

Electromagnetic Theory II: Homework 9

Due: 21 February 2025

1 Energy flow for uniform fields

Various sources produce the electric field

$$\mathbf{E} = E_0 \hat{\mathbf{y}}$$

and the magnetic field

$$\mathbf{B} = B_0 \hat{\mathbf{x}}$$

where $E_0, B_0 > 0$. In the following consider a square “window” with sides of length L .

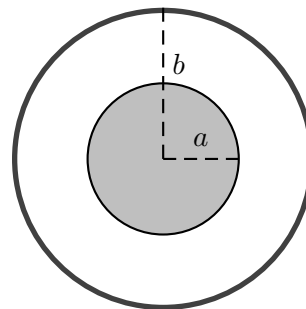
- Suppose that the window lies in the xy plane. Determine the rate at which energy flows through it.
- Suppose that the window lies in the a plane that makes a 45° angle between the both xy and the xz planes. Determine the rate at which energy flows through it.
- Suppose that the window lies in the xz plane. Determine the rate at which energy flows through it.

Now consider a hemisphere of radius R and for which $z > 0$ and which has a flat base in the xy plane.

- Determine the rate at which energy flows through the curved surface of the hemisphere.
- Determine the rate at which energy flows through the base of the hemisphere.
- At what rate does energy accumulate in the hemisphere?

2 Moving charged coaxial cylinders

A very long coaxial cylinder arrangement consists of an interior conductor with radius a and an exterior cylindrical shell with radius b . The interior cylinder carries fixed uniform surface charge density $\sigma_a > 0$ and the exterior cylinder fixed surface charge density $\sigma_b < 0$. These are such that in any section along the length of the arrangement, the total charge is zero. The cylinder is dragged with constant velocity parallel to its axis and this does not change the charge distribution.



- Describe regions where energy flows and the direction in which it flows.
- Describe whether the potential energy stored in any region of space changes with time.

- c) Given any region of space, does it appear that the rate at which the energy flows into the region is zero or non-zero?

3 Rotating cylinder

A very long cylinder, with axis oriented along $\hat{\mathbf{z}}$ has uniform volume charge density. The cylinder rotates about the axis with angular velocity $\boldsymbol{\omega} = \omega\hat{\mathbf{z}}$ where $\omega > 0$ is a constant. Which of the following is true about the Poynting vector?

- i) $\mathbf{S} = 0$ everywhere.
- ii) $\mathbf{S} = 0$ inside; \mathbf{S} points along $\hat{\boldsymbol{\phi}}$ outside.
- iii) $\mathbf{S} = 0$ inside; \mathbf{S} points along $\hat{\mathbf{s}}$ outside.
- iv) $\mathbf{S} = 0$ inside; \mathbf{S} points along $\hat{\mathbf{z}}$ outside.
- v) \mathbf{S} points along $\hat{\boldsymbol{\phi}}$ inside; $\mathbf{S} = 0$ outside.
- vi) \mathbf{S} points along $\hat{\mathbf{s}}$ inside; $\mathbf{S} = 0$ outside.
- vii) \mathbf{S} points along $\hat{\mathbf{z}}$ inside; $\mathbf{S} = 0$ outside.