Tues: Discussion / quiz

EX: 94, 97, 99, 100, 104, 107, 110, 111

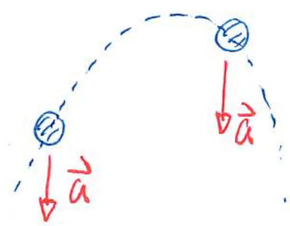
Projectile motion

Projectile motion is where an object only moves under the influence of Earth's gravity. Experiments show that:

Acceleration is constant and

$a_x = 0 \, m/s^2$

$a_y = -g = -9.8 \, m/s^2$

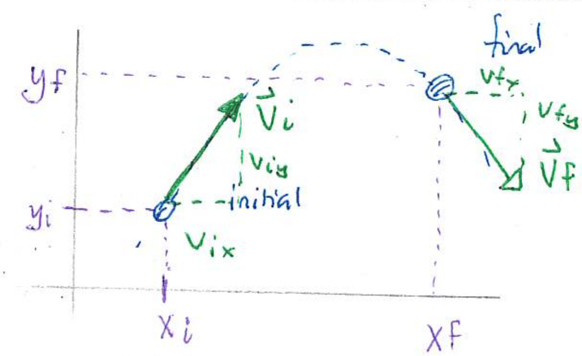


Then the two dimensional kinematic equations apply

$$v_{fx} = v_{ix} + a_x \Delta t \qquad v_{fy} = v_{iy} + a_y \Delta t$$

$$x_f = x_i + v_{ix} \Delta t + \tfrac{1}{2} a_x \Delta t^2 \qquad y_f = y_i + v_{iy} \Delta t + \tfrac{1}{2} a_y \Delta t^2$$

$$v_{fx}^2 = v_{ix}^2 + 2a_x(x_f - x_i) \qquad v_{fy}^2 = v_{iy}^2 + 2a_y(y_f - y_i)$$



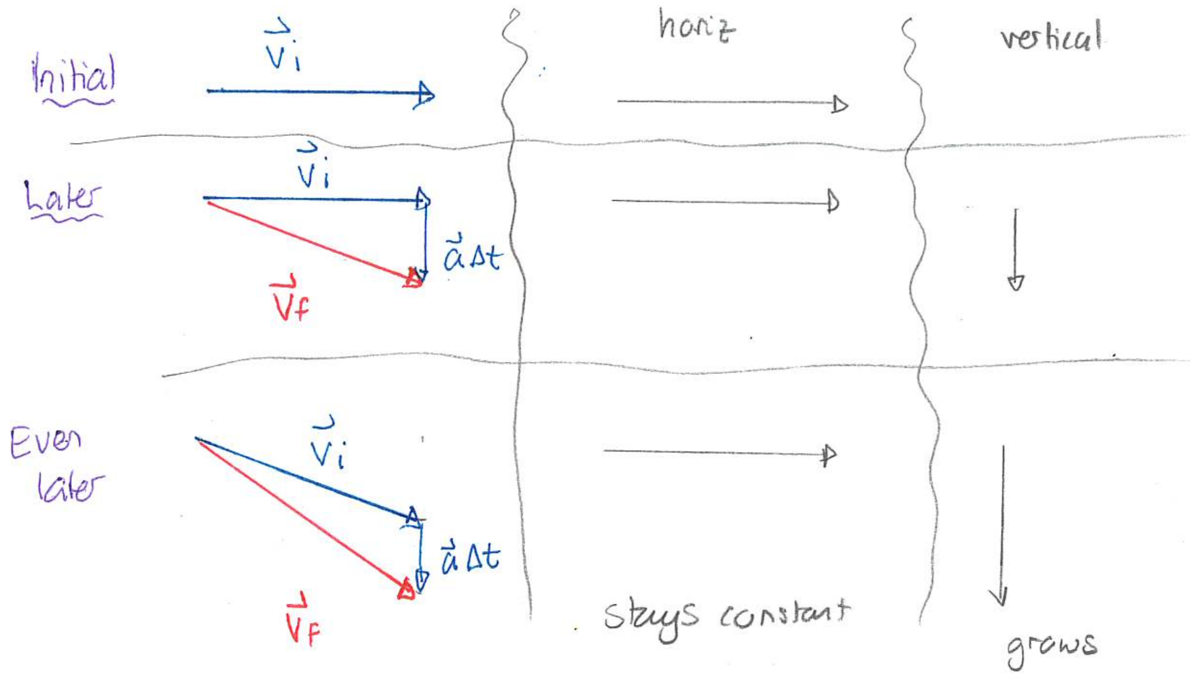
The horizontal and vertical components are independent. Note that $a_x = 0$ means that $v_{fx} = v_{ix}$ and thus the horizontal component of velocity is constant.

Warm Up 1

Demo: Ball vertical versus horizontal

Demo: Video U of Iowa site

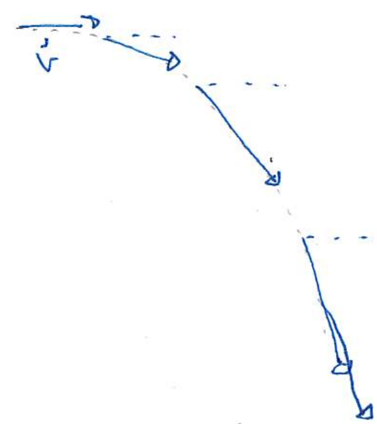
Consider the ball launched horizontally. Then the velocity evolves as:



Warm Up2

Quiz 1 50% -60%

Combining these gives a trajectory that is curved. One can show that this is exactly a parabola:



$$x_f = \cancel{x_i}^{0} + v_{ix}\Delta t + \frac{1}{2}\cancel{a_x \Delta t^2}^{0}$$

$$x_f = v_{ix}\Delta t \quad \Rightarrow \quad \Delta t = \frac{x_f}{v_{ix}}$$

Then

$$y_f = y_i + \cancel{v_{iy}\Delta t}^{0} + \frac{1}{2} a_y \Delta t^2$$

$$\Rightarrow \quad y_f = y_i - \frac{g}{2}\left(\frac{x_f}{v_{ix}}\right)^2$$

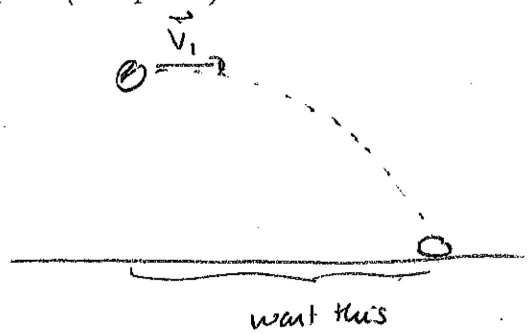
Let $y_f \rightarrow y$, $x_f \rightarrow x$ $\Big\} \Rightarrow \quad y = y_i - \frac{g}{2v_{ix}^2}x^2 \quad \Rightarrow \quad y = x^2(\text{const}) + \text{const}$

This is a parabola.

114 Aircraft dropping object

An aircraft flies horizontally with a constant speed of 600 km/h at a height of 1200 m above a flat surface. It drops an object from its underside; this object is supposed to hit a particular spot on the ground. How far (horizontally) from the spot must the aircraft be for the object to hit the spot? *(131Sp2025)*

Answer:



$x_i = 0\,m$ $\quad y_i = 1200\,m$

$x_f = ?$ $\quad y_f = 0\,m$

$v_{ix} = 167\,m/s$ $\quad v_{fx} =$

$v_{iy} = 0\,m/s$ $\quad v_{fy} =$

$a_x = 0\,m/s^2$ $\quad a_y = -9.8\,m/s^2$

Note: * not a straight line

* cannot set $\vec{v}_f = 0$ (that's no longer free fall)

Initial velocity $\vec{v}_i \rightarrow$ $\quad 600\,km/hr = \dfrac{600 \times 10^3\,m}{3600\,s} = 167\,m/s$

Need

$$x_f = \cancel{x_i}^{\,0} + v_{ix}\Delta t + \tfrac{1}{2}\cancel{a_x \Delta t^2} \Rightarrow \boxed{x_f = 167\,m/s\ \Delta t}$$

get Δt from vertical

$$\cancel{y_f}^{\,0} = y_i + \cancel{v_{iy}}^{\,0}\Delta t + \tfrac{1}{2} a_y \Delta t^2 \Rightarrow 0\,m = 1200\,m - \tfrac{1}{2}\,9.8\,m/s^2\,\Delta t^2$$

$$\Rightarrow 1200\,m = 4.9\,m/s^2\,\Delta t^2$$

$$\Rightarrow \boxed{\Delta t = \sqrt{1200\,m / 4.9\,m/s^2} = 16\,s}$$

Then $x_f = 167\,m/s \times 16\,s = 2.6 \times 10^3\,m = 2.6\,km$