Electromagnetic Theory II: Homework 4

Due: 4 February 2025

1 Work and dipoles

Consider the process of assembling electric dipoles from point charges and then changing their polarization. Suppose that a dipole consists of two point charges with charges $+q$ and $-q$.

a) Is the work required to assemble the dipole (i.e. the energy stored in the dipole) from two point charges infinitely far apart positive, negative or zero? Explain your answer.

b) If you were to calculate the electrostatic energy for a collection of free charges in the presence of dipoles, the general formula would give you the work done needed to assemble the free charges *and also the dipoles starting with charges infinitely far apart.* Given a situation where the charges in the dipoles were already assembled (perhaps with less polarization) does this formula correctly reflect the additional work done to assemble free charge in the presence of the dipoles? How might the additional work done differ? Explain your answer.

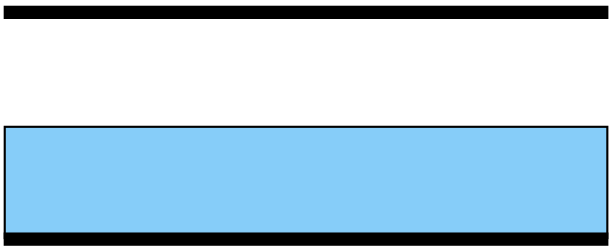
c) Suppose that the dipoles are assembled and the placed into an external electric that increases the polarization. Does the energy stored in the dipole increase, decrease or stay the same? Explain your answer.

2 Energy required to charge a capacitor

A parallel plate capacitor has plates of area A separated by distance d.

a) The space between the capacitor is filled with a dielectric material of permittivity ϵ. Determine an expression for the work required to charge the capacitor in terms of A, d, ϵ and ϵ_0 and the potential difference across the capacitor, ΔV.

b) Now consider a similar capacitor but between whose plates the dielectric only fills half of the space.



Again, suppose that the potential difference across the capacitor is ΔV. Determine an expression for the work required to charge the capacitor in terms of A, d, ϵ and ϵ_0.

3 Force on a dielectric on a cylindrical capacitor

A cylindrical capacitor consists of an cylindrical rod of radius a surrounded by an outer cylindrical shell of radius b. Each have length $L \gg a, b$. The capacitor plates are connected to a battery which provides potential difference ΔV.

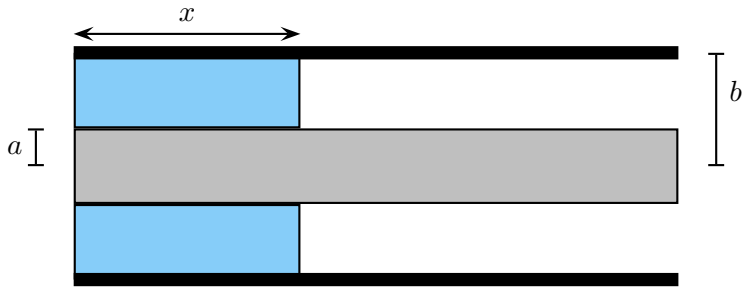
This problem uses the fact that the energy stored in a capacitor is always

$$W = \frac{1}{2} C (\Delta V)^2 .$$

a) Show that the capacitance of this arrangement is

$$C = 2\pi L \frac{\epsilon_0}{\ln (b/a)}.$$

Now suppose that the capacitor is partly filled by a dielectric with dielectric constant ϵ. The dielectric penetrates to depth x as illustrated.



b) Using qualitative arguments describe the direction of the force exerted by the capacitor on the dielectric.

c) Determine the capacitance of this arrangement. *Hint: This is effectively two capacitors connected in parallel.*

d) Determine the energy stored in this arrangement as a function of $x, L, a, b, \Delta V$ and the electric susceptibility.

e) In general the force associated with any energy, U, is

$$F = -\frac{dU}{dx}.$$

Use this to show that the force exerted by the capacitor on the dielectric has magnitude

$$F = \frac{\pi \epsilon_0}{\ln (b/a)} (\Delta V)^2 \chi_e.$$

f) Suppose that this capacitor is arranged vertically and that the bottom is immersed in water. The radius of the inner cylinder is 2.0 cm and the outer cylinder is 3.0 cm. Determine the voltage across the capacitor so that the water rises to a height of 5.0 mm.