Electromagnetic Theory II: Homework 5

Due: 7 February 2025

1 Torques between pairs of current loops

A small circular loop with radius a lies in the xy plane and is centered on the origin. It carries a counterclockwise current (when viewed down the $+z$ axis toward the origin). Another loop with radius b is placed at various locations a distance R away and is oriented in various directions. The loop is free to reorient itself in any direction but it center is fixed. Assuming $R \gg a, b$ the loops can be regarded as dipoles.

a) Suppose that the loop with radius b is placed along the z axis and lies in the xy plane. Explain whether the torque on this loop is zero or not and, if not, what the direction of the torque is.

b) Suppose that the loop with radius b is placed along the z axis and lies in the xz plane with the current counterclockwise (when viewed down the $+y$ direction toward the z-axis). Explain whether the torque on this loop is zero or not and, if not, what the direction of the torque is.

c) Consider the loop as oriented in part b) and released from rest. Describe how the direction of its magnetic dipole moment changes with time. If there is any type of friction present describe the direction of its magnetic dipole moment once it has settled.

2 Diamagnetic effects

An electron in an external uniform magnetic field will orbit in a circle. Suppose that the field points in the $\hat{\mathbf{z}}$ direction and the electron moves with speed v.

a) Determine an expression that relates the orbital speed to the radius of orbit and the magnitude of the field.

The orbiting electron does not exactly provide a steady current, but we will consider a simple approximate model which associates an effective current with the electron and the current magnitude is

$$I = \frac{e}{T}$$

where T is the orbital period of the electron.

b) Determine an expression for the effective magnetic dipole moment of the orbiting electron in terms of the external magnetic field. Which way does the dipole moment point? What happens to the dipole moment as the field increases?

The magnetic field produced by a current loop (of radius r) at its center has magnitude

$$B = \frac{\mu_0 I}{2r}.$$

c) Determine an expression for the magnetic field produced by the orbiting electron. Which way does this field point?

d) Determine an expression for the ratio of the magnetic field produced by the orbiting electron to the external field.

3 Stern-Gerlach experiment

Suppose that a beam of particles, each with mass M and velocity $\mathbf{v} = v_x\hat{\mathbf{x}}$, enters a region in which the magnetic field is

$$\mathbf{B} = (B_0 z)\,\hat{\mathbf{z}}$$

where $B_0 = 600\,\text{Tesla/m}$. This region extends in the x direction for distance L_B. A detector is placed L_D beyond the end of the magnetic field region. The setup is illustrated in Fig. 1.

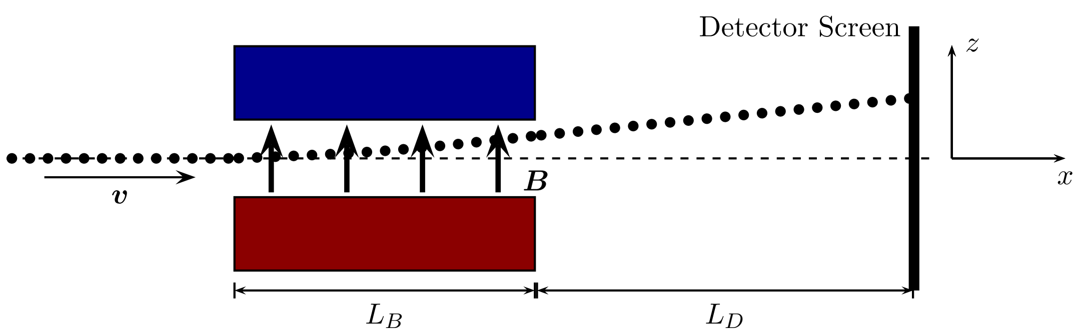


Figure 1: Question 3

a) Suppose that prior to entering the magnetic field, the z component of each particle's magnetic dipole moment has the same value, m_z. Find an expression, in terms of M and m_z, for the acceleration of the particles while they are in the region with non-zero magnetic field. Ignore all forces on any particle except that exerted by the magnetic field and assume that the orientation of the magnetic dipole moment vector remains constant as the particle passes through the field.

b) Assume that the particles follow trajectories governed by classical mechanics. Find an expression for the total deflection in the z direction in terms of M, m_z, v_x, L_B and L_D. Verify that it is proportional to m_z.

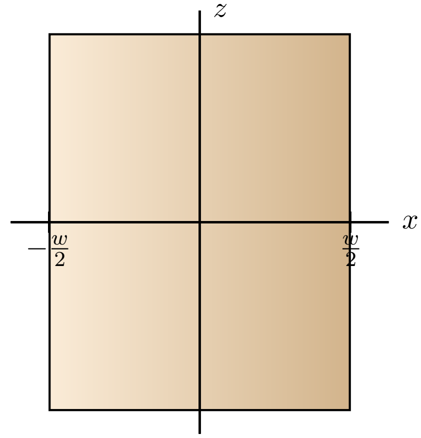
4 Bound current densities; gradient magnetization

Consider an infinite slab of material extending across the range $-w/2 \leqslant x \leqslant w/2$, $-\infty \leqslant y \leqslant \infty$, and $-\infty \leqslant z \leqslant \infty$. (this is parallel to the yz plane). The material has magnetization

$$\mathbf{M} = M_0 \frac{x}{a} \hat{\mathbf{z}}$$

where $M_0 > 0$ and $a > 0$ are constants. The following questions apply to a section that extends across the material and beyond its edges, for which

$$\begin{aligned} -w \leqslant x &\leqslant w, \\ y &= 0, \\ 0 \leqslant z &\leqslant h. \end{aligned}$$



a) Determine the bound current densities in the material.

b) Determine the total bound current that flows across the section described above.