

Lecture 5

Fri: HW by 5pm

Ex: 40, 41, 42, 44, 47, 48, 52, 56

Group Exercise for credit 3pts

Mon: Warm Up 3 D2L

Fri: SPS noon

Acceleration

Acceleration quantifies the rate at which velocity changes. The framework is:

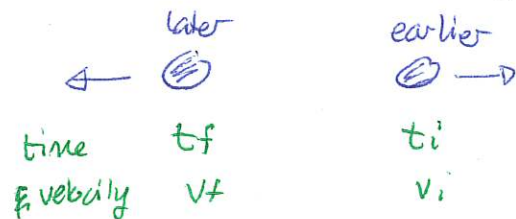
Conceptual Idea

Acceleration ~ rate of change of velocity

Preliminary definition

Observe object at two instants. The average acceleration over the interval is

$$a_{avg} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i}$$



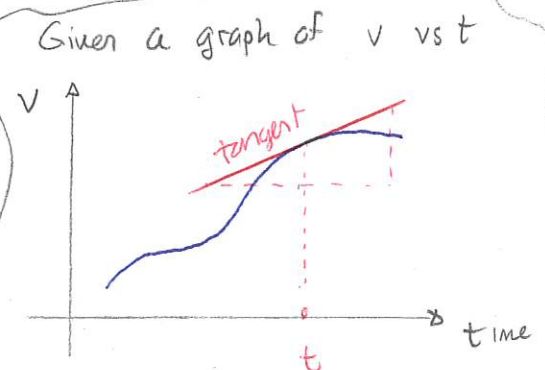
(Close to) Exact definition

The (instantaneous) acceleration is the limit of average acceleration over a vanishingly small interval

$$a = \lim_{\Delta t \rightarrow 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$$

units m/s^2

Given a formula for v vs t
acceleration = derivative of v w.r.t t



acceleration = slope of tangent to v vs t

Quiz 1 30% - 70%

Quiz 2 60% - 100%

Quiz 3 85% - 90%

Quiz 4

Motion with constant acceleration

In many situations the acceleration of an object is constant. Examples include objects in free fall or objects sliding down a ramp. In these cases we can relate kinematical quantities at one moment to those at another.

For example

Suppose the details about the initial situation are known

t_i, x_i, v_i

Physics will provide acceleration a

	initial (earlier)	final (later)
time	t_i	t_f
position	x_i	x_f
velocity	v_i	v_f

Want information at a later moment: x_f, v_f

The process can also:

- * take information at a later instant and determine information at an earlier instant
- * mix data from initial and final instants.

In general, only the elapsed time

$$\Delta t = t_f - t_i$$

matters

Quiz 5

In general $\Delta x \neq v_i \Delta t$. More correctly these quantities are related by the kinematic equations.

If an object moving in one dimension has constant acceleration, then.

$$v_f = v_i + a \Delta t$$

$$x_f = x_i + v_i \Delta t + \frac{1}{2} a \Delta t^2$$

$$v_f^2 = v_i^2 + 2a \Delta x = v_i^2 + 2a(x_f - x_i)$$

Proofs: First, consider $v_f = v_i + a \Delta t$.

If the acceleration is constant then $a = \frac{\Delta v}{\Delta t}$

$$\Rightarrow a \Delta t = \Delta v \Rightarrow \Delta v = a \Delta t$$

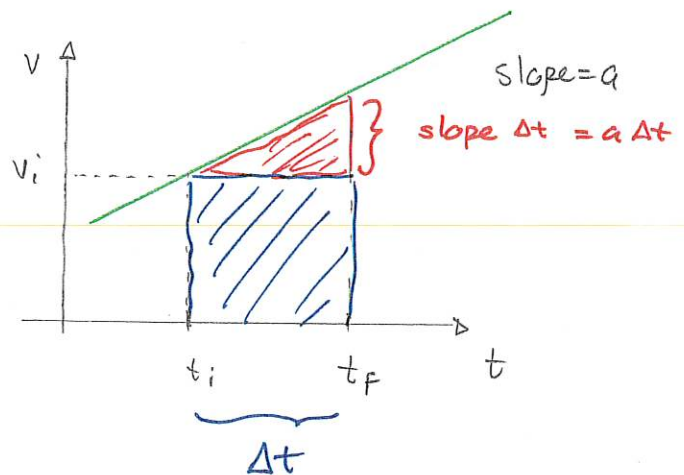
$$\Rightarrow v_f - v_i = a \Delta t \Rightarrow v_f = v_i + a \Delta t \quad \checkmark$$

Second, consider $x_f = x_i + \dots$ We can use a graphical proof

Then $\Delta x = \text{area under } v \text{ vs } t$

= area blue rectangle
+ area triangle

$$\begin{aligned} \text{Area blue rectangle} &= v_i (t_f - t_i) \\ &= v_i \Delta t \end{aligned}$$



Area red triangle = $\frac{1}{2} bh$

$$= \frac{1}{2} \Delta t (a \Delta t) = \frac{1}{2} a (\Delta t)^2$$

$$\text{Thus } \Delta x = v_i \Delta t + \frac{1}{2} a (\Delta t)^2$$

$$\Rightarrow x_f - x_i = v_i \Delta t + \frac{1}{2} a (\Delta t)^2 \Rightarrow x_f = x_i + v_i \Delta t + \frac{1}{2} a (\Delta t)^2 \quad \checkmark$$

Third $v_f^2 = v_i^2 \dots$

$$\text{From } v_f - v_i = a \Delta t \Rightarrow \Delta t = \frac{v_f - v_i}{a}$$

$$\begin{aligned} \text{Then } \Delta x &= v_i \Delta t + \frac{1}{2} a (\Delta t)^2 \\ &= v_i \left(\frac{v_f - v_i}{a} \right) + \frac{1}{2} a \left(\frac{v_f - v_i}{a} \right)^2 \end{aligned}$$

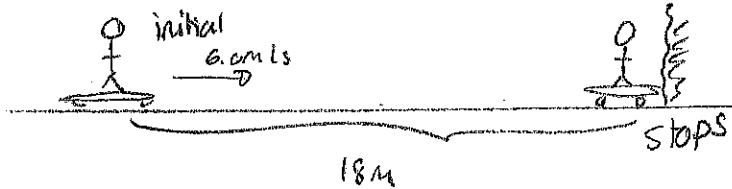
$$\Rightarrow a \Delta x = v_i (v_f - v_i) + \frac{1}{2} (v_f - v_i)^2$$

$$\Rightarrow 2a \Delta x = 2v_i v_f - 2v_i^2 + v_f^2 + v_i^2 - 2v_f v_i = v_f^2 - v_i^2 \quad \checkmark$$

49 Avoid the wall!

A skateboarder slides toward a wall. Initially the skateboarder is 18 m left of the wall and moving with speed 6.0 m/s to the right. The aim of this exercise will be to determine the minimum acceleration to barely avoid hitting the wall. (131F2024)

- a) Sketch the situation, illustrating the skateboarder at the initial instant and the instant just before reaching the wall.



List all relevant variables for the two instants:

$$\begin{array}{ll} t_i = 0 \text{ s} & t_f = \\ x_i = 0 \text{ m} & x_f = 18 \text{ m} \\ v_i = 6.0 \text{ m/s} & v_f = 0 \text{ m/s} \end{array}$$

- b) Determine the acceleration by selecting one of the kinematic equations, substituting and solving for a .

$$v_f^2 = v_i^2 + 2a\Delta x$$

$$\frac{v_f^2 - v_i^2}{2\Delta x} = a$$

$$a = \frac{(0 \text{ m/s})^2 - (6.0 \text{ m/s})^2}{2 \times 18 \text{ m}}$$

$$= \frac{-36 \text{ m}^2/\text{s}^2}{36 \text{ m}} \Rightarrow a = -1.0 \text{ m/s}^2$$

- c) Use one of the kinematic equations to determine the time that it takes for the skateboarder to reach the wall.

$$v_f = v_i + a\Delta t \Rightarrow \frac{v_f - v_i}{a} = \Delta t \Rightarrow \Delta t = \frac{0 \text{ m/s} - 6.0 \text{ m/s}}{-1.0 \text{ m/s}^2}$$

$$\Rightarrow \Delta t = 6.0 \text{ s}$$

- d) Would the equation

$$v = \frac{\Delta x}{\Delta t} \Rightarrow 6.0 \text{ m/s} = \frac{18 \text{ m}}{\Delta t} \quad \text{not, it would give } \Delta t = 3.0 \text{ s}$$

allow one to find the time taken to reach the wall correctly? Why or why not?

- e) Set up the moving man animation at:

<http://phet.colorado.edu/en/simulation/moving-man>

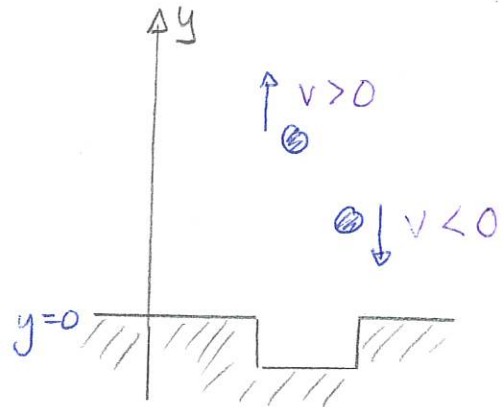
and run this to check your prediction. In order to verify that you have done this, use the animation to provide the times at which the man is 10 m to the left of the wall.

Vertical motion

Although we described kinematics for horizontal motion, the same system applies for vertical motion with these modifications:

- 1) position variable: y
- 2) velocity $v > 0 \Rightarrow$ moves up
 $v < 0 \Rightarrow$ moves down

The kinematic equations then apply with x replaced by y



Free fall

An example of vertical motion is free fall

Free fall motion \Rightarrow vertical motion (up or down) only under the influence of Earth's gravity.

Longstanding questions about this are:

- 1) Does the motion depend on the mass of the object?
- 2) Does the object accelerate constantly or not?
- 3) Does the acceleration depend on the state of motion (up/down / faster/slower)?

DEMO: Free fall demo