Electromagnetic Theory II: Homework 1

Due: 24 January 2025

1 Electric field produced by an infinitely long charged rod

An infinitely long cylindrical rod has radius R and, within the cylinder, charge density (in cylindrical coordinates)

$$\rho(r') = \alpha s'^n$$

where n is a non-negative integer and $\alpha > 0$ is constant. Determine the electric field at all points inside and outside the cylinder.

Your solution *must contain:*

i) A diagram of the charge distribution.

ii) A simplification of the electric field using symmetry arguments.

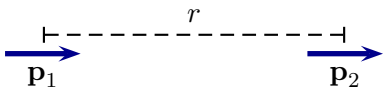
iii) An illustration and explanation of what Gaussian surface (the surface that appears in the surface integral) you are using. If this is the same as the surface of the charged rod then all that you will calculate is the field on the surface of the rod. That would be **incomplete and incorrect**. The illustration must indicate the enclosed charge.

iv) Evaluation of the surface integral.

v) Evaluation of the enclosed charge.

2 Forces and torques between dipoles

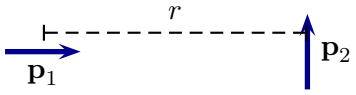
Consider two perfect electric dipoles as illustrated.



a) Determine the torque exerted by 1 on 2.

b) Determine the force exerted by 1 on 2.

Now suppose that the two dipoles are oriented as illustrated.



c) Determine the torque exerted by 1 on 2.

d) Determine the force exerted by 1 on 2.

3 Charged disks

Two disks lie in the same plane as illustrated. Charge is distributed in disk A according to (in cylindrical coordinates centered at the disk center)

$$\rho_A = \alpha \cos\phi'$$

and in disk B according to (in cylindrical coordinates centered at the disk center)

$$\rho_B = -\alpha \cos\phi'$$

where $\alpha > 0$. Assume that the separation between the disks is very large compared to the radius of each disk.

a) Use qualitative arguments to describe whether disk A exerts a net force on disk B. If so, what is the direction of the force?

b) Use qualitative arguments to describe whether disk A exerts a net torque on disk B. If so, what is the direction of the torque?

4 Dipole in a field

a) A dipole is placed in a uniform field (same at all points). Is it possible for the field to exert a net force on the dipole? If so, explain how the dipole would have to be oriented relative to the field.

b) A dipole is placed in a uniform field (same at all points). Is it possible for the field to exert a net torque on the dipole? If so, explain how the dipole would have to be oriented relative to the field.

c) A dipole is placed in a field $\mathbf{E} = (E_0)z\ \hat{\mathbf{x}}$ where $E_0 > 0$ is a constant. Is it possible for the field to exert a net force on the dipole? If so, explain how the dipole would have to be oriented relative to the field.

d) A dipole is placed in a field $\mathbf{E} = (E_0)z\ \hat{\mathbf{x}}$ where $E_0 > 0$ is a constant. Is it possible for the field to exert a net torque on the dipole? If so, explain how the dipole would have to be oriented relative to the field.