Electromagnetic Theory II: Homework 3

Due: 31 January 2025

1 Polarized spherical shell

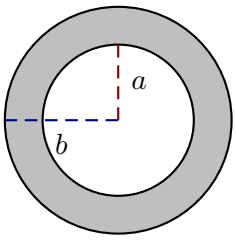
A spherical shell has inner radius a and outer radius b. The shell is polarized with polarization

$$\mathbf{P}(\mathbf{r}') = -\frac{\alpha}{r'^2}\hat{\mathbf{r}}$$

where $\alpha > 0$ has units of charge. Sketch the bound charge density inside on the surfaces of the shell and use the sketch (not a calculation) to predict the direction of the electric field at all locations.

2 Polarized cylindrical object

An infinitely long cylindrical pipe has inner radius a and outer radius b. The pipe consists of a material that has polarization, given in cylindrical coordinates, by



$$\mathbf{P}(\mathbf{r}') = \frac{\alpha}{s'^2}\hat{\mathbf{s}}$$

where $\alpha > 0$. There is no free charge present. Determine the electric displacement and the electric field at all points.

3 Point charge within a dielectric sphere

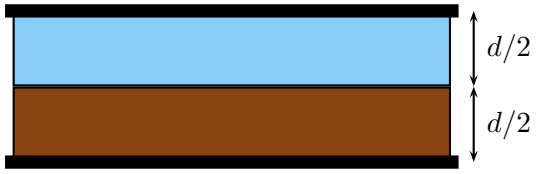
A point particle with charge q is located at the center of a sphere, with radius R, made of a linear dielectric material with permittivity ϵ.

a) Determine the electric displacement and electric field at all points.

b) Determine the volume bound charge density inside the sphere and the bound charge density on the surface of the sphere.

4 Parallel plate capacitor with two layers of dielectric

A parallel plate capacitor has plates of area A separated by distance d. The gap between the plates is filled with two linear dielectric materials, each of thickness $d/2$. The upper material has permittivity ϵ_1 and the other has permittivity ϵ_2. The free charge per unit area on the upper plate is $+\sigma_{\text{free}}$ and on the lower plate is $-\sigma_{\text{free}}$. Assume that the plates are so large that they can be approximated as infinite.



a) Determine the electric field at all points between the plates.

b) Determine the capacitance of this capacitor.

c) Determine expressions for the bound charge densities on the upper and lower surfaces of each dielectric.

5 Bound charge on a PVC dielectric

A parallel plate capacitor consists of two parallel metal disks, each with radius 0.10 m, and which are 0.0025 m apart. The disks are connected to a 120 V power supply.

a) Determine the total charge on each disk. *You can use results from parallel plate capacitors that we derived in class.*

The plates are disconnected from the power supply, without altering the charge on the plates. A sheet of PVC is placed between the plates, exactly filling the space between them and not protruding. The PVC is a linear dielectric with dielectric constant 3.18.

b) Determine the bound charge on the surfaces of the PVC sheet.

c) Suppose that the PVC were inserted while the capacitor was connected to the power supply. Would the bound charge on each of it's surfaces be larger than, smaller than or the same as that of part b)? Explain your answer.