

Electromagnetic Theory II: Final Exam

14 December 2016

Name: _____

Total: /75

Instructions

- There are 7 questions on 14 pages.
- Show your reasoning and calculations and always explain your answers.

For maximum credit you need to complete five out of the seven questions. If you attempt all seven questions you will be given the five highest scores that you attain on individual questions.

Physical constants and useful formulae

Permittivity of free space	$\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/\text{Nm}^2$
Permeability of free space	$\mu_0 = 4\pi \times 10^{-7} \text{ N/A}^2$
Charge of an electron	$e = -1.60 \times 10^{-19} \text{ C}$
Charge of a proton	$e = +1.60 \times 10^{-19} \text{ C}$
Speed of light	$c = 3.0 \times 10^8 \text{ m/s}$

Question 1

A particular charge distribution produces potentials (in spherical coordinates)

$$V = 0$$
$$\mathbf{A} = \frac{A_0}{r} \sin(\omega t) \hat{\phi}$$

where A_0 is a constant.

- a) Determine the electric and magnetic fields that this produces.

Question 1 continued ...

b) Determine the charge distributions that produce these fields.

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Question 2

Consider the potentials, given in cylindrical coordinates as

$$V = -\alpha \ln s$$
$$\mathbf{A} = \frac{\alpha t}{s} \hat{\mathbf{s}} + \beta \sin(\omega t) \hat{\boldsymbol{\phi}}$$

where α and β are constants.

a) Show that this potential is in the Coulomb gauge.

b) Let

$$\lambda = -\alpha t \ln s$$

generate a gauge transformation. Determine the potentials in this new gauge. Is this in either the Lorentz or the Coulomb gauge?

Question 2 continued ...

Question 3

An infinitely long wire lies along the z axis and carries current

$$I = \begin{cases} 0 & \text{if } t < 0 \\ I_0 e^{t/\tau} & \text{if } t > 0 \end{cases}$$

where $\tau > 0$ is a constant with units of time. The charge density along the wire is always zero.

- a) Suppose that the potentials (and eventually fields) are to be determined at a point in the xy plane a distance s from the wire. Describe, in terms of s and t , which portions of the current in the wire contribute to the vector potential at this location at time $t > 0$. Use these to *set up* an integral, in terms of s, t and other variables for the field (*you do not need to evaluate the integral*).

Question 3 continued ...

b) Explain *how* you could determine the electric field produced by the current.

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Question 4

An electric dipole in the xy plane oscillates in such a way that the dipole moment is

$$\mathbf{p} = p_0 [\cos(\omega t)\hat{\mathbf{x}} + \sin(\omega t)\hat{\mathbf{y}}]$$

where p_0 is the magnitude of the dipole moment and ω is a constant frequency.

- a) Determine expressions for the radiation electric and magnetic fields at any point along the z axis.

Question 4 continued ...

b) Determine an expression for the total power radiated by the dipole (in all directions).

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Question 5

Consider a classical electron, with mass m , that orbits around a fixed proton in a circular orbit with radius r .

- a) Let U be the total energy of the particle. Show that the rate at which the energy changes is

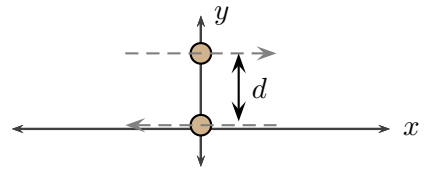
$$\frac{dU}{dt} = -\frac{\mu_0 e^6}{96c\pi^3\epsilon_0^2 m^2} \frac{1}{r^4}$$

- b) Explain *how* you could use this to determine the lifetime of a classical atom.

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Question 6

Two identical charged particles, each with charge $+q$, move with the same constant speeds, v in opposite directions along lines parallel to the x axis. At one instant the charges are situated as illustrated and the questions below refer to this instant.



- a) Determine the fields that the charge at the origin produces at the location of the other charge.

Question 6 continued ...

b) Determine the force exerted by the charge at the origin on the other charge.

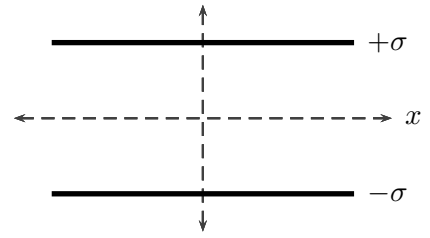
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Question 7

The electric field produced by a uniformly charged infinite sheet of *stationary charge* is

$$\mathbf{E} = \frac{\sigma}{2\epsilon_0} \hat{\mathbf{n}}$$

where $\hat{\mathbf{n}}$ is a normal vector perpendicular to and away from the sheet and σ is the charge per unit area. Consider two parallel sheets, where the upper sheet has charge density σ and the lower sheet has charge density $-\sigma$. Both sheets *move* with constant speed v along the $+x$ axis.



- a) Determine the electric and magnetic fields in the rest frame of the two sheets.

Question 7 continued ...

- b) Determine the electric and magnetic fields in the frame in which the sheets move with speed v along the $+x$ axis.

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