Phys 131 2025

Lecture 9

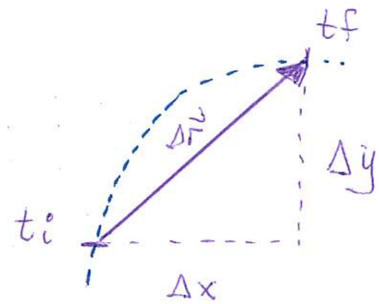
Mon: Warm Up 4 (D2L) by 9am Group Exercise : credit!

Tues: Discussion/quiz

Ex: 94, 97, 99, 100, 104, 107, 110, 111

Velocity in two dimensions

For an object moving in two dimensions, position, displacement and velocity are all vectors. For velocity



$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}\hat{\imath} + \lim_{\Delta t \to 0} \frac{\Delta y}{\Delta t}\hat{\jmath} = \frac{dx}{dt}\hat{\imath} + \frac{dy}{dt}\hat{\jmath}$$

$\Delta t = t_f - t_i$

and we have that velocity has two components.

$$\vec{v} = v_x\hat{\imath} + v_y\hat{\jmath}$$

horizontal component

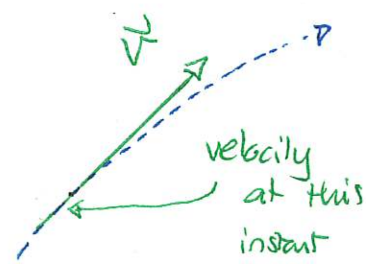
vertical component

Velocity has the following properties:

* The magnitude of velocity is the speed

$$v = \sqrt{v_x^2 + v_y^2}$$

* Velocity is tangent to the trajectory along the direction of motion



DEMO: PhET Ladybug 2D - ellipse
- show trace and $\vec{v}$

Quiz 1 40% – 90%

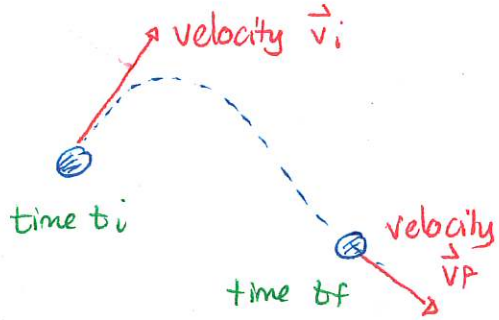
Quiz 2 20% – 50%

Quiz 3 50% – 80%

Acceleration in two dimensions

Acceleration describes the rate at which the velocity vector changes. We start with.

Observe the object at two instants.



$\Rightarrow$ The average acceleration over the interval is

$$\vec{a}_{avg} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t}$$

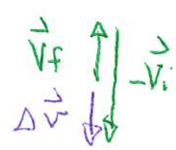
Note that average acceleration is a vector and this requires vector subtraction.

Example: A ball moves vertically up under Earth's gravity. Determine the acceleration vector as it moves up.

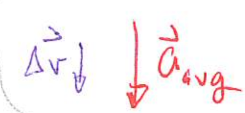
Answer:

① Sketch trajectory and velocity vector at two instants

$\rightarrow$ ② Subtract velocity to get $\Delta \vec{v} = \vec{v}_f - \vec{v}_i$



$\rightarrow$ ③ Direction of average accel is same as direction of $\Delta \vec{v}$



Quiz 4 30% – 80%

The true definition of acceleration is:

$$\boxed{\text{The acceleration of any object is} \quad \vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}}$$

Notes:

1) acceleration is a vector

$$\vec{a} = \underbrace{a_x}_{\frac{dv_x}{dt}} \hat{i} + \underbrace{a_y}_{\frac{dv_y}{dt}} \hat{j}$$

2) the direction of acceleration is the same as that of $\Delta\vec{v}$ but this is not always aligned with the direction of motion.

Constant acceleration

We will consider situations where the acceleration is constant. Then, regardless of the interval it will be exactly true that

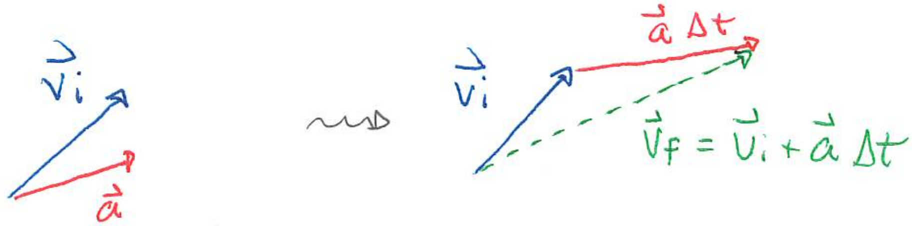
$$\vec{a} = \frac{\Delta \vec{v}}{\Delta t} \quad \Rightarrow \quad \Delta\vec{v} = \vec{a}\,\Delta t$$

$$\Rightarrow \quad \vec{v}_f - \vec{v}_i = \vec{a}\,\Delta t$$

Thus if acceleration is constant then

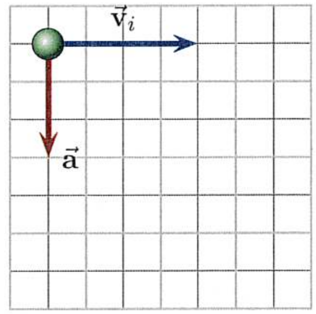
$\vec{v}_f = \vec{v}_i + \vec{a}\,\Delta t$ (ONLY for CONSTANT ACCELBRATION)

So we have



103 Constant vertical acceleration

A ball launches off a horizontal surface. At the moment of launch its velocity is $\vec{\mathbf{v}}_i$. At all later times it accelerates with a constant acceleration, $\vec{\mathbf{a}}$. The situation with the vectors drawn to scale is illustrated (for the velocity vector, the grid unit is the standard unit of velocity and for acceleration the grid unit is the standard unit of acceleration). *(131Sp2025)*

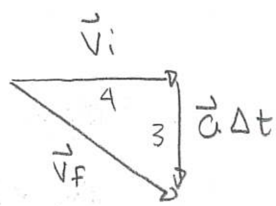


a) Draw, as accurately as possible, the velocity vector, $\vec{\mathbf{v}}_f$, at an instant 1.0 s after the initial instant.

b) Using $\vec{\mathbf{v}}_f$ describe whether the object is moving faster at the 1.0 s instant than at the initial instant.

c) Using $\vec{\mathbf{v}}_f$ describe the direction in which the object is moving at the 1.0 s instant.

a) $\vec{v}_f = \vec{v}_i + \vec{a}\,\Delta t$



b) $v_f = \sqrt{v_{fx}^2 + v_{fy}^2} = \sqrt{(4\,m/s)^2 + (3\,m/s)^2} = 5\,m/s$

c) angled downward

When acceleration is constant

$$\vec{v}_f = \vec{v}_i + \vec{a}\,\Delta t \Rightarrow \quad v_{fx} = v_{ix} + a_x \Delta t$$
$$v_{fy} = v_{iy} + a_y \Delta t$$

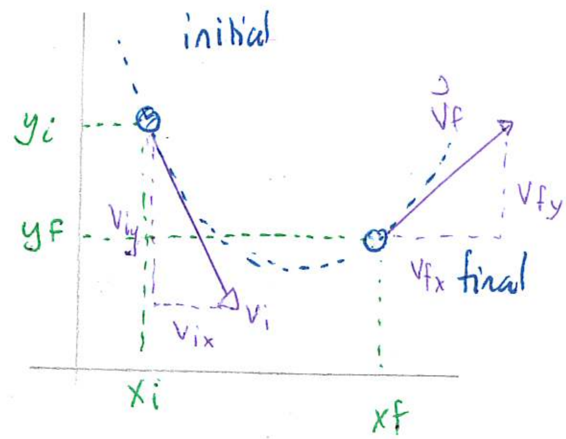
Continuing, using calculus, gives the <u>kinematic equations</u>

If <u>acceleration is constant</u> then

$$v_{fx} = v_{ix} + a_x \Delta t \qquad v_{fy} = v_{iy} + a_y \Delta t$$
$$x_f = x_i + v_{ix}\Delta t + \tfrac{1}{2} a_x \Delta t^2 \qquad y_f = y_i + v_{iy}\Delta t + \tfrac{1}{2} a_y \Delta t^2$$
$$v_{fx}^2 = v_{ix}^2 + 2a_x(x_f - x_i) \qquad v_{fy}^2 = v_{iy}^2 + 2a_y(y_f - y_i)$$

Only horizontal — Only vertical

Only Δt is in common



In these cases we see that the horizontal and vertical components of the motion are independent.

<u>Projectile motion</u>

Projectile motion is such that only Earth's gravity acts on an object. Then experimental observations show:

The acceleration of a projectile is constant and has components

$$a_x = 0\,\text{m/s}^2$$
$$a_y = -g = -9.80\,\text{m/s}^2$$

This is motion with constant acceleration.

<u>DEMO</u>: Cart + Ball Launcher.