Phys 312
Fall 2016

Electromagnetic Theory II: Class Exam 2

9 November 2016

Name: _____ **Total:** **/50**

Instructions

- There are 4 questions on 6 pages.
- Show your reasoning and calculations and always explain your answers.

Physical constants and useful formulae

Permittivity of free space	$\epsilon_0 = 8.85 \times 10^{-12}\,\mathrm{C}^2/\mathrm{Nm}^2$
Permeability of free space	$\mu_0 = 4\pi \times 10^{-7}\,\mathrm{N/A}^2$
Charge of an electron	$e = -1.60 \times 10^{-19}\,\mathrm{C}$
Charge of a proton	$e = +1.60 \times 10^{-19}\,\mathrm{C}$
Speed of light	$c = 3.0 \times 10^{8}\,\mathrm{m/s}$

Question 1

Consider an electromagnetic wave in free space. The complex representation of a particular wave is

$$\widetilde{\mathbf{E}} = E_0 e^{i(ky-\omega t)}\hat{\mathbf{z}}$$

where k, ω and $E_0 > 0$ are real. **By substituting into one of Maxwell's equations**, find an expression for $\dfrac{\partial \widetilde{\mathbf{B}}}{\partial t}$. Solve this equation to determine an expression for $\widetilde{\mathbf{B}}$ in terms of E_0, ω, k and constants.

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Question 2

Answer either part a) or part b) for full credit for this problem.

a) Light in water (index of refraction 1.33) is incident on glass (index of refraction 1.50). at an angle of 40° with respect to the normal. Determine whether light polarized perpendicular to the plane of incidence or light polarized parallel to the plane of incidence is reflected more intensely. Assume $\mu_{\text{water}} = \mu_{\text{glass}} = \mu_0$.

Question 2 continued ...

b) An electromagnetic wave propagates in a medium with conductivity σ, permeability μ and permittivity ϵ. The complex representation for the electric field is

$$\widetilde{\mathbf{E}}(z,t) = \widetilde{E}_0 e^{-\kappa z} e^{i(kz-\omega t)} \hat{\mathbf{x}}$$

where $\widetilde{E}_0$ is independent of x, y, z, t,

$$k = \omega\sqrt{\frac{\epsilon\mu}{2}}\left[\sqrt{1+\left(\frac{\sigma}{\epsilon\omega}\right)^2}+1\right]^{1/2} \quad \text{and} \quad \kappa = \omega\sqrt{\frac{\epsilon\mu}{2}}\left[\sqrt{1+\left(\frac{\sigma}{\epsilon\omega}\right)^2}-1\right]^{1/2}$$

Suppose that $\omega \gg \sigma/\epsilon$. Determine an approximate (non-zero) expression for the skin depth. Does the depth to which the wave penetrates depend on the frequency?

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Question 3

An arrangement of currents and charges produces the following potentials, given in cylindrical coordinates as

$$V = 0$$

and

$$\mathbf{A} = \begin{cases} 0 & \text{if } s < R \\ A_0 \cos(\omega t)\hat{\boldsymbol{\phi}} & \text{if } s > R \end{cases}$$

where A_0 and ω are constants.

a) Determine the electric and magnetic fields produced by these potentials.

b) In which direction do these fields transport energy? Explain your answer.

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Question 4

Answer either part a) or part b) for full credit for this problem.

a) Consider the potentials given Cartesian coordinates as

$$V = 0$$

and

$$\mathbf{A} = A_0 \cos(kz - \omega t)\hat{\mathbf{y}}$$

where A_0 and ω are constants. Describe whether this potential is in the Coulomb gauge, the Lorentz gauge or both. Explain your answer.

b) A stationary point particle is at the origin. At time t_1 it becomes positively charged and its charge stays constant until at a later time t_2 it becomes neutral again. An observer, who is stationary at a distance R from the origin can measure electric and magnetic fields at her location. At what times does she detect electric fields? At what times does she detect magnetic fields? Explain your answers.

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