe the direction of the magnetization and the direction of the bound currents.
- c) Determine the auxiliary field at all locations. If you use Ampère's law, provide an argument that constrains the direction of the auxiliary field first.

- d) Determine the magnetic field at all locations.
- e) Determine an expression for the bound current densities in the material.

Consider a finite version of this in which the material is partially inserted into the gap between the plates



- f) Without calculating all the fields and by using qualitative reasoning about the interactions between the currents in the sheets and the material describe the direction of the net force exerted on the material by the sheets if the material is: i) silver and ii) platinum. In each case you can assume that there is a thin insulating layer between the sheets and the material. *Hint: you will have to look and use the susceptibility of the materials.*

3 Solenoid filled with material

A very long cylindrical solenoid carries current I . The solenoid can be filled with various materials. Using magnetic susceptibility, answer the following. You will need to find values of the susceptibility; find a (*reliable!*) source for this information and cite the source.

- a) Does filling the solenoid with iron increase or decrease the magnetic field? By what factor will the field change?
- b) Does filling the solenoid with PVC increase or decrease the magnetic field? By what factor will the field change?

Whether or not a material increases or decreases the magnetic field is determined by the details of its atomic structure. This involves the arrangement of orbiting electrons and the spins within the material. Maybe you will work on this one day!