Electromagnetic Theory II: Class Exam I

25 February 2025

Name: ______________________________ **Total:** **/50**

Instructions

- There are 6 questions on 7 pages.
- Show your reasoning and calculations and always explain your answers.

Physical constants and useful formulae

Permittivity of free space	$\epsilon_0 = 8.85 \times 10^{-12}\,\mathrm{C^2/Nm^2}$
Permeability of free space	$\mu_0 = 4\pi \times 10^{-7}\,\mathrm{N/A^2}$
Charge of an electron	$e = -1.60 \times 10^{-19}\,\mathrm{C}$

$$\int \sin(ax)\sin(bx)\,\mathrm{d}x = \frac{\sin((a-b)x)}{2(a-b)} - \frac{\sin((a+b)x)}{2(a+b)} \quad \text{if } a \neq b$$

$$\int \cos(ax)\cos(bx)\,\mathrm{d}x = \frac{\sin((a-b)x)}{2(a-b)} + \frac{\sin((a+b)x)}{2(a+b)} \quad \text{if } a \neq b$$

$$\int \sin(ax)\cos(ax)\,\mathrm{d}x = \frac{1}{2a}\sin^2(ax)$$

$$\int \sin^2(ax)\,\mathrm{d}x = \frac{x}{2} - \frac{\sin(2ax)}{4a}$$

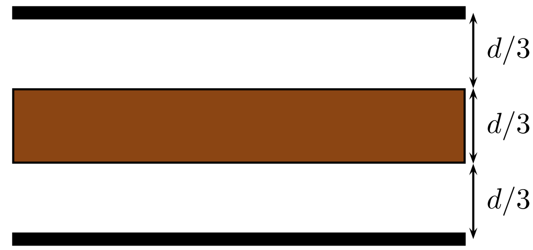
$$\int \cos^2(ax)\,\mathrm{d}x = \frac{x}{2} + \frac{\sin(2ax)}{4a}$$

$$\int x\sin^2(ax)\,\mathrm{d}x = \frac{x^2}{4} - \frac{x\sin(2ax)}{4a} - \frac{\cos(2ax)}{8a^2}$$

$$\int x^2\sin^2(ax)\,\mathrm{d}x = \frac{x^3}{6} - \frac{x^2}{4a}\sin(2ax) - \frac{x}{4a^2}\cos(2ax) + \frac{1}{8a^3}\sin(2ax)$$

Question 1

A parallel plate capacitor consists of two conducting plates separated by distance d. A linear dielectric, with permittivity ϵ, occupies the middle third of the region between its plates. A cross-sectional view of the arrangement is as illustrated. The area of the plates is A and the gap between the plates is sufficiently small for them to be considered infinite in extent.



a) Suppose that the capacitor plates are equally and oppositely charged. The free surface charge density on the upper plates is $+\sigma > 0$ and on the lower plate it is $-\sigma$. Determine $\mathbf{D}$ for all regions and use this to determine the electric field at all locations between the plates.

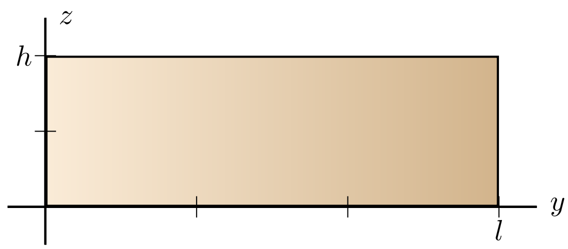
Question 1 continued . . .

b) Determine the capacitance of this arrangement.

/12

Question 2

A slab of material occupies the region $0 \leqslant x \leqslant w$, $0 \leqslant y \leqslant l$, and $0 \leqslant z \leqslant h$. The magnetization within the material is $\mathbf{M} = kz\hat{\mathbf{x}}$ where $k > 0$.



a) Suppose that the magnetization were produced by miniscule magnetic dipoles. Sketch some of these dipoles and use your sketch to predict the directions of the surface and volume bound currents.

b) Determine expressions for the bound surface (on each surface) and volume current densities.

/8

Question 3

A point dipole, with dipole moment $\mathbf{p}$ is placed in an electric field $\mathbf{E} = E_0 x\,\hat{\mathbf{z}}$ where $E_0 > 0$ is a constant. Which of the following (choose one) is true about the force $\mathbf{F}$ exerted by the field on the dipole?

i) If $\mathbf{p}$ is along $\hat{\mathbf{x}}$ then $\mathbf{F} = 0$.
ii) If $\mathbf{p}$ is along $\hat{\mathbf{x}}$ then $\mathbf{F} = F\hat{\mathbf{z}} \neq 0$.
iii) If $\mathbf{p}$ is along $\hat{\mathbf{x}}$ then $\mathbf{F} = F\hat{\mathbf{x}} \neq 0$.
iv) If $\mathbf{p}$ is along $\hat{\mathbf{z}}$ then $\mathbf{F} = F\hat{\mathbf{z}} \neq 0$.
v) If $\mathbf{p}$ is along $\hat{\mathbf{z}}$ then $\mathbf{F} = F\hat{\mathbf{x}} \neq 0$.

/4

Question 4

A rectangular piece of linear magnetic material has a surface in the xy-plane. There is free space above the surface. The magnetic field immediately above the surface is $\mathbf{B}_{\text{above}} = B\hat{\mathbf{x}}$ where $B > 0$. There is a surface current in the surface along $+\hat{\mathbf{y}}$. Which of the following (choose one) is true about the magnetic field immediately below the plane, $\mathbf{B}_{\text{below}}$?

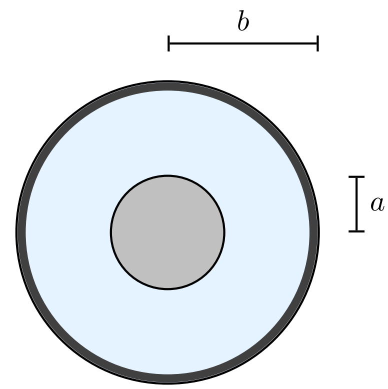
i) $B_{\text{below }z} = 0$ and $B_{\text{below }y} = 0$.
ii) $B_{\text{below }z} = 0$ and $B_{\text{below }y} \neq 0$.
iii) $B_{\text{below }z} \neq 0$ and $B_{\text{below }y} = 0$.
iv) $B_{\text{below }z} \neq 0$ and $B_{\text{below }y} \neq 0$.

Briefly explain your choice.

/6

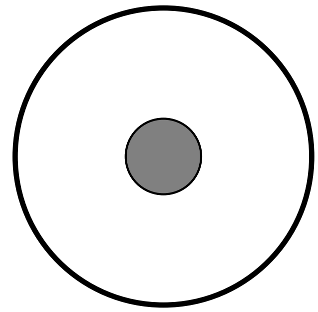
Question 5

A coaxial arrangement consists of two cylinders separated by a region filled with a linear magnetic material with magnetic susceptibility χ_m. The inner cylinder has radius a and the outer cylinder has radius b. The inner cylinder carries a uniformly distributed current I flowing out of the page. The outer cylinder carries a uniformly distributed current I into the page. Determine the magnetic field $\mathbf{B}$, in terms of I, χ_m, μ_0, and radial distance, at all points beyond the inner cylinder.



/12

Question 6



Two cylindrical conductors are concentric and arranged as illustrated (viewed along their axis). The radius of the inner conductor is a and of the outer conductor is b. The fields in the gap are

$$\mathbf{E} = \frac{\alpha}{s}\hat{\mathbf{s}} \quad \text{and} \quad \mathbf{B} = \frac{\beta}{s}\hat{\boldsymbol{\phi}}$$

where $\alpha, \beta > 0$. Determine the direction of flow of electromagnetic energy in the region between the cylinders and determine the total energy that flows per second through a closed cylindrical surface, whose axis is along that of the conductor's axis and which has radius $a \leqslant r \leqslant b$ and length L. *Note: The cylinder has three surfaces.*

/8