Fundamental Mechanics: Class Exam 3

8 November 2024

Name: Solution **Total:** /70

Instructions

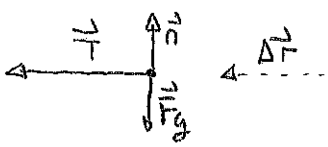
- There are 8 questions on 6 pages.
- Show your reasoning and calculations and always explain your answers.

Physical constants and useful formulae

$$g = 9.80\,\mathrm{m/s^2}$$

Question 1

A 2.0 kg box is initially at rest on a horizontal frictionless floor. Subsequently a rope pulls horizontally to the left with a constant force of 27 N and the box moves 3.0 m. Determine the work done by the rope, the work done by gravity and the work done by the normal force. Use these to determine the speed of the box after it has moved 3.0 m from its starting point.



In all cases $W = F\Delta r\cos\theta$

Rope: $W = T\Delta r\cos 0^\circ = 27\,\mathrm{N} \times 3.0\,\mathrm{m} = 81\,\mathrm{J}$

Normal: $W = n\Delta r\cos 90^\circ = 0\,\mathrm{J}$

Gravity: $W = F_g\Delta r\cos 90^\circ = 0\,\mathrm{J}$

$W_{net} = \Delta K \Rightarrow K_f - K_i = W_{net}$

$\frac{1}{2}mv_f^2 - \cancel{\frac{1}{2}mv_i^2} = W_{rope} = 81\,\mathrm{J}$

$\frac{1}{2} \times 2.0\,\mathrm{kg}\ v_f^2 = 81\,\mathrm{J}$

$v_f^2 = 81\,\mathrm{m^2/s^2}$

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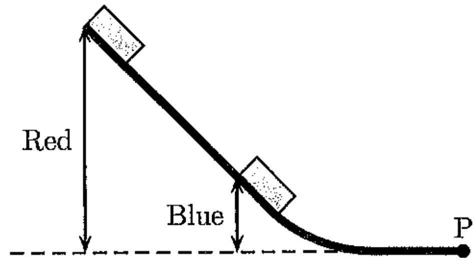
$v_f = \sqrt{81\,\mathrm{m^2/s^2}} \Rightarrow v_f = 9.0\,\mathrm{m/s}$

Question 2

Two identical blocks are released from rest on the same frictionless track. One of the blocks, which is red, is released from a point nine times as high as the other, which is blue. Let v_{red} be the speed of the red block when it reaches point P and v_{blue} be the speed of the blue block at the same point. Determine the ratio of these speeds $\frac{v_{\text{red}}}{v_{\text{blue}}}$. Ignore air resistance.



In both cases $\Delta E = 0 \Rightarrow E_f = E_i$

$\Rightarrow K_f + U_{gf} = K_i + U_{gi}$

$\Rightarrow \frac{1}{2}\cancel{m}v_f^2 + \cancel{m g y_f}^{0} = \cancel{\frac{1}{2}m v_i^2}^{0} + \cancel{m}g y_i$

$\Rightarrow \frac{1}{2}v_f^2 = g y_i$

$\Rightarrow v_f = \sqrt{2 g y_i}$

$\Rightarrow V_{red} = \sqrt{2g\, 9\, y_{i\,blue}}$

$= 3\underbrace{\sqrt{2g\, y_{i\,blue}}}_{V_{blue}}$

$\Rightarrow \frac{V_{red}}{V_{blue}} = 3$

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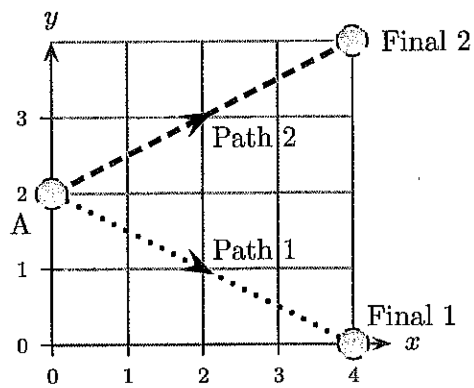
Question 3

Particles move along a horizontal surface. Both particles are initially at location A. Particle 1 moves along path 1. Particle 2 moves along path 2 where the axes are measured in meters. The same constant force, $\vec{F} = 15\,\text{N}\hat{\imath} - 30.0\,\text{N}\hat{\jmath}$ acts on both particles. Which of the following (choose one) is true of the work done by the force as each particles moves from point A to its final location?



i) The work done is zero for both particles.

ii) The work done is the same for both particles but is non-zero.

iii) The work done on particle 1 is smaller than that done on particle 2.

iv) The work done on particle 1 is larger than that done on particle 2.

$W = \vec{F}\cdot\Delta\vec{r}$ Path 1 $\Delta\vec{r} = 4.0\text{m}\hat{\imath} - 2.0\text{m}\hat{\jmath}$ /4

$\vec{F}\cdot\Delta\vec{r} = 4.0\times15\,\text{J} - 30(-2)\,\text{J} = 120\,\text{J}$

Path 2 $\Delta\vec{r} = 4.0\text{m}\hat{\imath} + 2.0\text{m}\hat{\jmath} \Rightarrow \vec{F}\cdot\Delta\vec{r} = 4.0\times15\,\text{J} - 30.0\times2.0\,\text{J}$

$= 0\,\text{J}.$

Question 4

A 0.50 kg box is launched at left end of the illustrated track with speed 5.0 m/s. A 10 m long section of the lower horizontal part of the track is rough and the coefficient of kinetic friction is 0.30. The rest of the track is frictionless and air resistance can be ignored. Determine the maximum height that the box reaches on the sloped section of track on the right.

$y_i = 2.5\,m$ $y_f = ?$

$v_i = 5.0\,m/s$ $v_f = 0\,m/s$

$\Delta E = W_{nc}$

$\Rightarrow E_f = E_i + W_{nc}$ +1

$K_f + U_{gf} = K_i + U_{gi} + W_{nc}$

$\cancel{\tfrac{1}{2} m v_f^2} + m g y_f = \tfrac{1}{2} m v_i^2 + m g y_i + W_{nc}$ +1 +1

$\cancel{m} g y_f = \tfrac{1}{2} \cancel{m} v_i^2 + \cancel{m} g y_i - \mu_k \cancel{m} g\, \Delta r$ +1

$y_f = \frac{1}{g}\left[\tfrac{1}{2} v_i^2 + g y_i - \mu_k g \Delta r\right]$

$= \frac{v_i^2}{2g} + y_i - \mu_k \Delta r$ +3

Rest +3

$= \frac{(5.0\,m/s)^2}{2 \times 9.8\,m/s^2} + 2.5\,m - 0.30 \times 10$

$y_f = 0.76\,m$

+10

$\boxed{10}$

$W_{nc} = W_{fric} + \cancel{W_{normal}}$

$= \underbrace{f_k}_{+1} \underbrace{\Delta r \cos 180°}_{+1}$

$= \mu_k m g \Delta r (-1)$

$\Sigma F_{iy} = 0 \Rightarrow n = mg$ +1

$= -\mu_k m g \Delta r$

$f_k = \mu_k n = \mu_k m g$ +1

$= -0.30 \times 0.50\,kg \times 9.8\,m/s \times 10\,m$ +2

$= -14.7\,J$

+3

$\boxed{6}$

~~Misses friction com~~

Misses friction completely −7

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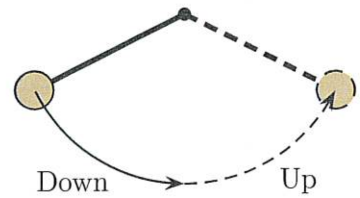
Question 5

A ball at the end of a string swings in a vertical plane in a circular arc. The ball swings down from the left and then up to the right. The highest points at the two ends of the arc are illustrated. Let $W_{\text{string down}}$ be the work done by the string as the ball swings down from the left high point to the lowest point. Let $W_{\text{string up}}$ be the work done by the string as the ball swings up from the lowest point to the right high point. Which of the following (choose one) is true?



i) $W_{\text{string down}} > 0$ and $W_{\text{string up}} > 0$.

ii) $W_{\text{string down}} > 0$ and $W_{\text{string up}} < 0$.

iii) $W_{\text{string down}} < 0$ and $W_{\text{string up}} > 0$.

iv) $W_{\text{string down}} < 0$ and $W_{\text{string up}} < 0$.

v) $W_{\text{string down}} = 0$ and $W_{\text{string up}} = 0$.

In both cases

$W = \vec{F} \cdot \Delta\vec{r}$

$= 0$

90°

$\vec{T}$

$\Delta\vec{r}$

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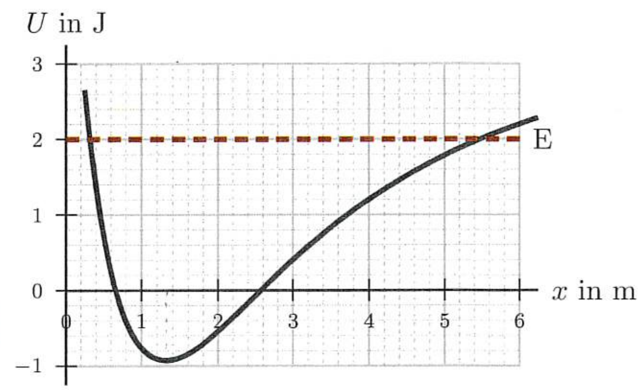
Question 6

A particle can move along the x axis and is subjected to the illustrated potential energy, U. The total energy of the particle is 2.0 J.



a) Describe the approximate location(s) where the force on the particle is zero. Explain your answer.

$F_x = -\frac{dU}{dx} = 0 \Rightarrow$ slope $= 0$] +2

Approx 1.3 m +1

b) Describe the approximate location(s) where the speed of the particle is zero. Explain your answer.

$K + U = E$ need $K = 0$ $\Rightarrow U = E$] +1

Approx 0.3 m and 5.5 m

+1 +1

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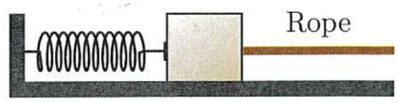
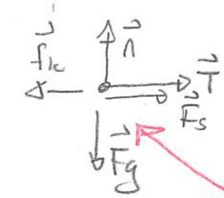
$y_i = 0\,m$ $\quad v_i = 0\,m/s$ $\quad \Delta s_i = 0.22\,m$

$y_f = 0\,m$ $\quad v_f = 0\,m/s$ $\quad \Delta s_f = 0\,m$

Question 7

A 4.00 kg box is held at rest against a spring, with spring constant 240 N/m. The spring is initially compressed by 0.22 m. The box is then released, and moves right while a rope pulls horizontally to the right with a constant tension of 20 N. At the same time, a constant kinetic friction force acts on the box; the coefficient of friction is 0.893. The box leaves the spring and continues moving right, eventually slowing to a stop. Determine the distance that the box travels from the moment of release until it stops.





$\Delta E = W_{nc} \Rightarrow E_f = E_i + W_{nc}$ (+1)

$W_{nc} = W_{fric} + W_{rope} + \cancel{W_{normal}}$

$W_{rope} = \overbrace{T\Delta r\cos 0^\circ}^{(+1)} = T\Delta r$ (+1)

$W_{fric} = f_k \Delta r \cos 180^\circ = -\mu_k m g\,\Delta r$ (+1)

$\sum F_{iy} = ma_y = 0 \Rightarrow n = mg$ (+1)

$f_k = \mu_k n = \mu_k m g$ (+1)

$(= 0.893 \times 4.00\,kg \times 9.80\,m/s^2 = 35.0\,N)$ (+1)

$E_f = E_i + W_{nc}$

$K_f + U_{gf} + U_{spf} = K_i + U_{gi} + U_{spi} + W_{nc}$

(+1) OR (+1)

$\cancel{\frac{1}{2}mv_f^2} + \cancel{mgy_f} + \cancel{\frac{1}{2}k\Delta s_f^2} = \cancel{\frac{1}{2}mv_i^2} + \cancel{mgy_i} + \frac{1}{2}k\Delta s_i^2 - \mu_k m g\,\Delta r + T\Delta r$

$$\left.\begin{aligned}
0 &= \tfrac{1}{2}k\,\Delta s_i^2 + \Delta r(T - \mu_k m g)\\
\Delta r\,(\mu_k m g - T) &= \tfrac{1}{2}k\,\Delta s_i^2\\
\Delta r &= \frac{k\,\Delta s_i^2}{2(\mu_k m g - T)} = \frac{240\,N/m \times (0.22\,m)^2}{2(35.0\,N - 20\,N)}
\end{aligned}\right\} +5$$

$\Delta r = 0.39\,m$

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Question 8

Two ice skaters, Zog and Geraldine, hold each other while skating right at a constant velocity of 6.0 m/s. Zog has mass 90 kg and Geraldine has mass 60 kg. At one moment Zog pushes Geraldine and after that she moves left with a constant velocity of 5.0 m/s.

a) Determine the velocity (including direction) of Zog after they separate.

$V_{Gi} = 6.0 m/s$ $\qquad$ $V_{Gf} = -5.0 m/s$

$V_{Zi} = 6.0 m/s$ $\qquad$ $V_{Zf} = ?$ $\quad$ +2

$p_{tot f} = p_{tot i}$ +1

+1

$m_G V_{Gf} + M_Z V_{Zf} = M_G V_{Gi} + M_Z V_{Zi}$

$\Rightarrow \quad 60kg(-5.0m/s) + 90kg\, V_{Zf} = (M_G + M_Z) V_i = 150kg \times 6.0m/s$

$\Rightarrow \quad -300 kgm/s + 90kg\, V_{Zf} = +900 kgm/s$ $\quad$ +2

$\Rightarrow \quad 90kg\, V_{Zf} = 1200kg\, m/s \Rightarrow V_{Zf} = 13.3m/s$

b) Is the total mechanical energy, $E = K + U_{\text{grav}}$, conserved during the process of separation? Explain your answer.

Before $\quad E_i = K_i = \frac{1}{2}(m_Z + m_G) v_i^2 = \frac{1}{2} 150kg\,(6.0m/s)^2 = 2700J$

After $\quad E_f = K_{fZ} + K_{fG} = \frac{1}{2} m_G V_{Gf}^2 + \frac{1}{2} M_Z V_{Zf}^2$

$= \frac{1}{2} 60 \times (-5)^2 + \frac{1}{2} 90 \times (13.3)^2$

$= 8710J$

actual calculation +3

It is not conserved

+1

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