Phys 131: Exercises

One-Dimensional Kinematics

1 Motion diagrams: car slowing and accelerating

A car moves to the right. Earlier in its motion it slows down and later it speeds up. Which of the following (choose one) best represents its location as time passes? (131Sp2025)



Briefly explain your choice.

2 Drone motion diagram

A drone (unmanned aircraft) hovers at one height above the ground. The drone is subsequently allowed to descend in a straight line toward the ground. It initially descends at an increasing rate, then at a steady rate and, a few feet before the ground, the descent rate decreases. Draw a motion diagram for the drone while it descends. (131Sp2025)

3 Motion diagrams: racing carts

Motion diagrams for two carts are as illustrated. (131Sp2025) Blue



- a) Is the speed of the red cart the same as or different to that of the blue cart? Explain your answer.
- b) Is the velocity of the red cart the same as or different to that of the blue cart? Explain your answer.

4 Motion diagrams and average velocity

A car moves from left to right and $0\,\mathrm{s}$ $5.0\,\mathrm{s}$ $10.0\,\mathrm{s}$ $15.0\,\mathrm{s}$ its position is recorded every 5.0 s. 50m10m20m30m40m 60m70m80m90mThe resulting motion diagram is illustrated. (131Sp2025)

- a) Determine the average velocity from 0s to 5.0s.
- b) Determine the average velocity from 5.0 s to 10.0 s.
- c) Determine the average velocity from 5.0 s to 15.0 s.
- d) Determine the average velocity from 0s to 15.0 s. Is this the average of the two velocities in the interval $0 s \rightarrow 5.0$ s and $5.0 s \rightarrow 15.0$ s?

5 Motion diagrams, speed and average velocity



6 Katie Ledecky splits

Katie Ledecky, one of the greatest female swimmers, won the women's 800 m freestyle race at the 2023 World Aquatics Championships. In the final, she covered the first 200 m in 1:58.29 (means 1 min and 58.29 s), the second 200 m in 2:03.63, the third 200 m in 2:04.00 and the final 200 m in 2:01.50. Determine her average speed for each 200 m section and also the entire race. Which section most resembled her speed for the entire race? (131Sp2025)

7 Bouncing hockey puck

A hockey puck can slide along a horizontal surface. Starting at an initial location, it slides right and takes 3.0 s to hit a board 18 m away. It bounces back to the left and reaches a final point 4.0 m from the board 2.0 s from the time it first hit the board. Determine the average velocity of the puck from the initial to final moments. (131Sp2025)

3

8 Red car, blue car, green car

Determine the average speed and the average velocity for cars moving in a straight line as follows:

- a) Red car moves right for 50 m in 10.0 s.
- b) Blue car moves right for 16 m in 8.0 s and then continues right for 34 m in 2.0 s.
- c) Red car moves right for 30 m in 5.0 s reverses and moves left for 20 m in 5.0 s. (131Sp2025)

9 Man versus dog

The following objects lie along a straight line: a bicycle, a coffee cup and a soccer ball. The distance from the coffee cup to the bicycle is 400 m and from the cup to the ball is 500 m. A man starts at the cup and travels in a straight line to the ball. This takes 200 s. A dog is initially at the cup and runs at constant speed to the bicycle, taking 50 s to do so. The dog immediately turns around and runs to ball; this takes the dog an additional 150 s. Consider the entire trip from the cup to the ball for each. Who has the larger average velocity for this entire trip? Explain your answer. (131Sp2025)



10 Displacement and average velocity

Various people move as described below over a total interval of 40 s. In each case determine the total displacement and the average velocity over the 40 s period. (131Sp2025)

- a) Anna takes a trip in two stages. In the first stage, lasting 20 s, she moves 30 m to the right. In the second stage, lasting 20 s, she moves 10 m to the left.
- b) Bill takes a trip in two stages. In the first stage, lasting 10 s, he moves 10 m to the left. In the second stage, lasting 30 s, he moves 30 m to the right.
- c) Explain whether displacement and average velocity capture all the information about the motion of each. If not, how might one modify the description of the motion in terms of displacement and velocity to describe the motion more accurately?

11 Average velocity, speed for motion with changing directions

- a) A person takes a trip, first walking 30 m to the right in 5.0 s and then returning to his starting point in another 10 s. Determine the average velocity of the person for the entire trip.
- b) Anna takes a trip in two stages. First she moves 100 m right in 40 s. She briefly stops and then she moves 120 m to the left in another 60 s. Determine her average velocity for each stage of the trip and also for the entire trip.

c) Bill takes a trip in two stages. First he moves 200 m left in 100 s. He briefly stops and then he moves 150 m to the right in another 100 s. Determine his average velocity for each stage of the trip and also for the entire trip. (131Sp2025)

12 Average velocity and direction of motion

- a) An object moves in such a way that during a certain period the average velocity of the object is negative. Is it possible that at the end of the period the object is to the right of the origin of the coordinate system/axis? Explain your answer.
- b) An object moves in such a way that over a certain period the average velocity of the object is positive. Is it possible that at some point during the period the object moves to the left? Explain your answer. (131Sp2025)

13 Average velocity and displacement down a field

Three people, Alice, Bob and Charlie leave one end of a 100 m long field at the same instant and travel in the same direction toward the other end. (131Sp2025)

- a) Alice travels at a constant speed of 8.0 m/s for half the distance and then at 4.0 m/s for the remaining half. Determine Alice's average velocity for the entire trip.
- b) Bob travels at a constant speed of 10.0 m/s for half the distance and then at 2.0 m/s for the remaining half. Determine Bob's average velocity for the entire trip.
- c) Charlie travels for half of the time at a constant speed of 8.0 m/s for the remaining half of the time at a constant speed of 4.0 m/s. Determine Charlie's average velocity for the entire trip.
- d) Determine the order in which they arrive at the other end of the field.

14 El Capitan climb

The rock climbing route called "The Nose" ascends the El Capitan formation in Yosemite National Park. The elevation gain on the climb is 2900 ft and the quickest ascent took 1 hr, 58 min, 7 s. Determine the average speed for this ascent in m/s. (131Sp2025)

15 Average speed of the Moon

The Moon orbits the Earth in an ellipse with average radius 239,000 miles. It completes one orbit in 27 days, 7 hrs, 43 min, 11.5 s. Suppose that it orbits in a perfect circle with radius 239,000 miles. (131Sp2025)

- a) Determine the average speed of the Moon in mph.
- b) Determine the average speed of the Moon in m/s.

16 Running speed

Estimate the speed with which you could run a distance of about 100 yards. Convert your answer to standard metric units. (131Sp2025)

17 Grass growing

Estimate the speed with typical lawn grass that one might find on campus grows. Express your answer in standard metric units. Describe all of the steps that give your estimate. (131 Sp 2025)

18 Motion diagrams and position vs. time graphs, 1

A car moves from left to right and its position, measured in meters, is 10m 20m 30m40m 50m60m 70m 80m The resultrecorded every 5.0 s. ing motion diagram is illustrated. (131Sp2025)

- a) Produce a table of numerical data for position versus time for the car for the duration of the motion.
- b) Produce a position versus time graph for the car for the duration of the motion. This graph must be drawn by hand using axes that are clearly labeled.

19 Motion diagrams and position vs. time graphs, 2

A car moves from left to right and its position, measured in meters, is recorded every 5.0 s. The resulting motion diagram is illustrated. (131Sp2025)



90m

- a) Produce a table of numerical data for position versus time for the car for the duration of the motion.
- b) Produce a position versus time graph for the car for the duration of the motion. This graph must be drawn by hand using axes that are clearly labeled.

20 Hockey puck trip time

A hockey puck travels along a horizontal surface to the right for a distance of 12 m at speed $5.0 \,\mathrm{m/s}$. It hits a board and bounces to the left traveling a distance of $20 \,\mathrm{m}$ at speed $4.0 \,\mathrm{m/s}$. Determine the total time for this trip. (131Sp2025)

21 Angry ant on a stick

An angry ant walks along a straight stick. The graph illustrates the ant's position vs. time. (131Sp2025)

- a) Describe in words qualitatively how the ant moves during the period from 0s to 5s. Include as much detail without using any numbers.
- b) During which period is the ant's speed largest? During which period is it smallest? Explain your answers.



$\begin{array}{c} x \\ 4.0m \\ 3.0m \\ 2.0m \\ 1.0m \\ 0s \\ 1.0s \\ 2.0s \\ 3.0s \\ 4.0s \\ 5.0s \\ 6.0s \\ 5.0s \\ 6.0s \\ t \end{array}$

22 Slug on a stick

A slippery slug crawls along a straight stick. The graph illustrates the slug's position vs. time. Answer the following, giving explanations for each answer. (131Sp2025)

- a) Determine the velocity of the slug at 1.0 s.
- b) Determine the velocity of the slug at 4.0 s.

23 Ant and bug on a stick

An ant and a bug walk along a straight stick. The graph illustrates their position vs. time. Answer the following, explaining each answer. (131Sp2025)

- a) When and where, if ever, are the ant and the bug at the same location?
- b) When, if ever, do the ant and the bug have the same speed?
- c) When, if ever, does the ant move faster than the bug?



24 Red ant on a stick

A red ant walks along a straight stick. The graph illustrates the ant's position vs. time. Answer the following, giving explanations for each answer. (131Sp2025)

- a) During which times is the ant moving right? 2.0m During which times is it moving left?
- b) When, if ever, is the velocity of the ant 0 m/s?
- c) How does the speed of the ant at 1.0 s compare to its speed at 4.0 s?
- d) How does the velocity of the ant at 1.0s compare to its velocity at 4.0s?

25 Timid tick on a stick

A timid tick walks along a straight stick. The graph illustrates the tick's position vs. time. In each of the following explain your answers. (131Sp2025)

- a) Determine the velocity of the tick at 0.5 s.
- b) Determine the velocity of the tick at 1.5 s.
- c) Determine the velocity of the tick at 3.0 s.
- d) Determine the velocity of the tick at 4.5 s.



A bug walks along a straight stick. The graph illustrates the bug's position vs. time. In each of the following, explain your answers. (131Sp2025)

- a) Does the bug ever return to the position that it occupied initially (t = 0 s)? If so, when?
- b) Does the bug ever reverse direction? If so when?
- c) Does the bug ever have zero velocity? If so, when?







27 Ant and bug on a stick: speeds

An ant and a bug walk along straight sticks. The solid graph illustrates the ant's position vs. time. The ⁴ dashed graph indicates the bug's position vs. time. Answer the following, giving explanations for each answer. ³ (131Sp2025)

- a) At what time(s) are the ant and bug at the same location?
- b) Which is moving faster at 2s?
- c) Do the ant and bug ever have the same velocity? If so when?

28 Graceful ladybug on a stick

A ladybug insect walks gracefully back and forth along a straight stick. A graph of position vs. time for its motion $^{2.0}$ is illustrated with several instants labeled A, B, C and D. Explain your answers for the following. (131Sp2025) 0

- a) At which of these instants is the ladybug moving right?
- b) At which of these instants is the ladybug slowing down?
- c) Rank the moments in order of increasing instantaneous speed.
- d) Sketch a graph of velocity versus time for the ladybug.

29 Agitated cat on a wall

An increasingly agitated cat walks back and forth along a straight wall. A graph of position vs. time for its motion ² is illustrated with several instants labeled A, B, C and D. Explain your answers for the following. (131Sp2025)

- a) At which of these instants is the cat moving right?
- b) Rank the instants in order of increasing instantaneous speed.
- c) Describe an interval during which the cat slows down. Provide approximate initial and final times for this interval.
- d) Sketch a graph of velocity versus time for the cat from 0s to 3s.







30 Bead sliding along a wire

A bead slides along a straight piece of wire. After an initial moment it slides right with decreasing speed, coming to a brief stop. Immediately after this it slides right with increasing speed. Taking right as positive, which of the following represents a graph of *veloc-ity versus time* for the bead? Explain your choice. (131Sp2025)



31 Crawling slug

A slug crawls along a straight wire, starting at x = 0.0 m at t = 0.0 s. A graph of the slug's velocity versus time is illustrated. Use the graph to answer the following. (131Sp2025)

- a) Determine the displacement of the slug from $t = \frac{1}{2}$.
- b) How is the displacement of the slug from t = 0.0 s to t = 4.0 s related to the shaded area between the graph and the horizontal axis (v = 0.0 m/s)?
- c) Assuming that the answer to the previous question is true in general, determine the displacement of the slug from t = 4.0 s to t = 8.0 s.
- d) Is the displacement of the slug from t = 8.0 s to t = 10.0 s positive or negative? How might this relate to the shaded area from t = 8.0 s to t = 10.0 s?

32 Stick insect on a stick

A stick insect walks back and forth along a straight stick. A graph of velocity vs. time for its motion is 10 illustrated. (131Sp2025)

- a) Determine the displacement of the stick insect from t = 0 s to t = 2 s.
- b) Determine the displacement of the stick insect from t = 2 s to t = 4 s.
- c) Determine the displacement of the stick insect $_{\text{-10.0m/s}}$ from $t=0\,\mathrm{s}$ to $t=4\,\mathrm{s}.$





33 Ladybug on a stick

A ladybug insect walks back and forth along a straight stick. A graph of velocity vs. time for its motion is illustrated. The ladybug is initially at x = -1.0 m. (131Sp2025)

- a) Determine the position of the ladybug at t = 1 s.
- b) Determine the position of the ladybug at t = 3 s.
- c) Determine the position of the ladybug at t = 4 s.
- d) Determine the position of the lady bug at $t=5\,{\rm s.}^{-4.0{\rm m/s}}$
- e) Describe in words how the position of the ladybug evolves with time during the illustrated interval.

34 Slithering worm

A worm is initially at rest and subsequently slithers along a straight wire. The worm's velocity increases steadily from 0.000 m/s to 0.015 m/s over a period of 120 s. Determine the distance traveled by the worm during this period. *Hint: A graph will help!* (131Sp2025)

35 Instantaneous velocity as a limit

An atom is trapped in such a way that it can move back and forth along one straight line. Its position is tracked as time passes and is represented by the function $x = t^2 - 2t - 1$ (the coefficients all have units such that x is in units of meters and t in seconds). (131Sp2025)

- a) Produce a list of positions at every 0.5 s from t = 0.0 s to t = 4.0 s. Use this data to plot an accurate graph of position versus time for t = 0.0 s $\leq t \leq 4.0$ s. The graph must be drawn accurately enough to draw and calculate slopes of tangent lines.
- b) Determine the velocity of the atom at 3.0 s by using a tangent line construction for the graph of position versus time.

The aim of the next parts of this problem is to determine the instantaneous velocity at 3.0 s.

- c) Use the function of position versus time to determine the average velocity over the time interval from t = 3.0 s to t = 3.1 s.
- d) Use the function of position versus time to determine the average velocity over the time interval from t = 3.00 s to t = 3.01 s.
- e) Does the value of average velocity at 3.0 s appear to approach a limit as the time interval decreases? If so what does the limit appear to be?
- f) Use the derivative of position to determine the exact instantaneous velocity at 3.0 s. Does the result agree with your answer to the previous part?
- g) At what time is the instantaneous velocity exactly zero? Explain your answer.



36 Velocity as a derivative, 1

Suppose that the position of an object is

$$x = (5 \text{ m/s}^2) t^2 + (3 \text{ m/s}) t$$

Determine the velocity of the object at t = 3 s. (131Sp2025)

37 Velocity as a derivative, 2

Suppose that the position of an object is

$$x = (0.25 \,\mathrm{m/s^3}) t^3 + (6 \,\mathrm{m/s}) t$$

Determine the velocity of the object at t = 4 s. (131Sp2025)

38 Velocity as a derivative, 3

Suppose that the position of an object is

$$x = (2 \text{ m/s}^3) t^3 + (6 \text{ m/s}^2) t^2$$

Determine the velocity of the object at t = 2 s. Is this the same as x/t? (131Sp2025)

39 Wandering ant

An ant walks along a straight stick. The graph illustrates the ant's velocity vs. time. Answer the following, giving explanations for each answer. (131Sp2025)

- a) How does the magnitude of the ant's acceleration at 1.5 s compare (larger, smaller, same) to its acceleration at 2.5 s?
- b) How does the magnitude of the ant's acceleration at 0.5 s compare (larger, smaller, same) to its acceleration at 2.5 s?
- c) How does the magnitude of the ant's acceleration at 0.5 s compare (larger, smaller, same) to its acceleration at 4.0 s?



40 Accelerating bug on a stick

A bug walks along a straight stick. The graph illustrates the bug's velocity vs. time. At t = 0 s the bug is at the x = 2.0 m mark. (131Sp2025)

- a) Determine the position, velocity and acceleration of the bug at 1.0 s.
- b) Determine the position, velocity and acceleration of the bug at 3.0 s.
- c) Determine the position, velocity and acceleration of the bug at 5.0 s.

41 Insects on sticks

An ant, a tick and a bug walk along straight sticks. The solid graph illustrates the ant's velocity vs. time. The dashed graph indicates the tick's velocity vs. time. The dotted line indicates the bug's velocity versus time. (131Sp2025)





- a) Which of the following is true during the period from 0s to 4s? Explain your answer.
 - i) The acceleration of the ant is the same as that of the tick.
 - ii) The acceleration of the tick is always larger than that of the ant.
 - iii) The acceleration of the tick is always smaller than that of the ant.
 - iv) The acceleration of the tick is sometimes smaller than that of the ant, sometimes larger.
- b) Which of the following is true during the period from 0s to 4s? Explain your answer.
 - i) The acceleration of the ant is the same as that of the bug.
 - ii) The acceleration of the bug is always larger than that of the ant.
 - iii) The acceleration of the bug is always smaller than that of the ant.
 - iv) The acceleration of the bug is sometimes smaller than that of the ant, sometimes larger.

42 Cyclist versus skateboarder

A skateboarder moves down a gentle slope. She passes a cyclist, who is at rest and at this moment the skater's speed is 4.0 m/s. At an instant 10s later the skater's speed is 10.0 m/s and the cyclist's is 8.0 m/s. During this period, who has the greater acceleration? Explain your answer. (131Sp2025)

43 Acceleration sign

A bicycle can move east (positive) or west (negative). (131Sp2025)

- a) If the bicycle moves east can the acceleration be negative? Explain your answer.
- b) If the bicycle moves west can the acceleration be positive? Explain your answer.

44 Ant and bug on sticks

An ant and a bug walk along straight sticks. The solid graph illustrates the ant's position vs. time. The dashed graph indicates the bug's position vs. time. (131Sp2025)

- a) For the bug and, separately, the ant, which of the following is true during the period from 0s to 4s? Explain your answers.
 - i) Velocity is zero at all times.
 - ii) Velocity is positive at all times.
 - iii) Velocity is negative at all times.
 - iv) Velocity is first positive and later negative.
 - v) Velocity is first negative and later positive.
- b) For the bug and, separately, the ant, which of the following is true during the period from 0s to 4s? Explain your answers.
 - i) Acceleration is zero at all times.
 - ii) Acceleration is positive at all times.
 - iii) Acceleration is negative at all times.
 - iv) Acceleration is first positive and later negative.
 - v) Acceleration is first negative and later positive.



45 Moving man, 1

A man in an animation walks along a horizontal line. Positions are marked in meters along the line with positive numbers to the right of the origin and negative to the left. His positions and velocities are recorded at equally spaced intervals in time. The data is:

Time in s	Position in m	Velocity in m/s
0.0	-5.00	-4.00
2.0	-7.00	2.00
4.0	3.00	8.00

- a) Determine the man's average acceleration from 0.0s to 2.0s.
- b) Determine the man's average acceleration from 2.0 s to 4.0 s. (131Sp2025)

46 Person walking left and right

A person walks along a horizontal line. Positions are marked in meters along the line with positive numbers to the right of the origin and negative to the left. His velocity is recorded at equally spaced intervals in time. The data is:

Time in s	Velocity in m/s
10.00	-1.00
10.10	-0.80
10.20	-0.60
10.30	-0.40
10.40	-0.20
10.50	0.00
10.60	0.20
10.70	0.40
10.80	0.60
10.90	0.80

- a) During which period is the person moving left? During which period is the person moving right?
- b) Determine the person's acceleration while moving left.
- c) Determine the person's acceleration while moving right.
- d) Does the person's acceleration appear to change during the period from 10.10 s to 10.90 s?
- e) At what moment does the person reverse direction? According to the data is the acceleration zero or not at this moment? (131Sp2025)

47 Bungee jumper

A bungee jumper falls downward stretching the cord, reaching a low point, after which the cord pulls him up again. His velocity is recorded at equally spaced intervals in time. The data is:

Time in s	Velocity in m/s
10.0	-20.0
10.5	-15.0
11.0	-10.0
11.5	-5.0
12.0	0.0
12.5	5.0
13.0	10.0
13.5	15.0
14.0	20.0

- a) During which period is the man falling? When is he rising?
- b) By how much does the man's velocity change per second? Is this change constant throughout the recorded motion?
- c) Determine the man's acceleration while he is falling and also while he is rising. Are these accelerations the same or not?
- d) What is the man's acceleration (according to the data) at his low point? (131Sp2025)

48 Moving man activity

Go to the moving man animation at:

http://phet.colorado.edu/en/simulation/moving-man

Run the moving man animation. Click on the charts tab. Set the position to 0.00 m, the velocity to -5.00 m/s and the acceleration to 2.00 m/s^2 . Run the animation, stopping it just before the man hits the wall. The animation will have recorded the motion. Check the playback button at the bottom. You can slide the light blue bar left and right to get data for the motion. Gray zoom icons at the right will let you rescale the charts. (131Sp2025)

- a) Consider the interval from 2.0 s to 3.0 s. Describe the motion verbally during this time.
- b) How does the speed of the man at 2.0s compare to that at 3.0s? Explain your answer.
- c) How does the velocity of the man at 2.0 s compare to that at 3.0 s? Explain your answer.
- d) Will the average acceleration over the interval from 2.0 s to 3.0 s be positive, negative or zero? Explain your answer.
- e) If the acceleration is not zero, does it vary during this interval? Explain your answer.
- f) Determine the average acceleration over the interval from 2.0 s to 3.0 s.

49 Person moving with constant acceleration

A person is initially at rest and subsequently moves right with a constant acceleration. The person's reaches speed 6.0 m/s at a point 9.0 m to the right of the starting location. The aim of this exercise will be to determine the time taken to reach this point. A first step will be to determine the acceleration of the person. (131Sp2025)

a) Sketch the situation, illustrating the person at the two instants described above. List all relevant variables for the two instants:

$t_{ m i} =$	$t_{\rm f} =$
$x_{\mathrm{i}} =$	$x_{\rm f} =$
$v_{\rm i} =$	$v_{\rm f} =$

- b) Determine the acceleration by selecting one of the kinematic equations, substituting and solving for a.
- c) Using a different kinematic equation, find the time that it takes the person to reach speed $6.0 \,\mathrm{m/s}$.
- d) Suppose that you had tried to find the time taken to reach speed $6.0 \,\mathrm{m/s}$ by using

$$v = \frac{\Delta x}{\Delta t} \Rightarrow 6.0 \,\mathrm{m/s} = \frac{9.0 \,\mathrm{m}}{\Delta t}.$$

What time does this give? Does it agree with the answer that you obtained to the previous part? Is it correct?

50 Avoid the wall!

A skateboarder slides toward a wall. Initially the skateboarder is 18 m left of the wall and moving with speed 6.0 m/s to the right. The aim of this exercise will be to determine the minimum acceleration to barely avoid hitting the wall. (131Sp2025)

a) Sketch the situation, illustrating the skateboarder at the initial instant and the instant just before reaching the wall.

List all relevant variables for the two instants:

$t_{ m i} =$	$t_{\rm f} =$
$x_{\rm i} =$	$x_{\rm f} =$
$v_{\rm i} =$	$v_{\rm f} =$

- b) Determine the acceleration by selecting one of the kinematic equations, substituting and solving for a.
- c) Use one of the kinematic equations to determine the time that it takes for the skateboarder to reach the wall.
- d) Would the equation

$$v = \frac{\Delta x}{\Delta t} \quad \Rightarrow \quad 6.0 \,\mathrm{m/s} = \frac{18 \,\mathrm{m}}{\Delta t}$$

allow one to find the time taken to reach the wall correctly? Why or why not?

e) Set up the moving man animation at:

http://phet.colorado.edu/en/simulation/moving-man

and run this to check your prediction. In order to verify that you have done this, use the animation to provide the times at which the man is 10 m to the left of the wall.

51 Braking car, 1

A car travels to the right with speed 30.0 m/s (about 67 mph). The car brakes and slows to a stop with constant acceleration. It does this in a distance of 80.0 m (about 260 ft). The aim of this exercise is to find the time taken to do this. (131Sp2025)

a) Sketch the situation, illustrating the car at two key instants.

List all relevant variables for the two instants:

b) Determine the acceleration using one of the kinematic equations. Write down the equation, substitute from your list of variables and solve for *a*. (is acceleration positive or negative?)

Answer: $-5.63 ms^2$

c) Using a different kinematic equation, find the time that it takes the car to stop. Write down the equation, substitute from your list of variables and solve for time.
 Answer: 5.33 s

52 Braking car, 2

A car travels to the right with speed 24.6 m/s (about 55 mph). The car brakes, coming to a stop in 50 m (about 165 ft). The acceleration during the braking process is constant. (131Sp2025)

- a) Determine the acceleration of the car while it brakes.
- b) Determine the time taken for the car to stop.

53 Accelerating aircraft

An aircraft accelerates from rest to a speed of 120.0 m/s. It does this with a constant acceleration of 4.00 m/s^2 . The aim of this exercise is to find the distance traveled by the aircraft during this process. (131Sp2025)

- a) Sketch the situation, illustrating the aircraft at two key instants. List all relevant variables for the two instants and list the acceleration.
- b) Determine the distance traveled by the aircraft using one of the kinematic equations. Write down the equation, substitute from your list of variables and solve for the relevant variable.

Answer: 1800 m

c) Using a different kinematic equation, find the time that it takes for the aircraft to reach this speed. Write down the equation, substitute from your list of variables and solve for time.

Answer: 30 s

d) Would it have worked to use $v = \frac{\Delta x}{\Delta t} \Rightarrow 120.0 \text{ m/s} = \frac{1800 \text{ m}}{\Delta t}$?

54 Accelerating cart

A cart travels in one direction. At an initial instant it passes the 4.0 m mark while traveling to the right with speed 15 m/s. (131Sp2025)

- a) Determine the location of the cart at an instant 6.0 s later if its acceleration is 3.0 m/s^2 .
- b) Determine the location of the cart at an instant 6.0 s later if its acceleration is -3.0 m/s^2 .

55 Accelerating shrew

A shrew (a small mammal) travels in one direction. At an initial instant it travels to the right with speed 3.0 m/s. It subsequently moves with constant acceleration and after traveling an additional 2.0 m to the right it reaches a speed of 7.0 m/s. Determine the time taken to do this. (131Sp2025)

56 Moving sloth

A sloth (a tree-climbing mammal) travels along a straight branch. At an initial instant it travels to the right with speed 0.030 m/s. It subsequently crawls with constant acceleration and after an additional 5.0s has traveled an additional 0.40 m to the right. Determine its speed at the end of this period. (131Sp2025)

57 Car starting from rest

A car is initially at rest and subsequently moves in one direction with constant acceleration. After 12 s its speed is 50 mph. Determine the total distance that it has traveled in this time. (131Sp2025)

58 Feasible Acceleration?

Suppose that one aims to push a cart along a horizontal surface, starting from rest, with a constant acceleration. Could one sustain this over a period of several seconds? This problem will try to help you answer this. When doing this, assume that the cart is light enough that one can easily exert the necessary force. Explain your answers, preferably with calculations, and describe any assumptions you made. (131Sp2025)

- a) Your friend claims that they can push the cart with a constant acceleration of 3.0 m/s^2 for a period of 10 s. Considering the state motion of the cart at the end of the 10 s period, does this seem reasonable?
- b) Suppose that you pushed the cart, starting from rest along a 100 m long straight track. What is the highest acceleration that you could feasibly attain? *Hint: Consider the highest speed you could attain at the end of the track.*

59 Reversing cart

At an initial instant a cart travels to the left with speed 12 m/s. Subsequently the cart's acceleration is 4.0 m/s^2 . (131Sp2025)

- a) How long does it take for the cart to reverse its direction of travel?
- b) How far does the cart travel before it reverses its direction of travel?

60 Racing cyclists

Two cyclists can ride along a straight road. Juliette moves right with a constant speed of 20 m/s. She passes Elisa, who is at rest. At the moment that Juliette passes, Elisa begins to move, accelerating at a constant rate of 2.0 m/s^2 . Elisa maintains this for 12.0 s and after that she moves with constant velocity. (131Sp2025)

- a) Explain how you can be sure that Elisa eventually passes Juliette.
- b) Determine the time (after Elisa starts to move) that it takes Elisa to catch up to Juliette.
- c) Determine the distance traveled by each when Elisa catches up with Juliette.

61 Runway design, 1

The takeoff speed for an aircraft under certain conditions is 260 km/h. During takeoff, one wants the acceleration to be as low as possible. (131Sp2025)

- a) Suppose that the runway has length 1500 m. Determine the aircraft's minimum acceleration so that it can take off on this runway.
- b) Suppose that the acceleration is only $0.20g = 1.96 \text{ m/s}^2$. Determine the minimum length of the runway so that the aircraft could take off successfully.

Note: It is useful to describe acceleration in terms of g as this is correlated to what humans can feel. An acceleration of g is what you feel when jumping off a diving board. An acceleration of around 6g will cause a person to blackout.

62 Aircraft takeoff acceleration

The takeoff speed for an aircraft under certain conditions is 260 km/h. A typical runway has length 3500 m. (131Sp2025)

- a) Suppose that the aircraft starts at rest somewhere along the runway and uses 2/3 of the runway length to takeoff. Determine the acceleration of the aircraft, assuming that it is constant.
- b) It is useful to describe acceleration in terms of g as this is correlated to what humans can feel. An acceleration of g is what you feel when jumping off a diving board. An acceleration of around 6g will cause a person to blackout. What multiple of g is the aircraft's acceleration?

c) Determine the fraction of the runway that the aircraft would have to use so that the acceleration were g.

63 Jumping cat

A cat jumps, launching itself vertically. During it's subsequent motion up and down, air resistance is negligible. In the following let the upward direction be positive. (131Sp2025)

- a) Which of the following is true in the period after the cat's feet have left the ground and before the cat reaches its highest point above the ground?
 - i) Acceleration is zero at all times.
 - ii) Acceleration is positive at all times.
 - iii) Acceleration is negative at all times.
- b) Which of the following is true in the period while the cat drops back to the ground?
 - i) Acceleration is zero at all times.
 - ii) Acceleration is positive at all times.
 - iii) Acceleration is negative at all times.
- c) Which of the following is true when the cat is at is highest point above the ground?
 - i) Acceleration is zero at all times.
 - ii) Acceleration is positive at all times.
 - iii) Acceleration is negative at all times.

64 Ball launched vertically

A ball is launched vertically from Earth's surface with speed 10 m/s. The aim of the first part of this exercise is to determine the maximum height reached by the ball and time taken to reach the maximum height. (131Sp2025)

a) Sketch the situation, illustrating the ball at two key instants.

List all relevant variables, including the acceleration, for the two instants:

b) Determine the maximum height reached by the ball by using one of the kinematic equations. Write down the equation, substitute from your list of variables and solve for the maximum height variable.

Answer: 5.1 m

c) Determine the time taken to reach the maximum height by using one of the kinematic equations. Write down the equation, substitute from your list of variables and solve for the maximum height variable.

Answer: 1.0 s

The second part of this exercise aims to find the speed of the ball just before returning to the ground using the fall from its maximum height.

- d) Sketch the situation for the falling ball, illustrating the ball at two key instants. List all relevant variables, including the acceleration, for the two instants:
- e) Determine the speed of the ball just before hitting the ground the ball by using one of the kinematic equations. Write down the equation, substitute from your list of variables and solve for the velocity variable.

Answer: velocity: -10 m/s, speed: 10 m/s.

65 Ball maximum height

A ball is launched vertically upward from Earth's surface with speed 25 m/s. (131Sp2025)

- a) Determine the maximum height reached by the ball.
- b) Determine the speed of the ball when it is 16 m above Earth's surface.

66 Bullet return speed

A bullet is fired vertically upward from Earth's surface with various speeds. The bullet will return to Earth at its launch position. (131Sp2025)

- a) Suppose that the bullet is launched with speed 600m/s. Ignoring air resistance, determine the speed of the bullet just before it hits the ground.
- b) Is the speed with which the bullet returns the same as that with which it is launched (regardless of the launch speed)? Explain your answer.

67 Thrown rock

A rock is thrown vertically down from a bridge. It leaves the hand with speed 20 m/s and hits the water 15 m below. Determine its speed at the instant before it hits the water. (131Sp2025)

Answer: 26 m/s

68 Ball thrown from above the ground, 1

A ball is thrown vertically upwards, leaving the hand at a height of 1.2 m above the ground. It hits the ground 2.5 s after leaving the hand. (131Sp2025)

- a) Determine the speed with which the ball left the hand.
- b) Determine the maximum height above the ground reached by the ball.

69 Ball thrown from above the ground, 2

A ball is thrown vertically upwards, leaving the hand with speed 4.0 m/s at a height of 1.2 m above the ground. Determine the speed of the ball just before it hits the ground. (131Sp2025)

70 Diver

A diver stands on a diving board that is 10 m above the surface of the water. The diver launches herself vertically up, leaving the board with a speed of 5.0 m/s. (131Sp2025)

- a) Determine the time taken for the diver to hit the water. Answer: 2.0 s
- b) Determine the diver's speed when she hits the water. Answer: 15 m/s

71 Jumping flea

A flea is at rest on a bed and launches itself vertically. A child watching this thinks that the flea is airborne for a total time of about 2s. Determine the maximum height above the bed that the flea would reach if this were true. Ignore air resistance. Is it plausible that the flea could have been airborne for that long? What might be a more realistic range of times? (131Sp2025)

Answer: 4.9 m

72 Penny and a well

A penny is held at rest at the top of a well. The penny is released, falls freely and takes 1.75 s to hit the water below. Determine the depth (distance from the top to the water) of the well. (131Sp2025)

Answer: 15 m

73 Parachuting package

A package is released from rest at a height of 100 m above Earth's surface. It falls freely until it is 40 m above Earth's surface. At that instant it deploys a parachute and after this is falls with a constant speed. The aim of this exercise is to determine the time taken to reach Earth.

To do this we will calculate the time taken for the free fall motion, Δt_A , and separately that time taken to fall with the parachute open, Δt_B .

The aim of the first part of this exercise is to determine Δt_A . (131Sp2025)

a) Sketch the situation, illustrating the ball at two key instants that will allow one to determine the time for the free fall portion of the motion.

List all relevant variables for the two instants and the acceleration.

b) Determine Δt_A by using one of the kinematic equations. Write down the equation, substitute from your list of variables and solve for Δt_A . **Answer:** 3.5 s.

The second part of this exercise aims to find Δt_B .

c) Sketch the situation for the falling ball, illustrating the ball at two key instants that will allow one to determine the time for the parachuting portion of the motion. List all relevant variables for the two instants and the acceleration.

- d) There is one quantity that one can obtain from the free fall part of the motion that will be needed to analyze the parachuting portion of the motion. Identify and compute this and insert it in to the list of variables from the previous part.
- e) Now determine Δt_B by using one of the kinematic equations. Write down the equation, substitute from your list of variables and solve for Δt_B . **Answer:** 1.2 s.
- f) Determine the total time taken to fall. Answer: 4.7 s.

74 Rocket flight

A rocket is at rest on the ground. Its engine then fires, producing a constant acceleration of 40 m/s^2 for a period of 15 s and propelling the rocket vertically. The engine then stops and the rocket continues to move upward. (131Sp2025)

- a) Determine the maximum altitude (height) reached by the rocket.
- b) Determine the time taken by the rocket to reach its maximum altitude.
- c) Determine the time taken for the rocket to fall back to the ground after reaching its maximum height.

Answer: a) 23 km b) 76 s c) 69 s

Vectors

75 Vector addition and subtraction: graphical, 1

Two displacement vectors, $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$, are illustrated. (131Sp2025)

- a) Sketch $\vec{\mathbf{C}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$. Determine the magnitude and direction of $\vec{\mathbf{C}}$.
- b) Let A be the magnitude of \vec{A} , let B be the magnitude of \vec{B} and C be the magnitude of \vec{C} . In this case, do these magnitudes satisfy C = A + B? Explain your answer.



76 Vector addition and subtraction: graphical, 2

Two displacement vectors, $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$, are illustrated. (131Sp2025)

- a) Sketch $\vec{\mathbf{C}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$. Determine the magnitude and direction of $\vec{\mathbf{C}}$.
- b) Sketch $\vec{\mathbf{C}} = \vec{\mathbf{A}} \vec{\mathbf{B}}$. Determine the magnitude and direction of $\vec{\mathbf{C}}$.
- c) Sketch $\vec{\mathbf{C}} = 2\vec{\mathbf{A}} \frac{1}{2}\vec{\mathbf{B}}$. Determine the magnitude and direction of $\vec{\mathbf{C}}$.



77 Vector addition and subtraction: graphical, 3

Two displacement vectors, $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$, are illustrated. (131Sp2025)

- a) Sketch $\vec{\mathbf{C}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$. Determine the magnitude and direction of $\vec{\mathbf{C}}$.
- b) Sketch $\vec{\mathbf{C}} = \vec{\mathbf{A}} \vec{\mathbf{B}}$. Determine the magnitude and direction of $\vec{\mathbf{C}}$.
- c) Sketch $\vec{\mathbf{C}} = 2\vec{\mathbf{A}} \frac{1}{2}\vec{\mathbf{B}}$. Determine the magnitude and direction of $\vec{\mathbf{C}}$.



78 Vector algebra: conceptual

Consider two vectors labeled $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$. (131Sp2025)

- a) Suppose that $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ are perpendicular. Explain whether it is possible that $\vec{\mathbf{A}} \vec{\mathbf{B}} = 0$.
- b) Suppose that the magnitudes satisfy B = 2A. Let $\vec{\mathbf{C}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$. Explain whether it is possible that the magnitude of $\vec{\mathbf{C}}$ satisfies C = A. Explain whether it is possible that the magnitude of $\vec{\mathbf{C}}$ satisfies C = 2A.

79 Adding two vectors

Consider displacement vectors $\vec{\mathbf{A}}$, whose magnitude is 4.0 m, and $\vec{\mathbf{B}}$, whose magnitude is 3.0 m. Their directions are not specified. Let $\vec{\mathbf{C}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$. (131Sp2025)

- a) What is the maximum possible magnitude for \vec{C} ? How must the vectors be arranged to give this?
- b) What is the minimum possible magnitude for \vec{C} ? How must the vectors be arranged to give this?

80 Vector components: conceptual

Consider a vector $\vec{\mathbf{A}}$ in two dimensions. (131Sp2025)

- a) Is it possible that either A_x or A_y is larger than the magnitude, A? Explain your answer.
- b) Is it possible that both components of $\vec{\mathbf{A}}$ can be non-zero and the magnitude of $\vec{\mathbf{A}}$ is zero? Explain your answer.

81 Vector components: algebraic, 1

Displacement vectors, $\vec{\mathbf{A}}, \vec{\mathbf{B}}$, and $\vec{\mathbf{C}}$ are illustrated. Their magnitudes are A = 15 m, B = 20 m and C = 25 m. (131Sp2025)

- a) Determine the x and y components of each vector.
- b) Determine the components of $\vec{\mathbf{D}} = \vec{\mathbf{A}} + \vec{\mathbf{B}} + \vec{\mathbf{C}}$. Determine the magnitude of $\vec{\mathbf{D}}$.



Determine the components of $\vec{\mathbf{A}}$, whose magnitude is 20.0 m. (131 Sp 2025)



y

 $\vec{\mathbf{A}}$

x

 $\vec{\mathbf{B}}$

 30°

 $\vec{\mathbf{C}}$

83 Vector addition: graphical and algebraic method, 1

Displacement vectors, $\vec{\mathbf{A}}, \vec{\mathbf{B}}$, and $\vec{\mathbf{C}}$ are illustrated. Let $\vec{\mathbf{D}} = \vec{\mathbf{A}} + \vec{\mathbf{B}} + \vec{\mathbf{C}}$. (131Sp2025)

- a) Using the graph sheet below, determine $\vec{\mathbf{D}}$ graphically via the head-to-tail method. Use the result to determine the magnitude of $\vec{\mathbf{D}}$.
- b) List the horizontal and vertical components of each of \vec{A} , \vec{B} , and \vec{C} and use these components to determine the components of \vec{D} . Use the result to determine the magnitude of \vec{D} .





84 Vector addition: algebraic method, 1

Two displacement vectors, $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ are illustrated. (131Sp2025)

- a) Determine the components of $\vec{\mathbf{A}}$.
- b) Determine the components of $\vec{\mathbf{B}}$.
- c) Determine the components of $\vec{\mathbf{C}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$.
- d) Determine the magnitude of $\vec{\mathbf{C}}$.

85 Vector addition: algebraic method, 2

Two displacement vectors, $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ are illustrated. (131Sp2025)

- a) Determine the components of $\vec{\mathbf{A}}$.
- b) Determine the components of $\vec{\mathbf{B}}$.
- c) Determine the components of $\vec{\mathbf{C}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$.
- d) Determine the magnitude of $\vec{\mathbf{C}}$.





86 Vector addition: algebraic method, 3

Two displacement vectors, $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ are illustrated. Determine the magnitude of $\vec{\mathbf{C}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$. (131Sp2025)



87 Vector addition: algebraic method, 4

Two displacement vectors, $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ are illustrated. (131Sp2025)

- a) Determine the components of $\vec{\mathbf{C}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$.
- b) Determine the magnitude of $\vec{\mathbf{C}}$.



Two displacement vectors, $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ are illustrated. (131Sp2025)

- a) Determine the components of $\vec{\mathbf{C}} = \vec{\mathbf{A}} \vec{\mathbf{B}}$.
- b) Determine the magnitude of $\vec{\mathbf{C}}$.

89 Vector algebra using components

Displacement vectors, $\vec{\mathbf{A}}, \vec{\mathbf{B}}, \vec{\mathbf{C}}$, and $\vec{\mathbf{D}}$ are illustrated. Their magnitudes are A = 2.0 m, B = 2.5 m, C = 3.0 m and D = 1.5 m. (131Sp2025)

- a) Determine the x and y components for each vector.
- b) Determine the x and y components for $\vec{\mathbf{E}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$. Determine the magnitude of $\vec{\mathbf{E}}$.
- c) Determine the x and y components for $\vec{\mathbf{E}} = \vec{\mathbf{B}} + \vec{\mathbf{C}}$. Determine the magnitude of $\vec{\mathbf{E}}$.
- d) Determine the x and y components for $\vec{\mathbf{E}} = \vec{\mathbf{B}} + \vec{\mathbf{D}}$. Determine the magnitude of $\vec{\mathbf{E}}$.
- e) Determine the x and y components for $\vec{\mathbf{E}} = \vec{\mathbf{B}} + \vec{\mathbf{C}} + \vec{\mathbf{D}}$. Determine the magnitude of $\vec{\mathbf{E}}$.



y

90 Vector subtraction using components

Displacement vectors, $\vec{\mathbf{A}}, \vec{\mathbf{B}}$ are illustrated. Their magnitudes are A = 3.16 m and B = 2.83 m. (131Sp2025)

- a) Determine the components of $\vec{\mathbf{C}} = \vec{\mathbf{A}} \vec{\mathbf{B}}$. Determine the magnitude of $\vec{\mathbf{C}}$.
- b) Determine the components of $\vec{\mathbf{C}} = 4\vec{\mathbf{A}} 3\vec{\mathbf{B}}$. Determine the magnitude of $\vec{\mathbf{C}}$.



91 Vector algebra using unit vectors

Displacement vectors, $\vec{\mathbf{A}}, \vec{\mathbf{B}}$, and $\vec{\mathbf{D}}$ are illustrated. Let $\vec{\mathbf{D}} = 3\vec{\mathbf{A}} + 2\vec{\mathbf{B}} + \vec{\mathbf{C}}$. Their magnitudes are A = 2.0 m, B = 3.5 m and D = 6.0 mExpress each of the vectors in terms of unit vectors and use these to determine $\vec{\mathbf{C}}$. (131Sp2025)

92 Marching soldier

A soldier marches around a playing field whose edges are along North-South (N-S) and East-West (E-W) lines. The soldier starts in the southwest corner, marches in a straight line in the direction 30° N of E for 40 m. He then marches straight south for 12 m. Finally he marches in a straight line in the direction 50° N of W for 30 m. After this, how far is the soldier from his starting point? (131Sp2025)

93 Successive displacements

An insect crawls along a horizontal two dimensional surface in two straight stages. First the insect walks 8.0 cm at an angle of 45° south of west. Second, the insect walks 10.0 cm in a straight line at an angle 60° south of east. Determine an expression for the displacement of the insect and how far the insect ends up from its starting point. (131Sp2025)



Two dimensional kinematics

94 Velocity vectors from motion

Particles move along the illustrated trajectories. Their locations are illustrated at instants 4.0 s apart. The grid units are meters. For each case determine the average velocity vector for the particle over the entire interval and express it in terms of the conventional unit vectors. (131Sp2025)



95 Ants on a table

Two ants move on a flat surface between two grains of sugar. Their trajectories are illustrated. The ants take the same time to move between the grains but follow different trajectories. Which of the following is true about the average velocities of the ants for the entire trip? Explain your answer. (131Sp2025)

- i) The average velocities are the same.
- ii) The average velocity of ant A is smaller than that for ant B.
- iii) The average velocity of ant A is larger than that for ant B.
- iv) The average velocities are different but one cannot say which is larger.





96 Model moon motion

A moon of a planet in a demonstration model of a planetary system travels a trajectory described by the position vector

$$\vec{\mathbf{r}}(t) = 2.0 \operatorname{m} \cos\left(\omega t\right) \hat{\mathbf{i}} + 2.0 \operatorname{m} \sin\left(\omega t\right) \hat{\mathbf{j}}.$$

where $\omega = 0.50\pi \,\mathrm{rad} \,\mathrm{s}^{-1}$ (note that this expresses the arguments of the trigonometric functions in terms of radians). (131Sp2025)

- a) Determine expressions for the horizontal and vertical coordinates x(t) and y(t) for the moon from t = 0s to t = 2.0s.
- b) Sketch the trajectory of the moon as accurately as possible on the illustrated grid.



- c) Determine the displacement vector from t = 1.0 s to t = 1.5 s and use this to get the average velocity vector from t = 1.0 s to t = 1.5 s.
- d) Determine the displacement vector from t = 1.0 s to t = 1.1 s and use this to get the average velocity vector from t = 1.0 s to t = 1.1 s.
- e) Determine the displacement vector from t = 1.0 s to t = 1.01 s and use this to get the average velocity vector from t = 1.0 s to t = 1.01 s.
- f) What do you results suggest about the direction of the instantaneous velocity vector at t = 1.0 s? Explain your answer.

97 Acceleration vectors from motion

Two particles move along the illustrated trajectories. Their locations and speeds are illustrated at instants 0.25 s apart. For each case determine the acceleration vector for the particle. (131Sp2025)



98 Ball moving in two dimensions

A ball travels along a trajectory described by the position vector

$$\vec{\mathbf{r}}(t) = (t-2)\,\hat{\mathbf{i}} + \left(-\frac{(t-2)^2}{2} + 4\right)\hat{\mathbf{j}}.$$

from t = 0 to t = 4. (131Sp2025)

- a) Determine expressions for the horizontal and vertical coordinates x(t) and y(t) for the ball.
- b) Sketch the trajectory of the ball as accurately as possible on the illustrated grid.
- c) Determine the displacement vector from t = 1.5 to t = 2 and use this to get the average velocity vector from t = 1.5 to t = 2.



- d) Determine the displacement vector from t = 2 to t = 2.5 and use this to get the average velocity vector from t = 2 to t = 2.5.
- e) Determine an expression for the instantaneous velocity vector and use this to obtain the average acceleration from t = 1.8 to t = 2.2. What does this imply for the direction of the acceleration at t = 2?

99 Bouncing hockey puck

A hockey puck slides along a horizontal surface toward a board, hitting it at an angle and bouncing off with unchanged speed. The view from above is as illustrated. (131Sp2025)

- a) Draw the velocity vectors of the puck just before and just after hitting the board, use these to draw the vector $\Delta \vec{\mathbf{v}}$, and use the result to draw the direction of the acceleration vector.
- b) If the puck traveled backwards along the same path (i.e. diagonally from top left to the board and then toward the bottom left), what would the direction of the acceleration vector be? Explain your answer.


100 Sliding ice cube

An ice cube slides back and forth along a bowl with a parabolic cross section. It moves with the same speeds at the indicated earlier and later moments, which are symmetrical about the lowest point in the parabola. Determine the direction of the average acceleration, using velocity. (131Sp2025)



101 Ants moving along a curved path

Various ants follow the same path on a horizontal surface, starting at point 1. The path is as illustrated. Ant A moves with a constant speed, ant B gradually speeds up and ant C gradually slows. (131Sp2025)

- a) Draw the velocity vector at points 1, 2, 3 and 4.
- b) Does any of the ants have zero acceleration at all times? Explain your answer.

102 Velocity and acceleration vectors and motion

Velocity and acceleration vectors for two particles at one instant are illustrated. For each case, describe whether the particle is speeding up, slowing down or moving with constant speed at the illustrated instant. Also describe how the direction of the particle's motion changes. Explain your answers. Note that the question refers to what is happening at the illustrated instant. The answers could be different if the particles were observed significantly later. (131Sp2025)





104 Constant acceleration in two dimensions

A ball can slide along a horizontal surface. At an initial instant its velocity is $\vec{\mathbf{v}}_i$. At all later times it accelerates with a constant acceleration, $\vec{\mathbf{a}}$. The situation with the vectors drawn to scale as viewed from above is illustrated (for the velocity vector, the grid unit is the standard unit of velocity and for acceleration the grid unit is the standard unit of acceleration). (131Sp2025)

- a) Draw, as accurately as possible, the velocity vector at an instant 1.0 s after the initial instant.
- b) Let v_0 denoted the speed at the initial instant. Determine an expression for the speed, in terms of v_0 , at an instant 1.0 s after the initial instant.
- c) Determine the time for the speed v to be twice the speed at the initial moment.



A ball launches off a horizontal surface. At the moment of launch its velocity is $\vec{\mathbf{v}}_i$. At all later times it accelerates with a constant acceleration, \vec{a} . The situation with the vectors drawn to scale is illustrated (for the velocity vector, the grid unit is the standard unit of velocity and for acceleration the grid unit is the standard unit of acceleration). (131Sp2025)

- a) Draw, as accurately as possible, the velocity vector, $\vec{\mathbf{v}}_f$,
- at an instant 1.0s after the initial instant. b) Using $\vec{\mathbf{v}}_f$ describe whether the object is moving faster at the 1.0s instant than at the initial instant.
- c) Using $\vec{\mathbf{v}}_f$ describe the direction in which the object is moving at the 1.0 s instant.



 $\vec{\mathbf{v}}_i$

 $\vec{\mathbf{a}}$



105 Motion with constant acceleration

A ball can slide along a horizontal surface. At an initial instant its velocity, $\vec{\mathbf{v}}_0$, is as illustrated. At all later times it accelerates with a constant acceleration, $\vec{\mathbf{a}}$, as illustrated. (131Sp2025)

- a) Draw, as accurately as possible, the velocity vector at an instant 0.5 s after the initial instant.
- b) Draw, as accurately as possible, the velocity vector at an instant 1.0 s after the initial instant.
- c) Which of the following best represents the trajectory of the particle? Explain your answer.



106 Dropping a coin on a ship

A person is on the deck of a ship. While the ship is at rest in a harbor, the person drops a coin from rest and the coin lands on the deck of the ship. The person marks the spot where the coin lands. Later the ship sails with constant speed in a straight line and the person returns to the same spot and drops the coin again. Does the coin land at the original mark, in front of it or behind it? Explain your answer. (131Sp2025)

107 Throwing a ball on a train

A person is inside a train that moves horizontally in a straight line with speed v_{train} . The person launches a ball vertically with with speed, v_{launch} (from the perspective of the person in the train). Let v be the speed of the ball at the moment of launch as viewed from outside the train. Which of the following is true? Explain your answer. (131Sp2025)

i) $v = v_{\text{train}}$

11)
$$v = v_{\text{launch}}$$

- iii) $v = v_{\text{train}} + v_{\text{launch}}$
- iv) $v = \sqrt{v_{\text{train}}^2 + v_{\text{launch}}^2}$



108 Jumping monkey

A monkey jumps leaving the Earth at an angle of 45° from the surface. Which of the following is true when the monkey reaches the highest point along the trajectory? Explain your answer. (131Sp2025)

- i) The acceleration of the monkey is zero.
- ii) The acceleration of the monkey is horizontal in the forwards direction.
- iii) The acceleration of the monkey is horizontal in the backwards direction.
- iv) The acceleration of the monkey is vertical downward.
- v) The acceleration of the monkey is vertical upward.

109 Red vs. blue ball

Two balls, one red and the other blue, are on a horizontal table. The are made to roll off the table. At the instants that they leave the speed of the blue ball is four times that of the red ball. Which of the following is true regarding the times taken to hit the horizontal floor on which the table stands? Explain your answer. (131Sp2025)

- i) The time taken for blue ball is a quarter of that for the red ball.
- ii) The time taken for blue ball is half that for the red ball.
- iii) The time taken for blue ball is the same as that for the red ball.
- iv) The time taken for blue ball is twice that for the red ball.
- v) The time taken for blue ball is four times that for the red ball.

110 Heavier vs. lighter ball

Two balls, one heavier and the other lighter, roll off the edge a horizontal table. (131Sp2025)

- a) Suppose that the balls are launched with the same speed. Which of the following is true? Explain your answer.
 - i) The time taken for the heavier ball to hit the floor is the same as the time taken for the lighter ball.
 - ii) The time taken for the heavier ball to hit the floor is larger than the time taken for the lighter ball.
 - iii) The time taken for the heavier ball to hit the floor is smaller than the time taken for the lighter ball.
- b) Suppose that the balls are launched with the same speed. Which of the following is true? Explain your answer.
 - i) The distance traveled by the heavier ball is the same as that traveled by the lighter ball.
 - ii) The heavier ball travels further than the lighter ball.
 - iii) The lighter ball travels further than the heavier ball.

111 Balls launched off horizontal surfaces

Two balls, one red and the other blue, are on horizontal surfaces above a floor. They are each launched horizontally off the surface with the same speed. The height of the blue ball surface above the floor is nine times that of the red ball surface. Let $t_{\rm red}$ be the time between the moment that the red ball leaves the table until it hits the floor; similarly $t_{\rm blue}$ represents the same for the blue ball. Let $\Delta x_{\rm red}$ be the horizontal distance between the point where the red ball leaves the table and hits the ground. (131Sp2025)

- a) Suppose that the balls are launched with the same speed. Which of the following is true? Explain your answer.
 - i) $t_{\text{blue}} = \frac{1}{9} t_{\text{red}}$
 - ii) $t_{\text{blue}} = \frac{1}{3} t_{\text{red}}$
 - iii) $t_{\text{blue}} = t_{\text{red}}$
 - iv) $t_{\rm blue} = 3 t_{\rm red}$
 - v) $t_{\rm blue} = 9 t_{\rm red}$
- b) Suppose that the balls are launched with the same speed. Which of the following is true? Explain your answer.
 - i) $\Delta x_{\text{blue}} = \frac{1}{9} \Delta x_{\text{red}}$
 - ii) $\Delta x_{\text{blue}} = \frac{1}{3} \Delta x_{\text{red}}$
 - iii) $\Delta x_{\text{blue}} = \Delta x_{\text{red}}$
 - iv) $\Delta x_{\text{blue}} = 3 \Delta x_{\text{red}}$
 - v) $\Delta x_{\text{blue}} = 9 \Delta x_{\text{red}}$
- c) Suppose that the blue ball is launched with a slower speed than the red ball. Describe how this would affect the answers of the previous questions.

112 Running off a roof

A person runs with speed 8.0 m/s off a flat roof that is 3.0 m above the ground. First suppose that the person travels horizontally at the moment that he leaves the roof. Determine how far horizontally from the edge of the roof the person will land. (131Sp2025)

a) Sketch the situation with the "earlier" instant being that at which the person leaves the roof and the "later" instant being the moment just before the person hits the ground.
 List as many of the variables as possible. Use the format:

$t_i =$	$t_f =$
$x_i =$	$x_f =$
$y_i =$	$y_f =$
$v_{ix} =$	$v_{fx} =$
$v_{iy} =$	$v_{fy} =$
$a_x =$	$a_y =$

- b) Sketch the velocity vector at the earlier moment and use this to determine the components of $\vec{\mathbf{v}}_i$. Enter these in the list above.
- c) Identify the variable needed to answer the question of the problem. Select and write down a kinematic equation that contains this variable and attempt to solve it. You should see to solve the variable describing the horizontal position, you first need the value for another, currently unknown variable. Which variable is this?
- d) Use the vertical aspects of the object's motion to solve for this other unknown variable and use this result to answer the question of this problem.

Suppose that the person ran and jumped from the building at an angle of 30° above the horizontal. This will change how far the person travels. Before answering that question, we ask, what is the maximum height above the ground reached by the person for this running jump?

- e) Sketch the velocity vector at the earlier moment and use this to determine the components of $\vec{\mathbf{v}}_i$. Reconstruct the list of variables for the problem.
- f) Sketch the velocity vector at the instant when the person reaches his highest point. Use this to add additional information to the list of variables for the problem.
- g) Use the kinematic equations to determine the maximum height that the person reaches.
- h) Determine the speed of the person at the maximum height.
- i) Determine how far horizontally from the edge of the roof the person will land.

113 Projectile motion range

A person runs with speed 8 m/s off a flat roof that is 3.0 m above the ground. The person can launch himself at various angles and the purpose of this exercise is to determine the speed with which the person hits the ground and where on the ground the person lands (for various angles of launch). First suppose that the person travels horizontally at the moment that he leaves the roof. It was found that the person lands 6.3 m from the base of the building. (131Sp2025)

a) Sketch the situation with the "earlier" instant being that at which the person leaves the roof and the "later" instant being the moment just before the person hits the ground. List as many of the variables as possible. Use the format

	$t_f =$
$x_i =$	$x_f =$
$y_i =$	$y_f =$
$v_{ix} =$	$v_{fx} =$
$v_{iy} =$	$v_{fy} =$
$a_x =$	$a_y =$

- b) Draw the velocity vector at the earlier instant and use this to determine the components of $\vec{\mathbf{v}}_0$. Enter these in the list above.
- c) Draw the velocity vector at the later instant. Describe whether the components are positive, negative or zero.
- d) The speed of the object is the magnitude of the velocity vector. So to determine the speed, you will first need to determine the components of the velocity vector. Use the kinematic equations to determine the x and y components of the velocity at the later instant. You must start by writing the equation that you use, before substituting.
- e) Sketch the velocity vector at the instant just before the person hits the ground and indicate the values of the x and y components on the sketch. Use this to determine the speed of the person just before hitting the ground.

Now suppose that the person travels with the same speed but launches himself at an angle of 45° from the roof.

- f) Using the same "earlier" and later instants as before, list as many of the variables as possible.
- g) Draw the velocity vector at the earlier moment and use this to determine the components of $\vec{\mathbf{v}}_0$.
- h) Use the kinematic equations to determine the x and y components of the velocity at the later instant.
- i) Sketch the velocity vector at the instant just before the person hits the ground and indicate the values of the x and y components on the sketch. Use this to determine the speed of the person just before hitting the ground.

- j) Determine the time taken to hit the ground.
- k) Determine how far the person lands from the building.

114 Aircraft dropping object

An aircraft flies horizontally with a constant speed of 600 km/h at a height of 1200 m above a flat surface. It drops an object from its underside; this object is supposed to hit a particular spot on the ground. How far (horizontally) from the spot must the aircraft be for the object to hit the spot? (131Sp2025)

Answer: 2610 m

115 Rifle shot

A rifle fires a bullet. At the moment that the bullet leaves the rifle it travels horizontally with speed 300 m/s. Determine how far the bullet will drop by the time that it reaches a vertical wall that is 100 m from the end of the rifle that is closest to the wall. (131Sp2025)

Answer: −0.54 *m*

116 Ball thrown horizontally

A ball is thrown, leaving the hand horizontally at a height of 2.0 m above the ground. It lands a horizontal distance of 5.0 m from where it left the hand. (131Sp2025)

- a) Determine the time from when the ball leaves the hand until it hits the ground.
- b) Determine the speed with which the ball leaves the hand.

117 Rock thrown horizontally

You throw a rock, which leaves your hand 1.5 m above the ground. You would like it to land a horizontal distance of 18.0 m from where it left the hand. How fast must you throw the ball? (131Sp2025)

Answer: 33 m/s

118 Jumping over a ditch

A dog attempts to jump over a ditch, which is 2.0 m wide. The dog launches itself from one edge of the ditch with speed 5.0 m/s at an angle of 30° from the horizontal. The aim of this exercise is to determine whether the dog will reach the other side of the ditch. First we will find out whether the dog reaches its maximum height before or after it is above the middle of the ditch. (131Sp2025)

a) Sketch the situation with the "earlier" instant being that at which the dog launches and the "later" instant being the moment when it reaches its highest point. List as many of the variables as possible. Use the format

$$t_i = t_f =$$

$$x_i = x_f =$$

$$y_i = y_f =$$

$$v_{ix} = v_{fx} =$$

$$v_{iy} = v_{fy} =$$

$$a_x = a_y =$$

- b) Draw the velocity vector at the earlier instant and use this to determine the components of $\vec{\mathbf{v}}_i$. Enter these in the list above.
- c) Draw the velocity vector at the later instant. Describe whether the components are positive, negative or zero and enter as much information about these in the list above.
- d) Determine the horizontal distance traveled by the dog by the time that it reaches its maximum height.
- e) As the dog descends from its maximum height back to the ground, how much further does it travel? Does it reach the other side of the ditch?

119 Launching off a ski ramp, 1

A ski ramp is arranged as illustrated. A skier launches off the ramp with a speed of 15 m/s. Initially the aim of this exercise is to determine the maximum height reached by the skier and the velocity at this point. A later goal is to determine the distance at which the skier lands from the bottom of the ramp. (131Sp2025)

a) Sketch the situation with the "earlier" instant being that at which the skier launches and the "later" instant being the moment when she reaches its highest point. List as many of the variables as possible. Use the format

$t_0 =$	$t_1 =$
$x_0 =$	$x_1 =$
$y_0 =$	$y_1 =$
$v_{0x} =$	$v_{1x} =$
$v_{0y} =$	$v_{1y} =$
$a_x =$	$a_y =$

b) Draw the velocity vector at the earlier instant and use this to determine the components of $\vec{\mathbf{v}}_0$. Enter these in the list above.

 20°

 $4.0\,\mathrm{m}$

- c) Draw the velocity vector at the later instant. Describe whether the components are positive, negative or zero and enter as much information about these in the list above.
- d) Determine the time taken to reach the maximum height and then the horizontal distance traveled by the skier to reach her maximum height. Determine the velocity at this point.

You will now consider the motion from the highest point back to the ground.

e) Repeat the problem set-up with the "earlier" instant being that at which the skier is at maximum height and the "later" instant being the moment *just before* she reaches hits the ground. Determine the time taken for this portion of the motion and use it to determine the horizontal distance from the base of the ramp to the skier's landing point.

120 Launching off a ski ramp, 2

A ski ramp is arranged as illustrated. A skier launches off the top of the ramp with a speed of 28 m/s. Determine the horizontal distance at which the skier lands from the bottom of the ramp. (131Sp2025)



121 Jumping grasshopper

A grasshopper jumps, leaving the ground at an angle of 70° from the horizontal. The grasshopper reaches a height of 0.60 m. (131Sp2025)

- a) Determine the speed with which the grasshopper leaves the ground.
- b) Determine how far the grasshopper lands from where it jumped.

122 Jumping lemur

A lemur (a type of primate) jumps, leaving the ground at an angle of 40° from the horizontal with speed 6.0 m/s. (131Sp2025)

- a) Determine the maximum height reached by the lemur.
- b) Determine the amount of time for which the lemur is airborne.
- c) Determine the horizontal distance traveled by the lemur from the moment it leaves the ground until it returns to the ground.

123 Diver splashdown

A diver launches off a platform at an angle of 55° above the horizontal and with speed 8.0 m/s. The platform is 3.0 m above the surface of a pool. (131Sp2025)

- a) Determine the diver's speed just before hitting the water.
- b) Determine the time taken by the diver to hit the water.

124 Stone thrown from a bridge

A person stands on a bridge over a small lake. The person throws a stone with speed 18.0 m/s at an angle of 40° above the horizontal. The stone leaves the hand at a height 3.0 m above the surface of the water. (131Sp2025)

- a) Determine the time taken for the stone to reach the highest point in its trajectory.
- b) Determine the time taken for the stone to hit the water.
- c) Determine the horizontal distance traveled by the stone between the throw and when it hits the water.

125 Ball launched from a cliff

A person hits a ball from the top of a cliff. The balls leaves at height of 12 m above the surface of the water at angle 30° above the horizontal. It hits the water 2.5 s later. (131Sp2025)

- a) Determine speed with which the ball is launched.
- b) Determine the horizontal distance traveled by the ball.

126 Cannonball range

A cannonball is fired from the ground (assume that the ball leaves from the ground level) and must hit a target on the ground a distance of 500 m away. Assume that the ground is horizontal and ignore air resistance. (131Sp2025)

- a) Suppose that the cannonball leaves at an angle of 50° from the horizontal. Determine the speed with which it must be fired to hit the target.
- b) Suppose that the cannonball is fired with speed 150 m/s. Determine the angle above the horizontal at which it must be fired so that it hits the target.
- c) (*Challenging*) Determine the minimum speed with which the cannonball must be fired so that it can hit the target if the angle is adjusted correctly.

127 Angry tennis player

An angry tennis player hits a tennis ball into the air toward the fence that surrounds the court. The tennis player is a horizontal distance of 20 m from the fence, which is 10 m high. The tennis player hits the ball with speed 28 m/s at an angle of 70° above the horizontal from a height of 1.2 m above the ground. Does the ball reach the fence? Does it pass over the top of the fence? Explain your answer. (131Sp2025)

128 Water chute and bucket

Water slides down a chute that is at an angle of 35° above the horizontal. It leaves the end of the chute with speed 3.0 m/s. A bucket with sides 30 cm high and diameter 20 cm is placed beyond the chute so that it catches the water. Determine where the left edge of the bucket be should placed so that the water will land in the bucket. Provide the entire range of possible locations. (131Sp2025)



129 Grain chute

Grain pours off a chute that is at an angle of 25° above the horizontal. It leaves the end of the chute with speed 5.0 m/s. Determine how far it travels horizontally. (131Sp2025)



130 Merry-go-round

A merry-go-round is a large flat disk that spins around a vertical axis through its center. A child is at the edge of a merry-go-round with radius 3.0 m. The merry-go-round spins so that the child's acceleration is 1.5g. Determine the period and frequency of orbit for this to occur. (131Sp2025)

131 Rotating mountain bike wheel

One standard mountain bike wheel has rims with diameter 29in. A cyclist rides the bike with a constant speed of 15 mph. Determine the speed and centripetal acceleration of a point on the rim in metric units. (131Sp2025)

132 Vinyl turntable

A turntable that plays vinyl records rotates at 33 rpm. The radius if the turntable is 0.15 m. (131Sp2025)

- a) Determine the frequency of the turntable.
- b) Determine the period of the turntable.
- c) Determine the speed of a point on the edge of the turntable.

133 Ball on a string

A ball swings with constant speed on the end of a 50 cm long string. Determine the period of orbit (time for one circle) such that the ball's acceleration is 2g. (131Sp2025)

134 Orbiting object

An object orbits in a circle with constant speed. The radius of the orbit is r and the object completes on orbit in time T (the period of orbit). Determine an expression for the object's acceleration in terms of T and r. (131Sp2025)

135 Rotating dolls

Two dolls sit in fixed places on a disk that rotates about its center. Doll A is halfway from the axle to the rim. Doll B is at the rim. The wheel rotates at a uniform constant rate. Let v_A be the speed of A, v_B be the speed of B, a_A the acceleration of A, etc. In each of the following, explain your answer. (131Sp2025)

- a) Which of the following is true?
- i) $v_A = \frac{1}{4} v_B$ ii) $v_A = \frac{1}{2} v_B$ iii) $v_A = v_B$ iv) $v_A = 2v_B$ v) $v_A = 4v_B$ b) Which of the following is true?
 - i) $a_A = \frac{1}{4} a_B$ ii) $a_A = \frac{1}{2} a_B$
 - $a_A = 2 a_A$
 - iii) $a_A = a_B$
 - iv) $a_A = 2a_B$
 - v) $a_A = 4a_B$

136 Acceleration on Grand Mesa versus Grand Junction

Consider standing in two places: one in Grand Junction and the other on top of Grand Mesa. The altitude of Grand Junction is roughly 1380 m and Grand Mesa 3200 m. The aim of this exercise is to determine the difference in velocity and acceleration between the two locations. Grand Junction's location latitude is about 39° North. Note that all objects rotate about an axis through Earth's poles. (131Sp2025)

- a) Determine the speed with which a person in Grand Junction moves as Earth (radius 6400 km at sea level) rotates.
- b) Determine the speed with which a person on top of Grand Mesa moves.
- c) Determine the acceleration of a person in Grand Junction.
- d) Determine the acceleration of a person on the Grand Mesa. Are the accelerations noticeably different?



137 Acceleration on Earth's surface

People stand on Earth's surface and are at rest *relative to Earth*. Earth has a radius of 6.4×10^3 km and spins about its poles at a rate of one revolution every 24 hrs. (131Sp2025)

- a) Determine the acceleration of a person at Earth's equator.
- b) Another person stands at a location much closer to the North pole. Is this person's acceleration the same as, larger than or smaller than that of a person at the equator? Explain your answer.

138 Centripetal acceleration

The formula for centripetal acceleration is

$$a = \frac{v_t^2}{r}.$$

In this exercise you will use calculus to prove that this is true. (131Sp2025)

a) Consider an object whose position is represented by the vector (from the origin)

$$\vec{\mathbf{r}} = r\cos\left(\omega t\right)\hat{\mathbf{i}} + r\cos\left(\omega t\right)\hat{\mathbf{j}}.$$

By considering this at various times, show that it represents the position of a particle traveling in a circle of radius r and with angular velocity ω .

b) Determine the tangential velocity vector

$$\vec{\mathbf{v}}_t = \frac{d\vec{\mathbf{r}}}{dt}$$

 $\vec{\mathbf{a}} = \frac{d\vec{\mathbf{v}}_t}{dt}.$

and the acceleration vector

c) Show that

$$\vec{\mathbf{a}} = -\omega^2 \vec{\mathbf{r}}$$

and use this to show that

$$a = \omega^2 r = \frac{v_t^2}{r}.$$

Dynamics

139 Hockey pucks sliding horizontally

Three identical hockey pucks slide horizontally across a frictionless sheet of ice and they maintain the indicated speeds during a particular period. Let F_A be the magnitude of the force acting on A during this period, F_B be the magnitude of the force acting on B, etc, Which of the following is true? Explain your answer. (131Sp2025)

- i) $F_B = 2F_A$ and $F_C = 4F_A$
- ii) $F_A = 2F_B$ and $F_A = 4F_C$
- iii) $F_A = F_B = F_C \neq 0$
- iv) $F_A = F_B = F_C = 0$

140 Forces on an ant and a bug

An ant and a bug walk along straight wires. The graph illustrates their velocities vs. time. They have 4 the same mass. Answer the following, giving explanations for each answer. (131Sp2025) 3

- a) How does the net force on the ant at 2.0 s com- 2 pare (same, larger, four times as large,etc,) to the net force on the ant at 4.0 s?
- b) How does the net force on the ant compare (same, always larger, sometimes larger, always smaller,...) to the net force on the bug?
- c) Is the net force exerted on the ant ever the same as that exerted on the bug? If so, explain when.





141 Moving carts

Three identical carts move horizontally along tracks. Their speeds at two instants 5.0s apart are indicated. Let F_A be the magnitude of the net force acting on A during this interval, F_B be the magnitude of the net force acting on B, etc, Which of the following is true? Explain your answer. (131Sp2025)

- i) $F_A > F_B > F_C$.
- ii) $F_B = F_C > F_A$.
- iii) $F_B = F_C < F_A$.
- iv) $F_A = F_B = F_C \neq 0$



142 Riding a bicycle

Suppose that you ride a bicycle at constant speed in straight lines across various surfaces. In each of the following, explain your answers. (131Sp2025)

- a) If you ride at a constant speed along a flat horizontal surface, which of the following is true?
 - i) The net force on the bicycle is zero.
 - ii) The net force on the bicycle is in the direction of motion.
 - iii) The net force on the bicycle is opposite to the direction of motion.
 - iv) Whether the net force is zero or not depends on the speed.
- b) If you ride at a constant speed along a slanted flat uphill surface, which of the following is true?
 - i) The net force on the bicycle is zero.
 - ii) The net force on the bicycle is in the direction of motion.
 - iii) The net force on the bicycle is opposite to the direction of motion.
 - iv) Whether the net force is zero or not depends on the speed.

143 Pushing carts

Zog and Geraldine (his wife) each push a cart along a horizontal surface where friction is negligible. Both carts are initially at rest. Zog takes the cart with mass 25 kg and exerts a force of 400 N on it for a period of 4.0 s and he then collapses and stops pushing. Geraldine has to push a cart of mass 50 kg and she is also able to exert a force of 400 N on it. Geraldine claims that it is possible for the speed of her cart to eventually reach the speed of Zog's cart. Is this true? Explain your answer. (131Sp2025)

144 Forces on a cart sliding horizontally

A cart can slide horizontally left or right. (131Sp2025)

- a) Suppose that the cart slides to the left with increasing speed. Which of the following is true about the net force, $\vec{\mathbf{F}}_{net}$, acting on the cart? Explain your choice.
 - i) $\vec{\mathbf{F}}_{net} = 0$
 - ii) $\vec{\mathbf{F}}_{net} \neq 0$ and points right.
 - iii) $\vec{\mathbf{F}}_{net} \neq 0$ and points left.
- b) Suppose that the cart slides to the left with decreasing speed. Which of the following is true about the net force acting on the cart? Explain your choice.
 - i) $\vec{\mathbf{F}}_{net} = 0$
 - ii) $\vec{\mathbf{F}}_{net} \neq 0$ and points right.
 - iii) $\vec{\mathbf{F}}_{net} \neq 0$ and points left.

145 Moving carts

Consider three carts with different masses. Different forces act on each cart and each is expressed as a multiple of some basic force, F. The mass of cart A is 8 times the mass of cart C. The mass of cart B is 3 times the mass of cart C. Rank the carts in order of increasing acceleration. Explain your answer. (131Sp2025)



146 Pickax

A pickax consists of a tapered wooden handle and a metal head that slides over the wooden handle. An exaggerated simplified cross section view is illustrated. The head sometimes works loose. One way to secure it is to hold the handle vertical with the head at the bottom and then drop the handle. Use Newton's laws to explain why even letting the handle hit the ground gently is likely to secure the head more tightly than hitting the head from above. (131Sp2025)



147 Force, acceleration and velocity

The exercise will investigate the assertion:

"The larger the force on an object, the larger the object's velocity."

Consider two carts, labeled A and B, each with mass 3.0 kg that can slide left or right. The net force on cart A is 12.0 N to the right and the net force on cart B is 6.0 N to the right. It would appear that the velocity of A is twice the velocity of B. Consider the situation where the velocity of each cart at 0.0 s is exactly 0.0 m/s and that the forces act on the carts at all later times. Explain your answers in the following. (131Sp2025)

- a) Determine the velocity of each cart at 1.0 s, 2.0 s, 3.0 s, 4.0 s and 5.0 s.
- b) Is the "velocity of A twice the velocity of B?"
- c) Is the "velocity of A at 2.0s twice the velocity of B at 4.0s?" Does this suggest a need to be specific with statements about velocity?

Now consider a different situation where the velocity of A at 0.0 s is exactly -8.0 m/s and the velocity of B is 0 m/s.

- d) Determine the velocity of each cart at 1.0 s, 2.0 s, 3.0 s, 4.0 s and 5.0 s.
- e) Is the "velocity of A twice the velocity of B?"
- f) Is the "velocity of A at 3.0s twice the velocity of B at 3.0s?"
- g) Is the statement: "The larger the force on an object, the larger the object's velocity" true or not in general?

148 Sledding dog

A 37.0 kg dog sits on a 3.0 kg sled that can slide along a horizontal frictionless sheet of ice. At one instant the dog and sled are moving right with speed 6.0 m/s. For the next 4.0 s a person exerts a constant 80 N force on the dog/sled in the same direction as they move. (131Sp2025)

- a) Determine the velocity of the dog/sled at the end of the 4.0 s period.
- b) Determine the distance traveled by the dog/sled during the 4.0 s period.

149 Force on an accelerating box

A 50 kg box is initially at rest on a horizontal sheet of ice. Subsequently someone pulls on the box with a constant horizontal force. The box moves 15 m during the first 10 s while the force is applied. Determine the magnitude of the force. (131Sp2025)

150 Moving a sleepy dog

A sleepy 30 kg dog lies on a horizontal surface. The dog is initially at rest. Subsequently a child pushes horizontally on the dog with a 150 N force. There is no friction between the dog and the surface and forces in the vertical direction are irrelevant to its motion. (131Sp2025)

- a) Determine how far the dog moves after the child has pushed for 3.0 s.
- b) Determine the speed of the dog after the child has pushed for 3.0 s.
- c) If the child continues pushing for twice as long will the speed be twice as much? Will the distance traveled be twice as much? Explain your answers.

151 Citrus fruit microjets

Citrus fruit emit aromatic oils when the peel is disrupted. This can occur in the form of microscopic scale jets of fluid, that had been stored in small glands within the fruit's peel. Recent studies have shown that the jets emitted from lemons leave with speed about 7.0 m/s. The aim of this problem is to estimate the force exerted on the jet emitted from a single gland. The volume of a typical gland in a lemon is 6.0×10^{-10} m³. Assuming the fluid has a similar density to water, this would imply that the fluid has mass 6.0×10^{-7} kg. (131Sp2025)

- a) Suppose that this entire fluid was accelerated from rest to $7.0 \,\mathrm{m/s}$ across a distance equal to the radius of a sphere with the volume of the gland. Show that the radius is $5.2 \times 10^{-4} \,\mathrm{m}$. Determine the acceleration of the fluid.
- b) Determine the net force on the fluid. How does this compare to the gravitational force on the fluid? Does gravity matter in this case?

Detailed analyses of this are much more sophisticated and eventually consider pressure within the glands. More details can be found in Smith, et.al. PNAS, **115** (26), E5887–E5895 (2018).

152 Pushing a curling stone across a frozen ice sheet

A 20 kg curling (a type of sport) stone lies at rest on a horizontal sheet of ice. The stone is pushed by a player in a straight line with a constant horizontal force for a distance of 3.0 m and is then released. It travels in a straight line for a further distance of 42 m in 20 s. There is no spin on the stone. Determine the magnitude of the force exerted by the player on the stone. (131Sp2025)

153 Forces and two dimensional motion

A 2.0 kg rock slides along a horizontal surface. At the moment that it passes the x = 1.0 m, y = 0.0 m mark it is moving with the illustrated velocity. During the next 4.0 s a constant force $8.0 \text{ h}\hat{\mathbf{i}}$ acts on the rock. (131Sp2025)

- a) Determine the rock's position and velocity at the instant 4.0 s later.
- b) Describe and sketch, as accurately as possible, the trajectory of the rock while the force acts on it.

154 Net force vector: two forces

Two forces, $\vec{\mathbf{F}}_1$ and $\vec{\mathbf{F}}_2$ act on an object and are as illustrated. Their magnitudes are $F_1 = 400$ N and $F_2 = 500$ N. (131Sp2025)

- a) Determine the x and y components of the net force.
- b) Determine the magnitude of the net force acting on the object.

155 Net force vector: three forces

Three forces, $\vec{\mathbf{F}}_1, \vec{\mathbf{F}}_2$ and $\vec{\mathbf{F}}_3$ act on an object and are as illustrated. Their magnitudes are $F_1 = 40 \text{ N}, F_2 = 25 \text{ N}$ and $F_3 = 15 \text{ N}.$ (131Sp2025)

- a) Determine the x and y components of the net force.
- b) Determine the magnitude of the net force acting on the object.







156 Pulling a box

A 20.0 kg box can move along a frictionless horizontal surface. A person pulls on the box with the indicated force. Initially assume that the force pulls horizontally; later we will consider a general angle. (131Sp2025)



- a) Draw a free body diagram for the box.
- b) Write Newton's Second Law in its component form, i.e. write

$$F_{\text{net }x} = \Sigma F_x = \cdots$$

$$F_{\text{net }y} = \Sigma F_y = \cdots$$

Insert as much information as possible about the components of acceleration at this stage. You will return to these equations soon; they will generate the algebra that eventually gives you the acceleration and the normal force.

c) List all the components of all the forces, using one of the two formats below.

$F_{gx} = \cdots$	Force	$x \operatorname{comp}$	$y \operatorname{comp}$
$F_{gy} = \cdots$ $n_x = \cdots$	$ec{\mathbf{F}}_{g}$		
$n_y = \cdots$	ñ		
$F_{\text{person } x} = \cdots$ $F_{\text{person } y} = \cdots$	$ec{\mathbf{F}}_{ ext{person}}$		

Use these to determine expressions for the components of the net force.

- d) Use Newton's second law in component form to relate the acceleration components to the forces.
- e) Determine the normal force on the box.
- f) Determine the acceleration of the box.
- g) Now suppose that $\theta = 25^{\circ}$ and repeat the entire exercise to determine the normal force and the acceleration.

157 Rope pulling a box

A 45.0 kg box can move along a frictionless horizontal surface. A rope pulls on the box with tension T at angle θ . (131Sp2025)

- a) Do you expect that the normal force exerted by the floor depends on the tension and angle of rope pull?
- b) Draw a free body diagram for the box.
- c) Write Newton's Second Law in its component form and insert as much information as possible about the components of acceleration at this stage. These equations will generate the algebra that eventually gives you the acceleration and the normal force.
- d) List all the components of all the forces.
- e) Use Newton's second law in component form to relate the acceleration components to the force components.
- f) Determine an expression for the acceleration of the box.
- g) Determine an expression for the normal force on the box. Does the expression support your expectations about how normal force depends on the tension and the angle of pull?
- h) Determine the acceleration and normal force when the rope pulls horizontally with force $275\,\mathrm{N}.$
- i) Determine the acceleration and normal force when the rope pulls midway between horizontally and vertically 275 N.



158 Dynamics of a single object

A 5.0 kg box can move along a frictionless horizontal surface. A person exerts a force at the illustrated angle. The aim of this exercise is to use Newton's laws to determine the acceleration of the box and the normal force exerted by the surface (provided that the box stays on the surface). (131Sp2025)



The *entire collection* of these steps is called "applying Newton's laws of mechanics to determine the acceleration of the block."

- a) Draw a free body diagram for the block.
- b) Write Newton's Second Law in its component form, i.e. write

$$F_{\text{net }x} = \Sigma F_x = \cdots \tag{1}$$

$$F_{\text{net }y} = \Sigma F_y = \cdots \tag{2}$$

Insert as much information as possible about the components of acceleration at this stage. You will return to these equations shortly; they will generate the algebra that eventually gives you the acceleration and the normal force.

- c) Determine the magnitude of the gravitational force. Let n be the magnitude of the normal force. Do you think that n = mq?
- d) List all the components of all the forces, using one of the two formats below.

$F_{gx} = \cdots$	Force	$x \operatorname{comp}$	$y \operatorname{comp}$
$F_{gy} = \cdots$ $n_x = \cdots$	$ec{\mathbf{F}}_{g}$		
$n_y = \cdots$	ñ		
	:		

- e) Use Eq. (1) to obtain an equation relating various quantities that appear in this problem. Do the same with Eq. (2). Solve these for the acceleration and the magnitude of the normal force. Is n = mg?
- f) Suppose that rather than pull up, the person pushed down on the box at the same angle from the left and with the same force. Would the acceleration and normal forces differ from the case where the person pulled up?
- g) You may have noticed that the acceleration does not depend on the normal force. This is only true if there is no friction. It turns out that when friction is present, the magnitude of friction force increases as the normal force increases. Knowing this, would pulling up or pushing down give a larger acceleration?

159 Multiple forces and motion

A 20.0 kg box can move along a frictionless horizontal surface. Three people pull horizontally on the box with the indicated forces. (131Sp2025)



- a) Draw a free body diagram for the box.
- b) Write Newton's Second Law in its component form, i.e. write

$$F_{\text{net }x} = \Sigma F_x = \cdots$$

$$F_{\text{net }y} = \Sigma F_y = \cdots$$

Insert as much information as possible about the components of acceleration at this stage. You will return to these equations soon; they will generate the algebra that eventually gives you the acceleration and the normal force.

c) List all the components of all the forces, using one of the two formats below.

$F_{1x} = \cdots$	Force	$x \operatorname{comp}$	$y \operatorname{comp}$
$F_{1y} = \cdots$ $F_{2x} = \cdots$	$ec{\mathbf{F}}_1$		
$F_{2y} = \cdots$	$ec{\mathbf{F}}_2$		
$F_{3y} = \cdots$	$\vec{\mathbf{F}}_3$		

Use these to determine the components of the net force.

- d) Use Newton's second law, which you wrote earlier, to determine the components of the acceleration of the box.
- e) If the box were moving right while these forces act would it slow down or speed up?
- f) If the box were moving left while these forces act would it slow down or speed up?

160 Lowering hippos

Two hippos are lowered vertically by cables. The hippos have the same mass. The red hippo is lowered at a constant speed of 7.0 m/s and the green hippo at a constant speed of 14.0 m/s. Let $T_{\rm red}$ be the tension in the rope lowering the red hippo and $T_{\rm green}$ be the tension in the rope lowering the green hippo. Which of the following is true? Explain your answer. (131Sp2025)



- i) $T_{\text{green}} < T_{\text{red}}/2$
- ii) $T_{\text{green}} = T_{\text{red}}/2$
- iii) $T_{\rm red}/2 < T_{\rm green} < T_{\rm red}$
- iv) $T_{\text{green}} = T_{\text{red}}$
- v) $T_{\rm red} < T_{\rm green} < 2T_{\rm red}$
- vi) $T_{\text{green}} = 2T_{\text{red}}$
- vii) $T_{\text{green}} > 2T_{\text{red}}$

161 Lowering a box with ropes

Two ropes are attached to the top of a 75 kg box. They both pull vertically up with the same tension. The box moves downwards with constant downward acceleration of 2.5 m/s^2 . Determine the tension in each rope. (131Sp2025)

162 Lowering a bunch of bananas

A bunch of bananas has mass 40 kg. Starting from rest, they are lowered from a ship by a rope through a distance of 12.0 m. The rope pulls vertically upward with tension 305 N and there is an upward air resistance force of 60 N. Determine the time taken to lower the bananas. (131Sp2025)

163 Box on the floor of an elevator

A 20 kg box sits on the floor of an elevator. (131Sp2025)

- a) During an initial 1.5s period the elevator moves upward with a speed that increases steadily from 0 m/s to 7.5 m/s. Determine the normal force exerted by the floor on the box during this period.
- b) During an later 2.5 s period the elevator moves upward with a speed that decreases steadily from 7.5 m/s to 0 m/s. Determine the normal force exerted by the floor on the box during this period.

164 Alice in an elevator

Alice, with mass 58 kg stands in an elevator, which moves vertically. The elevator accelerates (at a constant rate) from rest to speed of 4.0 m/s during a period of 1.5 s. It then moves with a constant velocity for a period of 10.0 s. Finally it slows to a stop at a constant rate during a period of 2.0 s. (131Sp2025)

- a) Determine the total distance traveled by Alice during the entire journey.
- b) Determine the normal force exerted by the elevator floor on Alice during each of the three stages of motion.

165 Free fall in an elevator

A phone with mass m sits on the floor of an elevator, which is initially at rest. The elevator cable snaps and the elevator and phone then undergo free fall (i.e. both move downward with an acceleration g). While they do this which is true of the magnitude of the normal force, n, acting on the phone? Explain your choice. (131Sp2025)

i) n = 0. ii) mg > n > 0. iii) n = mg.

iv) n > mg

166 Normal forces

Two identical boxes are at rest on a rough horizontal surface. A person pulls on each with the same force but in different directions on the boxes. On box A it pulls vertically and on box B it pulls at an angle of 45° from the vertical. Which of the following is true regarding the normal force exerted on A and that exerted on B? Explain your choice. (131Sp2025)



- i) $n_{\rm A} = n_{\rm B}$.
- ii) $n_{\rm A} < n_{\rm B}$.
- iii) $n_{\rm A} > n_{\rm B}$.

167 Accelerating box

A rope pulls on a 30 kg box which slides across a frictionless horizontal surface. The box moves along the surface to the right with acceleration 3.5 m/s^2 . (131Sp2025)

- a) Determine the tension in the rope.
- b) Determine the normal force exerted by the floor on the box.



168 Multiple forces and motion

The illustrated forces act on an 80.0 kg box that slides along a frictionless horizontal surface. (131Sp2025)

- a) Determine the acceleration of the box.
- b) Suppose that at one moment the box was moving left. Will it slow down, speed up or move with constant velocity in the next moments? Explain your answer.
- c) Suppose that at one moment the box was moving right. Will it slow down, speed up or move with constant velocity in the next moments? Explain your answer.

169 Multiple forces on a single object

A 25.0 kg crate can move along a frictionless horizontal surface. Two people exert forces on the crate as illustrated. (131Sp2025)

- a) Determine the acceleration of the crate.
- b) Determine the normal force on the crate.

170 Suspended ring in equilibrium, 1

A 2.50 kg ring is suspended from the ceiling and is held at rest by two ropes as illustrated. Rope 2 pulls horizontally. The aim of this exercise is to use Newton's 2nd Law to determine the tension in each rope. One piece of background information that you will need to answer this is that the magnitude of the gravitational force on an object of mass m is $F_q = mg$. (131Sp2025)

- a) Draw a free body diagram for the ring. Label the tension forces $\vec{\mathbf{T}}_1$ and $\vec{\mathbf{T}}_2$.
- b) Write Newton's 2nd Law in its component form, i.e. write

$$F_{\text{net }x} = \Sigma F_{ix} = ma_x \tag{3}$$

$$F_{\text{net }y} = \Sigma F_{iy} = ma_y \tag{4}$$

Insert as much information as possible about the acceleration. You will return to these equations shortly; they will generate the algebra that eventually gives you the tensions.

c) These equations require all components of all forces, including the two unknown tension forces. In order to manage these, you should express the components of each tension force in terms of its magnitude. When doing this denote the magnitude of the tension in rope 1 by T_1 and for rope 2, by T_2 .







d) List as much information as possible about each component for each force; each could be a number or an algebraic expression. Use one of the two formats below. ╗

$F_{gx} = \cdots$	Force	$x \operatorname{comp}$	$y \operatorname{comp}$
$F_{gy} = \cdots$ $T_{1x} = \cdots$	$ec{\mathbf{F}}_{g}$		
$T_{1y} = \cdots$	$ec{\mathbf{T}}_1$		
: :	:		

Ē

- e) Use Eq. (3) to obtain an equation relating various quantities that appear in this problem. Do the same with Eq. (4). You should get two expressions that contain the two unknowns T_1 and T_2 . Solve them for the unknowns.
- f) If you had one rope that is rated to break when the tension exceeds 30 N and another rated to break when the tension exceeds 40 N, which one would you use to suspend the object as illustrated above?

171 Suspended ring in equilibrium, 2

A 2.50 kg ring is suspended from the ceiling and by two ropes as illustrated. The aim of this exercise is to use Newton's 2^{nd} Law to determine the tension in each rope. (131Sp2025)

- a) Draw a free body diagram for the ring. Label the tension forces $\vec{\mathbf{T}}_1$ and $\vec{\mathbf{T}}_2$.
- b) Write Newton's 2nd Law in its component form, i.e. write

$$F_{\text{net }x} = \Sigma F_{ix} = ma_x \tag{5}$$
$$F_{\text{net }y} = \Sigma F_{iy} = ma_y \tag{6}$$



Insert as much information as possible about the acceleration. You will return to these equations shortly; they will generate the algebra that eventually gives you the tensions.

- c) These equations require all components of all forces, including the two unknown tension forces. In order to manage these, you should express the components of each tension force in terms of its magnitude. When doing this denote the magnitude of the tension in rope 1 by T_1 and for rope 2, by T_2 .
- d) List as much information as possible about each component for each force; each could be a number or an algebraic expression. Use one of the two formats below.

$F_{gx} = \cdots$	Force	$x \operatorname{comp}$	$y \operatorname{comp}$
$F_{gy} = \cdots$ $T_{1x} = \cdots$	$ec{\mathbf{F}}_{g}$		
$T_{1y} = \cdots$	$ec{\mathbf{T}}_1$		
:	÷		

Ē

e) Use Eq. (5) to obtain an equation relating various quantities that appear in this problem. Do the same with Eq. (6). You should get two expressions that contain the two unknowns T_1 and T_2 . Solve them for the unknowns.

172 Suspended ball

A 6.0 kg ball is suspended from the ceiling. A person pushes horizontally on the ball so that the ball is at rest with the rope at an angle of 70° from the horizontal. Determine the force exerted by the person. (131Sp2025)

173 Object suspended at rest

A 0.20 kg block is suspended by two ropes as illustrated. The tension in the left rope is T_1 and in the right rope it is T_2 . (131Sp2025)

- a) Is T_1 the same as or different to T_2 ? Explain your answer.
- b) Determine the magnitude of the tension in each rope.



174 Stretched clothesline

A clothesline (rope) initially droops approximately horizontally between two poles. A hanger is suspended at rest from the midpoint of the clothesline. (131Sp2025)

- a) Is it possible to stretch the clothes line so that it is perfectly horizontally while the hanger is suspended? Explain your answer using Newton's laws.
- b) The hanger is removed. Is it possible to stretch the clothes line so that it is perfectly horizontally? Explain your answer using Newton's laws.

175 Slackline

A 75.0 kg person is at the midpoint of a slackline (cord stretched between two supports). The angle between the slackline and the horizontal is 6.0° . Determine the tension in the slackline. (131Sp2025)



176 Dangling monkey

A 15.0 kg monkey hangs from a rope stretched between the tops of two equally tall supports in the illustrated configuration. Determine the tension in the rope left of the monkey and the tension in the rope right of the monkey. (131Sp2025)



177 Sledding down a slope

A sled and person, with combined mass 100 kg slide down a flat frictionless surface that is angled at 20° above the horizontal. (131Sp2025)

- a) Determine the acceleration of the sled and person.
- b) Determine the normal force on the sled and person.

178 Object pulled along a ramp

A 4.0 kg box can move along a frictionless ramp angled 30° from the horizontal. A person pulls on a rope which exerts a force of 15 N up the ramp parallel to its surface. The object of this exercise is to determine the acceleration of the box. (131Sp2025)



- a) Draw a free body diagram for the box.
- b) Describe the x and y axes that you will use.
- c) Write Newton's Second Law in vector form and also in its component form, i.e. write

$$F_{\text{net }x} = \Sigma F_{ix} = \cdots \tag{7}$$

$$F_{\text{net }y} = \Sigma F_{iy} = \cdots \tag{8}$$

Insert as much information as possible about the components of acceleration at this stage. The resulting equations will generate much of the algebra that follows.

- d) Determine the magnitude of the gravitational force.
- e) List all the components of all the forces, using one of the two formats below.

$F_{\mathrm{g}x} = \cdots$	Force	$x \operatorname{comp}$	$y \operatorname{comp}$
$F_{gy} = \cdots$ $n_x = \cdots$	$ec{\mathbf{F}}_{ ext{g}}$		
$n_y = \cdots$	ñ		
:	÷		

T

- f) Use Eq. (7) to obtain an equation relating various quantities that appear in this problem. Do the same with Eq. (8).Use the resulting equations to determine the acceleration of the box.
- g) Is it possible to say with certainty whether the box is moving up the ramp or down the ramp? Is either direction possible in this situation? If only one direction is possible, which is it?
- h) Suppose that the box is initially at rest. With the indicated applied force, how long will it take for the box to slide a distance of 2.0 m along the ramp?

179 Crate dragged up a ramp

A crate, with mass m, can move along a frictionless ramp angled 20° from the horizontal. A rope is attached to the crate and it pulls parallel to the ramp and upward with a tension T. (131Sp2025)

a) Draw a free body diagram for the box.



- b) Describe the x and y axes that you will use.
- c) Use the system of Newton's Second Law (i.e. component version of the law, acceleration and force components) to determine an expression for the acceleration of the crate.
- d) Consider a crate with mass 15 kg and a rope pulling with tension 75 N. Is it possible to say with certainty whether the crate is moving up the ramp or down the ramp? Is either direction possible in this situation? If only one direction is possible, which is it?
- e) Suppose that the crate is initially at rest. How long will it take for it to slide 1.5 m along the ramp?

180 Bathroom scale in a train

A bathroom scale reads the normal force that it exerts on the object on the scale. Suppose that a bathroom scale is fixed to the floor of a train and a person stands on the scale. The train travels at a constant speed along horizontal sections of track and then up and down inclined sections (the slopes of the inclined sections are the same). In the following explain your answers. (131Sp2025)

- a) Is the reading on the scale when traveling up the inclined section the same as, smaller than or larger than when traveling along the horizontal section?
- b) Is the reading on the scale when traveling down the inclined section the same as, smaller than or larger than when traveling up the inclined section?
- c) Would the reading on the scale while the train travels up the inclined section depend on the speed with which it travels?

181 Blocks on ramps

Two blocks with the same mass are at rest on the illustrated inclined surfaces. Which of the following is true about the magnitudes of the normal forces that act on the blocks? Explain your answer. (131Sp2025)

- i) $n_{\rm A} > n_{\rm B}$.
- ii) $n_{\rm A} = n_{\rm B}$.
- iii) $\frac{1}{2}n_{\rm B} < n_{\rm A} < n_{\rm B}$.
- iv) $n_{\rm A} = \frac{1}{2} n_{\rm B}$.
- v) $n_{\rm A} < \frac{1}{2} n_{\rm B}$.



182 Different boxes on the same ramp

Two boxes, one red (mass 3.0 kg) and the other blue (mass 6.0 kg) are each on frictionless ramps inclined at an angle of 10° from the horizontal. They are released from rest. Let a_{red} be the acceleration of the red box and a_{blue} that of the blue box. Which of the following is true? Explain your answer. (131Sp2025)

- i) $a_{\text{blue}} = \frac{1}{2} a_{\text{red}}$
- ii) $a_{\text{blue}} = a_{\text{red}}$
- iii) $a_{\text{blue}} = 2a_{\text{red}}$
- iv) $a_{\text{blue}} > 2a_{\text{red}}$

183 Snowboarder sliding up a ramp

An 80 kg snowboarder slides along a horizontal surface with a speed of 25 m/s and approaches a surface inclined at 10° above the horizontal. Both surfaces are frictionless and you can ignore air resistance. (131Sp2025)

- a) Determine the acceleration of the snowboarder along the inclined surface.
- b) Determine how far along the inclined surface the snowboarder slides before stopping.
- c) Determine the maximum vertical height ascended by the snowboarder along the ramp.
184 Launch force on a cannonball

A 40 kg cannonball is launched at an angle of θ above the horizontal. The cannonball is initially at rest at the ground level and reaches height h. The length of the cannon tube is L. The aim of this exercise is to determine the launching force exerted on the cannonball, assuming that it is constant and that it points parallel to the tube. (131Sp2025)



a) Show that the launch speed is

$$v_{\text{launch}} = \frac{\sqrt{2g\left(h - L\sin\theta\right)}}{\sin\theta}$$

b) Show that the magnitude of the acceleration of the cannonball, while in the cannon tube is

$$a = g \frac{h - L\sin\theta}{L\sin^2\theta}.$$

c) Show that the launching force exerted by the cannon has magnitude

$$F = \frac{mg}{L\sin^2\theta} \left(h - L\sin\theta\cos^2\theta \right).$$

185 Blow dart

A toy blow dart gun fires a 60 g dart through a tube. The tube has length 40 cm and is angled at 75° from the horizontal. The dart is propelled from rest by air which exerts a constant force and it reaches a height of 6.0 m above the upper end of the tube. Ignore air resistance. (131Sp2025)

- a) Determine the launch velocity of the dart.
- b) Determine the magnitude of the acceleration of the dart while in the tube.
- c) Determine the force exerted by the propelling air while the dart is in the tube.

186 Towing a car up a ramp, 1

A rope pulls a 2200 kg car up an icy ramp which is tilted at angle 10° above the horizontal. The rope pulls parallel to the ramp with tension 4500 N. Determine the acceleration of the car, ignoring friction and air resistance. (131Sp2025)

187 Towing a car up a ramp, 2

A rope pulls a 2500 kg car up an icy ramp which is tilted at angle 15° above the horizontal. The rope pulls with a constant force parallel to the ramp. The car is at rest at the bottom of the ramp and while the rope pulls it travels a distance 54 m along the ramp in 6.0 s. Determine the tension in the rope. Ignore friction and air resistance. (131Sp2025)

188 Sledding on a slope

A person in a sled is at rest at the top of a slope that is angled 15° above the horizontal. The length of the slope is an 800 m (about 0.5 mi). The combined mass of the person and sled is 90 kg. They are released from the top of the slope and slide straight down without pushing. While this happens the air exerts a constant force of 150 N exactly opposite to the direction in which they move. Ignore friction between the sled and the slope. (131Sp2025)

- a) Determine the acceleration of the person and sled. You must solve this by starting with a FBD, using Newton's second law, finding components,.... Simply looking up a formula is not adequate.
- b) Determine the time taken for the person and sled to reach the bottom of the slope.
- c) Determine the speed of the person and sled at the bottom of the slope.

189 Pushing a box horizontally

A box with mass m can move along a rough horizontal surface. A person pushes the box with a horizontal force with magnitude F_0 to the right and the box moves with constant speed v_0 to the right. (131Sp2025)

- a) Which of the following is true? Explain your answer.
 - i) $F_0 > mg$ regardless of the materials of the box and the surface.
 - ii) $F_0 = mg$ regardless of the materials of the box and the surface.
 - iii) $F_0 < mg$ regardless of the materials of the box and the surface.
 - iv) None of the above.
- b) Someone else pushes the box horizontally and it moves with constant speed larger than v_0 . Which of the following is true regarding the force exerted F in this case? Explain your answer.
 - i) $F < F_0$.
 - ii) $F = F_0$.
 - iii) $F > F_0$.
- c) A third person pushes the box horizontally with force $F = 3F_0$. Which of the following is true regarding the speed of the box, v, at all moments while this person pushes? Explain your answer.
 - i) $v = v_0$.
 - ii) $v_0 < v < 3v_0$.
 - iii) $v = 3v_0$.
 - iv) $v > 3v_0$.
 - v) None of the above.

190 Dragging a box horizontally

A 25 kg box can move along a rough horizontal surface. The coefficient of static friction between the box and surface is 0.30. A rope pulls horizontally on the box. Determine the minimum force needed to move the box from rest. (131Sp2025)



191 Dynamics of a single object with friction

A 15.0 kg box moves rightward along a horizontal surface. A rope pulls with a force at the illustrated angle. The coefficient of kinetic friction is 0.250. The primary aim of this exercise will be to determine the acceleration of the box. (131Sp2025)

- 50.0 N
- a) Draw a free body diagram for the block.
- b) Write Newton's Second Law in its component form, i.e. write

$$F_{\text{net }x} = \Sigma F_x = \cdots \tag{9}$$

$$F_{\text{net }y} = \Sigma F_y = \cdots \tag{10}$$

Insert as much information as possible about the components of acceleration at this stage. Can you describe in words what these equations are telling you to do?

- c) Determine the magnitude of the gravitational force. Let n be the magnitude of the normal force. Using this write an expression for the magnitude of the friction force. Do you know the exact number for the friction force at this point?
- d) List all the components of all the forces, using one of the two formats below.

$w_x = \cdots$	Force	$x \operatorname{comp}$	$y \operatorname{comp}$
$w_y = \cdots$ $n_x = \cdots$	$\vec{\mathbf{w}}$		
$n_y = \cdots$	ñ		
:	:		

- e) Use Eq. (9) to obtain an equation relating various quantities that appear in this problem. Do the same with Eq. (10). Does either give the acceleration immediately? Can one of then at least give the normal force immediately?
- f) Determine the normal force and use this result to find the acceleration.
- g) What tension would be required for the box to have acceleration 4.00 m/s^2 to the right?

192 Dynamics of a single object with friction, 2

A 15.0 kg box moves rightward along a horizontal surface. A rope pulls with a force at the illustrated angle. The coefficient of kinetic friction is 0.350. Use *all of* the steps of Newton's Second Law to determine the acceleration of the box. (131Sp2025)



193 Pushing a box at constant speed

A 10 kg box can move along a rough horizontal surface. You discover that when you push horizontally with a force of 50 N that the box moves with constant speed. Which of the following is true about the magnitude of the friction force? Explain your answer. (131Sp2025)

- i) $f = 10 \, \text{N}$
- ii) $f = 50 \,\mathrm{N}$
- iii) $f = 98 \,\mathrm{N}$
- iv) f is more than 50 N but less than 98 N.
- v) f is more than 98 N.

194 Pushing crates along a rough surface

Two people, Jordi and Agnes, each push horizontally on identical crates, which move at constant speeds along the same rough horizontal surface. Jordi pushes his crate faster than Agnes pushes her crate. Which of the following is true? Explain your answer. (131Sp2025)

- i) Jordi exerts a larger force than Agnes.
- ii) Jordi exerts a smaller force than Agnes.
- iii) Jordi exerts the same force as Agnes.

195 Dragon on a cart

A dragon rides on the horizontal surface of a wooden cart. The cart moves along another horizontal surface. The dragon does not slip on the cart's surface. (131Sp2025)

a) The cart moves with increasing speed to the right and so does the dragon. Use this to determine the direction of the dragon's acceleration and use that to determine the direction of the friction force exerted by the cart on the dragon.



b) The cart moves with decreasing speed to the left and so does the dragon. Use this to determine the direction of the dragon's acceleration and use that to determine the direction of the friction force exerted by the cart on the dragon.

196 Crate on a truck

A crate lies on the flat, horizontal bed of a truck. The truck moves along a horizontal surface. In the following the truck moves so that the crate does not slip along the bed of the truck. (131Sp2025)

- a) Is there a friction force between the crate and the truck when they both move north at the same constant speed along a straight horizontal road? Explain your answer.
- b) Suppose that the truck speeds up while heading North. In which direction does the friction force point when it does this? Explain your answer.
- c) Suppose that the truck slows down while heading North. In which direction does the friction force point when it does this? Explain your answer.

Explain your answers. Hint: Consider applying Newton's second law to the crate.

197 Braking car

A 2000 kg car moves along a horizontal concrete road with speed 24 m/s. At one moment the driver applies the brakes sharply and the tires slide without rotating across the concrete surface. The coefficient of static friction in this case is 0.90 and the coefficient of kinetic friction is 0.60. Air resistance provides a 3500 N force acting opposite to the car's motion. (131Sp2025)

- a) Determine the acceleration of the car.
- b) Determine the distance which the car travels while the brakes are applied and it skids.

198 Sliding oak box

A person tries to control a 16 kg wooden (oak) box that slides down a ramp by pulling on a rope attached to the box. The box speeds up as it slides down the ramp and then moves right along a horizontal wooden (oak) surface. At this stage the rope is at an angle of 25° above the horizontal and the tension in the rope is 85 N. The coefficient of kinetic friction between two oak surfaces is 0.32. Determine the acceleration of the oak box as it slides along the horizontal surface. (131Sp2025)



199 Dragging a crate

A crate with mass m is dragged along a rough horizontal surface by a rope that pulls with tension T. The rope can pull at various angles. Suppose that you want to produce the largest acceleration for a given tension. (131Sp2025)

- a) Explain what the drawbacks, in terms of attaining higher acceleration, would be of pulling at a larger angle.
- b) Explain what the benefits, in terms of attaining higher acceleration, would be of pulling at a larger angle.
- c) Determine an expression for the acceleration in terms of T, m, g, θ , and the coefficient of kinetic friction. Explain how you could use this to find the angle that would give the optimal acceleration.

200 Speed at the bottom of a rough ramp, 1

A 10 kg box can move along a 4.0 m long rough ramp angled 20° from the horizontal. The coefficient of kinetic friction between the box and the ramp is 0.25 and the coefficient of static friction is 0.30. The box is released from rest at the top of the ramp and moves down the ramp. The aim of this exercise is to determine the speed of the box at the bottom of the ramp. (131Sp2025)



- a) Draw a free body diagram for the box.
- b) Describe your choice of x and y axes.
- c) Write Newton's Second Law in vector form and also in its component form, i.e. write

$$F_{\text{net }x} = \Sigma F_x = \cdots \tag{11}$$

$$F_{\text{net }y} = \Sigma F_y = \cdots \tag{12}$$

where x and y refer to your specially chosen axes. Insert as much information as possible about the components of acceleration at this stage. The resulting equations will generate much of the algebra that follows.

- d) Determine expressions for the *magnitudes* of the gravitational and the friction forces.
- e) List all the components of all the forces, using one of the two formats below.

$F_{gx} = \cdots$	
$F_{gy} = \cdots$	
$n_x = \cdots$	
$n_y = \cdots$	
:	

Force	$x \operatorname{comp}$	$y \operatorname{comp}$
$ec{\mathbf{F}}_{ ext{g}}$		
ñ		
:		



- f) Use Eq. (11) to obtain an equation relating various quantities that appear in this problem. Do the same with Eq. (12). Use the resulting equations to find the acceleration of the box.
- g) Determine the speed of the box when it reaches the bottom of the ramp.
- h) Do these results depend on the mass of the box?
- i) What would be the minimum force, pushing parallel to the ramp, required to keep the box at rest?

201 Speed at the bottom of a rough ramp, 2

A 25 kg box can move along a 6.0 m long rough ramp angled 18° from the horizontal. The coefficient of kinetic friction between the box and the ramp is 0.25 and the coefficient of static friction is 0.30. The box is released from rest at the top of the ramp and moves down the ramp. (131Sp2025)



- a) Determine the speed of the box when it reaches the bottom of the ramp.
- b) Does the speed depend on the mass of the box?
- c) What would be the minimum force, pushing parallel to the ramp, required to keep the box at rest?

202 Perfect friction on a rough ramp

A box can move along a rough ramp which makes an angle θ from the horizontal and which has length L. The box is launched with speed v from the top of the ramp. The aim of this exercise is to determine the coefficient of friction needed to bring the box to a stop at the bottom of the ramp. (131Sp2025)



- a) The first part of the solution uses kinematics to assess the acceleration of the box. Using kinematics, and eventually dynamics, is greatly simplified by choosing an appropriate "x" and a "y" axis. These do not have to be along the usual vertical and horizontal directions. Regardless of the axes that you choose, the usual general kinematics and dynamics equations will be valid. Describe the "x" and "y" axes that you will use.
- b) Determine an expression for the magnitude of the acceleration of the box, a, in terms of variables relevant to this problem, such as L, θ, v , and possibly the mass of the box, m.
- c) Draw a free body diagram for the box.
- d) Write Newton's Second Law in vector form and also in its component form, i.e. write

$$F_{\text{net }x} = \Sigma F_{ix} = \cdots \tag{13}$$

$$F_{\text{net }y} = \Sigma F_{iy} = \cdots \tag{14}$$

where x and y refer to your specially chosen axes. Insert as much information as possible about the components of acceleration at this stage. The resulting equations will generate much of the algebra that follows.

- e) Determine expressions for the *magnitudes* of the gravitational and the friction forces.
- f) List all the components of all the forces, using one of the two formats below.

$F_{gx} = \cdots$	Force	$x \operatorname{comp}$	$y \operatorname{comp}$
$F_{gy} = \cdots$ $n_x = \cdots$	$ec{\mathbf{F}}_{g}$		
$n_y = \cdots$	ñ		
:	:		

- g) Use Eq. (13) to obtain an equation relating various quantities that appear in this problem. Do the same with Eq. (14). Use the resulting equations to find an expression for the coefficient of kinetic friction. Does the result depend on the mass of the box?
- h) Suppose that the ramp is a roof whose length is $5.0 \,\mathrm{m}$ and which is angled at 15° from the horizontal. If the box is pushed with speed $4.0 \,\mathrm{m/s}$, determine the coefficient of friction needed to stop the box at the bottom of the roof.

203 Box pushed up a rough ramp



θ

a = formula involving $m, \theta, F_{\rm p}, \mu_{\rm k}, g$ and constants.

The *entire collection* of these steps is called "applying Newton's laws of mechanics to determine the acceleration of the block." (131Sp2025)

- a) Draw a free body diagram for the block.
- b) Describe the x and y axes that you will use.
- c) Write Newton's Second Law in vector form and also in its component form, i.e. write

$$F_{\text{net }x} = \cdots$$
 (15)

$$F_{\text{net }y} = \cdots$$
 (16)

Insert as much information as possible about the components of acceleration at this stage. The resulting equations will generate much of the algebra that follows.

- d) Determine expressions for the magnitudes of all the friction and gravitational forces. Do you think that n = mg?
- e) List all the components of all the forces, using one of the two formats below.

$F_{gx} = \cdots$	Force	$x \operatorname{comp}$	$y \operatorname{comp}$
$F_{gy} = \cdots$ $n_x = \cdots$	ñ		
$n_y = \cdots$	÷		
:	÷		

f) Note that $F_{\text{net }x} = \sum_{i} F_{ix}$ (i.e. the *x* component of the net force is the sum of the *x* components of individual forces). Use this and Eq. (15) to obtain an equation relating various quantities that appear in this problem. Do the same with Eq. (16). Use the resulting algebraic expressions to get an expression for *n*. Is this *mg*? Use the resulting equations to get an expression for *a*.

204 Crate sliding along a rough ramp

A 15 kg crate can slide up or down a ramp at angle 25° from the horizontal. The coefficient of kinetic friction between the crate and the ramp is 0.30. (131Sp2025)

- a) Determine the crate's acceleration if it slides up the ramp.
- b) Determine the crate's acceleration if it slides down the ramp.

205 Crate pushed up a rough ramp

A 12 kg crate can slide up or down a ramp at angle 20° from the horizontal. The coefficient of kinetic friction between the crate and the ramp is 0.40. A hand pushes with a 120 N force parallel to the ramp in the uphill direction. Determine the crate's acceleration if it slides up the ramp. (131Sp2025)

206 Box held at rest on a rough ramp

A 30 kg block lies on a ramp at angle 25° from the horizontal. A person pushes parallel to the surface and uphill with a 80 N force. Determine the minumum coefficient of static friction so that the box stays at rest. (131Sp2025)

207 Box on a ramp

A 100 kg box is at rest on a ramp that is inclined at angle 25° from the horizontal. (131Sp2025)

- a) Determine the friction force on the box.
- b) Determine the minimum coefficient of static friction between the box and ramp that allows the box to stay at rest.







208 Brick on a slope

A 1.50 kg brick is at rest on an inclined surface. The angle between the surface and the horizontal is 10° . (131Sp2025)

- a) Determine the friction force that the surface exerts on the brick.
- b) Determine the minimum coefficient of static friction between these surfaces that allows for the brick to stay at rest.

209 Car on a slope

A car with mass 1500 kg is at rest on an incline. The coefficient of static friction between dry asphalt and rubber here is 0.90 and the coefficient of static friction between wet asphalt and rubber is 0.40. (131Sp2025)

- a) Determine the maximum angle of the incline so that the car will stay at rest when the surface is dry.
- b) Determine the maximum angle of the incline so that the car will stay at rest when the surface is wet.

210 Launching a box up a ramp

A ramp with length 2.0 m is inclined at an angle of 15° above the horizontal. A tiny box with mass 0.25 kg is launched with a very brief kick imparting speed 2.75 m/s at the base of the ramp. The coefficient of kinetic friction between the box and slope is 0.20. (131Sp2025)

- a) Determine the acceleration of the box as it slides up the ramp.
- b) Determine whether the box reaches the other end of the ramp or not.

211 Box pushed up a rough ramp

A 6.00 kg box slides up a ramp at angle 25.0° from the horizontal. The coefficient of kinetic friction between the surfaces is 0.450. A person pushes horizontally with a 80.0 N force as illustrated. Determine the acceleration of the box. (131Sp2025)



212 Book held against a wall

A 10.0 kg book lies against a vertical wall and a hand pushes on it as illustrated. The coefficient of static friction between the book and wall is 0.30. The coefficient of kinetic friction between the book and wall is 0.20. (131Sp2025)

- a) Determine the minimum force needed so that the book stays at rest on the wall.
- b) Determine the magnitude of the force exerted by the hand so that the book slides down the wall with constant speed.

213 Box on a sloped cart

A cart moves along a horizontal surface. The upper surface of the cart slopes at a 20° angle to the horizontal. A 3.0 kg box lies on the upper surface of the cart and is at rest relative to the cart. (131Sp2025)

- a) Suppose that the cart moves right with a constant speed of $4.5 \,\mathrm{m/s}$. Determine the magnitude and direction of the friction force between the cart and the box.
- b) Suppose that the cart moved left with the same speed. Would the magnitude and direction of the friction force be different from when it moved right?





214 Ice skaters

Two ice skaters, Alice with mass 55 kg and Bob with mass 85 kg, are initially at rest on a frictionless horizontal surface. Bob gently pushes on Alice and while he does this she has a constant acceleration of 0.020 m/s^2 . (131Sp2025)

- a) Determine the force exerted by Bob on Alice while she accelerates.
- b) Determine the force exerted by Alice on Bob while Alice accelerates.
- c) Determine the acceleration of Bob during the period while he is pushing on Alice.

215 Coin on a board

A silver coin, with mass m, lies on a horizontal board. Let $F_{\rm b \ on \ c}$ be the magnitude of the force exerted by the board on the coin. Let $F_{\rm c \ on \ b}$ be the magnitude of the force exerted by the coin on the board. Let $F_{\rm c \ on \ E}$ be the magnitude of the force exerted by the coin on the Earth. In the following situations, the coin is always in contact with the board. (131Sp2025)



- a) The board is held at rest. Which of the following is true? Explain your answer.
 - i) $F_{c \text{ on } b} = F_{b \text{ on } c} = mg.$ ii) $F_{c \text{ on } b} = F_{b \text{ on } c} > mg.$ iii) $F_{c \text{ on } b} = F_{b \text{ on } c} < mg.$ iv) $F_{c \text{ on } b} > F_{b \text{ on } c} = mg.$ v) $F_{c \text{ on } b} < F_{b \text{ on } c} = mg.$

b) The board is held at rest. Which of the following is true? Explain your answer.

- i) $F_{\rm c \ on \ E} = mg$.
- ii) $F_{\rm c \ on \ E} > mg$.
- iii) $F_{\rm c \ on \ E} < mg$.
- c) The board ascends with increasing speed. Which of the following is true? Explain your answer.
 - i) $F_{\rm c \ on \ b} = F_{\rm b \ on \ c} = mg$.
 - ii) $F_{\rm c \ on \ b} = F_{\rm b \ on \ c} > mg.$
 - iii) $F_{\rm c \ on \ b} = F_{\rm b \ on \ c} < mg.$
 - iv) $F_{\rm c \ on \ b} > F_{\rm b \ on \ c} = mg.$
 - v) $F_{\rm c \ on \ b} < F_{\rm b \ on \ c} = mg.$
- d) The board ascends with increasing speed. Which of the following is true? Explain your answer.
 - i) $F_{\rm c \ on \ E} = mg$.
 - ii) $F_{\rm c \ on \ E} > mg$.
 - iii) $F_{\rm c \ on \ E} < mg.$

216 Pushing crates in contact

Two crates can move along a horizontal surface. There is no friction between either crate and the surface. The crates maintain contact with each other. A hand pushes on the crate at the left. (131Sp2025)

- a) Which of the following is true? Explain your answer.
 - i) The force exerted by the smaller crate on the larger crate is less than the force exerted by the hand.
 - ii) The force exerted by the smaller crate on the larger crate is more than the force exerted by the hand.
 - iii) The force exerted by the smaller crate on the larger crate is the same as the force exerted by the hand.
- b) The positions of the two crates are interchanged so that the larger crate is to the left of the smaller crate. Which of the following is true? Explain your answer.
 - i) The force exerted by the larger crate on the smaller crate is less than the force exerted by the hand.
 - ii) The force exerted by the larger crate on the smaller crate is more than the force exerted by the hand.
 - iii) The force exerted by the larger crate on the smaller crate is the same as the force exerted by the hand.
- c) For either of these arrangements, the crates could either be moving left or else right. Does the direction of motion affect the answers to the previous questions? Explain your answer.

217 Pushing boxes in contact

Two boxes, with masses $m_A < m_B$, can move along a rough horizontal surface. The boxes maintain contact with each other. A hand pushes on the box at the left. Let $F_{A \text{ on } B}$ be the magnitude of the force that A exerts on B and $F_{B \text{ on } A}$ be the magnitude of the force that B exerts on A. (131Sp2025)



- a) The boxes move with constant speed to the right. Which of the following is true? Explain your answer.
 - i) $F_{A \text{ on } B} < F_{B \text{ on } A}$.
 - ii) $F_{A \text{ on } B} = F_{B \text{ on } A}$.
 - iii) $F_{A \text{ on } B} > F_{B \text{ on } A}$.
- b) The boxes move with constantly increasing speed to the right. Which of the following is true? Explain your answer.
 - i) $F_{A \text{ on } B} < F_{B \text{ on } A}$.
 - ii) $F_{A \text{ on } B} = F_{B \text{ on } A}$.
 - iii) $F_{A \text{ on } B} > F_{B \text{ on } A}$.



There is no friction between either crate and the surface.

The crates are connected by a rope. A hand pulls on the other rope with tension T. (131Sp2025)

218 Pushing boxes in contact

219 Dragging connected crates

(131Sp2025)

a) Which of the following is true? Explain your answer.

Two different crates can move along a horizontal surface.

Two boxes can move along a frictionless horizontal surface.

The boxes maintain contact with each other. A person pushes

with a 100 N force on the box at the left. Determine the force that the box at the left exerts on the box at the right.

- i) The tension in the connecting rope is the same as T.
- ii) The tension in the connecting rope is more than T.
- iii) The tension in the connecting rope is less than T.

b) Which of the following is true? Explain your answer.

- i) The tension in the connecting rope depends on whether the heavier crate is to the left.
- ii) The tension in the connecting rope does not depend on whether the heavier crate is to the left.

220 Connected objects: tension and acceleration

Two objects on a frictionless horizontal surface are connected by a massless rope. They are connected in different orders and pulled by another massless rope. (131Sp2025)

- a) Let a_1 be the acceleration for case 1 and a_2 the acceleration for case 2. Which of the following is true? Explain your choice.
 - i) $a_1 = a_2$
 - ii) $a_1 > a_2$
 - iii) $a_1 < a_2$
- b) Let T_1 be the tension in the rope connecting the blocks for case 1 and T_2 that for case 2. Which of the following is true? Explain your choice.
 - i) $T_1 = T_2 = 10 \,\mathrm{N}$
 - ii) $T_1 < T_2 < 10 \,\mathrm{N}$
 - iii) $T_2 < T_1 < 10 \,\mathrm{N}$
 - iv) $T_1 = T_2 < 10 \,\mathrm{N}$









221 Slipping stacked objects

Two boxes are stacked and move along a frictionless horizontal surface as illustrated. Block A has mass 2.0 kg and block B has mass 3.0 kg. A rope is attached to block B and pulls horizontally with a 50 N force. The coefficient of friction between block A and block B is 0.25. Determine the acceleration of each block, assuming that they both move right and that B moves faster than A. (131Sp2025)



222 Connected objects: tension and acceleration

Two boxes can move along a horizontal surface. There is no friction between either box and the surface. The boxes are connected by a rope. A hand pulls on the other rope with force 50 N. (131Sp2025)

- a) Determine the acceleration of each box.
- b) Determine the tension in the rope connecting the boxes.

223 Connected objects: friction

Two boxes can move along a horizontal surface. There is no friction between the 6.0 kg box and the surface. There is friction for the other box: the coefficient of static friction is 0.70 and the coefficient of kinetic friction is 0.50. The boxes are connected by a rope. A hand pulls on the other rope with force 50 N. (131Sp2025)

- a) Determine the acceleration of each box.
- b) Determine the tension in the rope connecting the boxes.

224 Connected objects separated by a spring

Two boxes can move along a horizontal surface. They are connected by a spring with spring constant 600 N/m. A person exerts a constant 100 N force pointing right on the box on the left. The boxes eventually move so that they maintain a fixed distance between them. (131Sp2025)

- a) Determine the distance by which the spring is compressed when they move like this.
- b) Suppose that the situation is repeated but a constant 100 N force is now exerted on the box on the right; this force points left. Is the compression of the spring the same as before? Explain your answer.







225 Level/suspended blocks without friction

Two blocks are connected by a string, which runs over a massless pulley. One block, with mass 3.0 kg is suspended and the other block, with mass 7.0 kg can move along a frictionless horizontal surface. The string connected to the block on the surface runs horizontally. (131Sp2025)



- a) Draw a free body diagram for the box on the surface.
- b) Write Newton's Second Law in component form for the box on the surface, i.e. write

$$F_{\text{net }x} = \Sigma F_x = \cdots \tag{17}$$

$$F_{\text{net }y} = \Sigma F_y = \cdots \tag{18}$$

Insert as much information as possible about the components of acceleration at this stage. The resulting equations will generate much of the algebra that follows.

- c) List all the components of all the forces for the box on the surface.
- d) Use Eqs. (17) and (18) and the components to obtain an equation relating the tension in the rope and the acceleration of the box. Can you solve this for acceleration at this stage?
- e) Repeat parts a) to d) for the suspended crate. Be careful about the acceleration!
- f) Combine the equations for the two objects to obtain the acceleration and the tension in the rope.

226 Level/suspended blocks without friction and a hand

Two blocks are connected by a string, which runs over a massless pulley. One block, with mass m_1 , is suspended and the other block, with mass m_2 , can move along a frictionless horizontal surface. The string connected to the block on the surface runs horizontally. A hand exerts a constant force on the block on the surface. This force has magnitude F_{hand} and points horizontally to the left. (131Sp2025)



- a) Draw a free body diagram for the box on the surface.
- b) Write Newton's Second Law in component form for the box on the surface, i.e. write

$$F_{\text{net }x} = \Sigma F_x = \cdots \tag{19}$$

$$F_{\text{net }y} = \Sigma F_y = \cdots \tag{20}$$

Insert as much information as possible about the components of acceleration at this stage. The resulting equations will generate much of the algebra that follows.

- c) List all the components of all the forces for the box on the surface.
- d) Use Eqs. (19) and (20) and the components to obtain an equation relating the tension in the rope and the acceleration of the box. Can you solve this for acceleration at this stage?
- e) Repeat parts a) to d) for the suspended crate. Be careful about the acceleration!
- f) Combine the equations for the two objects to obtain the acceleration and the tension in the rope. These should be of the form

a =expression with only $m_1, m_2, g, F_{\text{hand}}$ and constants.

227 Connected objects: Level/suspended blocks with friction

A 6.0 kg crate is suspended from a rope which runs over a massless pulley and is connected to a 4.0 kg box, which moves right along a rough horizontal surface. The coefficient of friction between the box and surface is 0.25. The string connected to the box on the surface runs horizontally. The aim of this exercise will be to determine the acceleration of the objects, ignoring any air resistance. (131Sp2025)



- a) Draw a free body diagram for the box on the surface.
- b) Write Newton's Second Law in component form for the box on the surface, i.e. write

$$F_{\text{net }x} = \Sigma F_x = \cdots \tag{21}$$

$$F_{\text{net }y} = \Sigma F_y = \cdots \tag{22}$$

Insert as much information as possible about the components of acceleration at this stage. The resulting equations will generate much of the algebra that follows.

- c) List all the components of all the forces for the box on the surface.
- d) Use Eqs. (21) and (22) and the components to obtain an equation relating the tension in the rope and the acceleration of the box. Can you solve this for acceleration at this stage?
- e) Repeat parts a) to d) for the suspended crate. Be careful about the acceleration!
- f) Combine the equations for the two objects to obtain the acceleration and the tension in the rope.

The analysis can be performed for objects of any mass. Let m_1 be the mass of the block on the surface, m_2 the mass of the suspended block and μ_k be the coefficient of friction between the block and the surface.

g) Determine an expression for the magnitude of acceleration of the blocks, a. This should be of the form

a = formula with only m_1, m_2, g, μ_k and constants.

228 Connected objects: Level/suspended blocks with friction, 2

A 3.0 kg box is suspended from a rope which runs over a massless pulley and is connected to a 5.0 kg crate, which moves right along a rough horizontal surface. The coefficient of kinetic friction between the crate and surface is 0.35. The string connected to the crate runs horizontally. (131Sp2025)

- a) Determine the acceleration of the objects, ignoring any air resistance.
- b) Determine the tension in the rope.
- c) Suppose that the surface is polished and this reduces the friction to the point that the acceleration is 0.25g. Determine the coefficient of friction in this case.

229 Stacked objects on a surface connected to a suspended object

Various boxes are arranged as illustrated. The 4 kg block lies on a table with a horizontal surface. (131Sp2025)

- a) Suppose that all the blocks are at rest. Is there a friction force between the 4 kg block and the table? If so, in which direction? Explain your answer.
- b) Suppose that all the blocks are at rest. Is there a friction force between the 2 kg block and the 4 kg block? If so, in which direction? Explain your answer.
- c) Suppose that the block on the table accelerates to the right and the 2 kg block does not slip relative to the 4 kg block. Is there a friction force between the 2 kg block and the 4 kg block? If so, in which direction? Explain your answer.

230 Connected objects: block dragged along a surface

A block on a surface can be dragged along a frictionless surface by a connecting rope in two ways. Either a person exerts a force on the rope or else a suspended block exerts a force on the rope. Suppose that the person exerts a 50 N tension force on the rope and that the weight (gravitational force on) of the suspended block is exactly 50 N. In which case is the acceleration of the block on the horizontal surface largest? Explain your answer. (131F2024)







231 Connected objects: direction of motion

A block can move along a horizontal frictionless surface. It is connected to a suspended object via a massless, frictionless pulley. A hand briefly pushes either left or right on the block so that it starts moving either left or right. The block then departs from the hand. (131Sp2025)



- a) Which of the following is true after the block has left the hand? Explain your answer.
 - i) The acceleration when moving left is the same as when moving right.
 - ii) The acceleration when moving left is larger than moving right.
 - iii) The acceleration when moving left is smaller than moving right.
- b) Which of the following is true about the tension in the string after the block has left the hand? Explain your answer.
 - i) The tension when moving left is the same as when moving right.
 - ii) The tension when moving left is larger than moving right.
 - iii) The tension when moving left is smaller than moving right.

232 Level/suspended blocks without friction

Two blocks are connected by a string, which runs over a massless pulley. A 10 kg block is suspended and a 5.0 kg block can slide along a frictionless horizontal surface. The string connected to the block on the surface runs horizontally. The blocks held at rest and then released. They move, constantly speeding up. Which of the following is true regarding the tension in the connecting string, T, while they move? Explain your choice. (131Sp2025)

i) T = 0.

- ii) 98 N > T > 0.
- iii) $T = 98 \,\mathrm{N}.$
- iv) $T > 98 \,\mathrm{N}$



233 Box dragged along a rough surface

A small box, with mass m, is connected to a suspended mass, with mass 3m, by a string, which runs over a massless pulley. The box can slide along a rough horizontal surface. The coefficient of static friction is μ_s and the coefficient of kinetic friction is μ_k , where $\mu_k < \mu_s$. In the following the only variables that are allowed to appear in expressions are: m, g, μ_k and μ_s . (131Sp2025)



- a) Determine an expression for the minimum coefficient of static friction such that the box does not move.
- b) The box is given a brief push to the left and it begins to move. Determine an expression for the acceleration of the box *after* this brief push and while it is moving.

234 Atwood's machine

Two blocks, with masses indicated, are connected by a string which runs over a massless pulley. The aim of this exercise is to determine the acceleration of the blocks. (131Sp2025)

- a) Draw a free body diagram for the block on the left.
- b) Write Newton's Second Law in component form for the *block on the left*, i.e. write

$$F_{\text{net }x} = \Sigma F_x = \cdots \tag{23}$$

$$F_{\text{net }y} = \Sigma F_y = \cdots$$
 (24)

Insert as much information as possible about the components of acceleration at this stage. The resulting equations will generate much of the algebra that follows.

c) List all the components of all the forces for the block on the left.

$F_{gx} = \cdots$	Force	$x \operatorname{comp}$	$y \operatorname{comp}$
$F_{gy} = \cdots$ $n_x = \cdots$	$ec{\mathbf{F}}_{g}$		
$n_y = \cdots$	ñ		
: :	:		

- d) Use Eqs. (23) and (24) and the components to obtain an equation relating the tension in the rope and the acceleration of the block on the left. Can you solve this for acceleration at this stage?
- e) Repeat parts a) to d) for the block on the right. Be careful about the acceleration!
- f) Combine the equations for the two objects to obtain the acceleration and the tension in the rope.



 m_1

235 Atwood's machine variation

Two blocks, with masses $m_1 > m_2$, are connected by a string which runs over a massless pulley. A hand exerts a constant downward force with magnitude F_{hand} on the block on the left. Determine an expression for the magnitude of acceleration of the blocks, *a*. The only variables allowed in the expression are m_1, m_2, g and F_{hand} . (131Sp2025)

236 Counterbalanced elevator

A 3000 kg elevator is connected to a 2000 kg block by a rope that runs over a pulley. Determine the acceleration of the elevator. (131Sp2025)



A block on a rough ramp is connected via a string to another suspended block. At one instant the block on the ramp is sliding up the ramp. (131Sp2025)

- a) Determine an expression for the acceleration of the block on the ramp if the ramp is inclined at an angle of 65° above the horizontal, the mass of the block on the ramp is 8.0 kg, the mass of the suspended block is 9.0 kg and the coefficient of kinetic friction is 1.2. Does the block on the ramp speed up or slow down?
- b) Determine a general expression for the acceleration of the block moving up the ramp if the ramp is inclined at an angle of θ above the horizontal and the coefficient of kinetic friction is μ_k .
- c) What condition must the masses satisfy for the block to speed up as it ascends the ramp?





 m_1



97

238 Connected objects: horizontal and a ramp

Blocks connected by a massless string are able to slide on the illustrated frictionless surfaces. The strings run parallel to the surfaces. Determine the acceleration of the block on the ramp. (131Sp2025)



239 Connected objects: horizontal and a rough ramp

Blocks connected by a massless string are able to slide on the illustrated surfaces. The strings run parallel to the surfaces. The horizontal surface is frictionless and the ramp has coefficient of kinetic friction 0.50. Determine the acceleration of the block on the ramp if it descends. (131Sp2025)



240 Connected objects: two ramps

Blocks connected by a massless string are able to slide on two frictionless ramps. (131Sp2025)

- a) Determine the acceleration of the blocks.
- b) Suppose that the block on the right were replaced by another block so that the two blocks slide with constant speed. What would the mass of the block on the right be?



241 Bug walking in a circle

A bug walks at a constant speed in a circular path on a horizontal surface. Which vector best illustrates the net force on the bug at the illustrated moment? Explain your choice. (131Sp2025)



242 Balls swinging on the end of strings

Various balls at the end of strings swing in circles, with the same speed, on a horizontal frictionless surface. The diagram illustrates this as viewed from above. Rank the cases in order of increasing tension in the string. Explain your answer. (131Sp2025)



243 Connected balls swinging in horizontal circles, 1

Two balls are connected by strings. They are on a horizontal frictionless surface and swing in horizontal circles about a central point; both strings point radially outward from the center as the balls move. The diagram illustrates this as viewed from above. (131Sp2025)

- a) Draw a free body diagram, as viewed from the side, for ball B at the moment when the balls are at the leftmost location of their motion.
- b) Draw the direction of the net force on the ball B at the moment when the balls are at the leftmost location of their motion.
- c) How does the tension in the inner string compare (larger, smaller, same) to that in the string connecting the two balls? Explain your answer.



244 Connected balls swinging in horizontal circles, 2

Two balls are connected by strings. They are on a horizontal frictionless surface and swing in horizontal circles about a central point; both strings point radially outward from the center as the balls move. The diagram illustrates this as viewed from above. (131Sp2025)

- a) Suppose that A has a larger mass than B. Is the tension in the inner string the same as, larger than or equal to the tension in the outer string? Explain your answer.
- b) Suppose that A has a smaller mass than B. Is the tension in the inner string the same as, larger than or equal to the tension in the outer string? Explain your answer.



c) Now consider two particular balls, one with mass 3.0 kg and the other with mass 5.0 kg. They swing in circles as illustrated in one of the following two arrangements: i) the 3.0 kg is further out and the 5.0 kg is closer in, or ii) the 5.0 kg is further out and the 3.0 kg is closer in. In each case they swing with the same angular velocity. Which arrangement (larger mass on the outside, smaller mass on the outside) results in a larger tension in the outer string? Explain your answer.

245 Merry-go-round dynamics

A 50 kg child sits at the edge of a merry-go-round with radius 2.5 m. The merry-go-round rotates with frequency 15 revolutions per minute. (131Sp2025)

- a) Determine the direction of the friction force on the child.
- b) Determine the magnitude of the friction force on the child.

246 Child swinging on ice

A parent and child are each on a horizontal sheet of ice. The parent is fixed to the ice and swings the child, who is connected by a horizontal rope to the parent. The child slides without any friction with constant velocity. (131Sp2025)

- a) Determine an expression for the tension in the rope in terms of the mass of the child, the length of the rope and the speed of the child.
- b) Determine an expression for the tension in the rope in terms of the mass of the child, the length of the rope and the period of motion of the child.
- c) Determine an expression for the period of motion of the child so that the tension felt by the parent is larger the force that the parent must exert to hold the child at rest off the ground.

247 Particle accelerator

The Large Hadron Collider (LHC) accelerates subatomic particles and then lets them collide with a target. During this trip the particles move through a circular tunnel with radius 2.80 km. One type of particle use in the LHC is a proton. Suppose that a proton travels this circle at 10% of the speed of light. Determine the net force on the proton. Note: to be strictly correct this needs to include the effects of special relativity. The approach here will give an approximate force. (131Sp2025)

248 Rotating Earth

Earth rotates, completing one rotation in a day (86400 s). Suppose that you stand on a scale at sea level at Earth's equator; the scale measures the normal force that it exerts and converts it into a reading in kilograms by dividing by g. In the following ignore air resistance. (131Sp2025)

- a) Explain why the force measured by the scale will not equal the gravitational force exerted by Earth.
- b) Assuming that your mass is 70.0 kg, determine the normal force and the scale reading.
- c) Suppose that Earth's rotation were to speed up. How will this affect the scale reading?
- d) If Earth were able to speed up, determine the time for one rotation such that the scale would read zero. How many hours would it take Earth to complete one rotation? What would happen to you if you were standing on the scale and Earth continued to speed up beyond this threshold?

249 Animals in a space station

A particular space station is essentially a giant circular hoop with radius 600 m. People and objects reside on the inside of the hoop. The hoop rotates at a constant rate. (131Sp2025)

- a) Consider 30 kg dog on the inside of the space station. The dog is fixed at one point on the floor. Determine the speed of the space station so that the dog feels a normal force the same as it would at rest on Earth.
- b) Determine the time taken for the space station of the previous part to complete one orbit.
- c) A 4.0 kg rabbit also sits on the floor of the space station. It would also like to feel the same normal force as it does on Earth. Would the time taken for the space station to complete one orbit have to change to do this; Explain your answer.



250 Science fiction space station

A science fiction movie features a space station which is essentially a giant circular hoop. People and objects reside on the inside of the hoop. The hoop rotates at a constant rate. (131Sp2025)

- a) Consider an object with mass m at rest relative to the surface of the space station floor. Determine an expression for the normal force exerted by the floor in terms of the radius and the time taken to complete one orbit.
- b) The floor will feel the same as Earth's surface if the normal force equals mg. Determine an expression for the time taken to complete one orbit so that this is true.



c) Imagine a space station with a radius of 5.0 km. Determine the time it should take to complete one rotation so that the space station floor feels like Earth's surface.

251 Car turning

A 3000 kg car travels along a flat (horizontal) road. The road curves with a radius of 350 m. The maximum speed with which the car can do the turn is 18 m/s. (131Sp2025)

- a) Determine the net force on the car. What force provides the net force?
- b) Determine the (minimum) coefficient of static friction that allows the car to follow this curve at the given speed.

252 Coin on a turntable

A coin with mass m sits on a horizontal turntable with radius R. The distance from the turntable axle to the coin is d. The coefficient of static friction between the coin an the turntable is μ_s and the coefficient of kinetic friction is μ_k . The turntable is initially at rest and slowly speeds up. Determine an expression for the maximum angular velocity of the turntable so that the coin does not slip. (131Sp2025)

253 Ball swinging in a vertical circle, 1

A 0.80 kg ball swings with in a vertical circle at the end of a 0.50 m long string. The speed of the ball at the highest point in the circle is 3.0 m/s. Determine the tension in the string at this moment. (131Sp2025)

