Phys 131 2025

Lecture 8

Fri: HW by 5pm

Ex 71, 72, 74, 78, 79, 89, 90a, 93

Mon: Warm Up 4 D2L

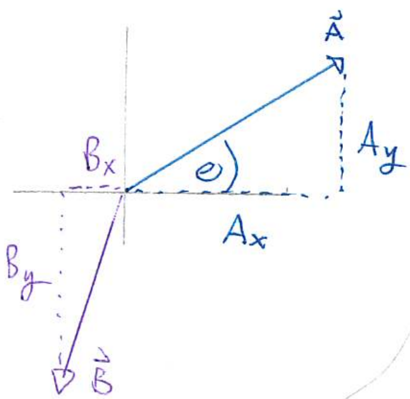
Thurs: Seminar

Vector algebra with components

The scheme for vector algebra is:

Task: Given vectors $\vec{A}, \vec{B}$

determine $\vec{D} = \alpha \vec{A} + \beta \vec{B}$



Strategy: get components of $\vec{D}$ via:

$$D_x = \alpha A_x + \beta B_x$$

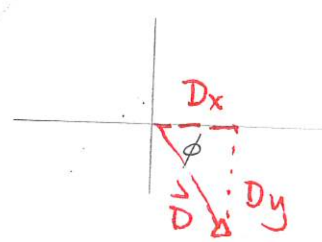
$$D_y = \alpha A_y + \beta B_y$$

Components of $\vec{A}, \vec{B}$

$A_x = A\cos\theta$ $\quad B_x = \ldots$

$A_y = A\sin\theta$ $\quad B_y = \ldots$

Construct $\vec{D}$



Magnitude $D = \sqrt{D_x^2 + D_y^2}$

angle via trig:

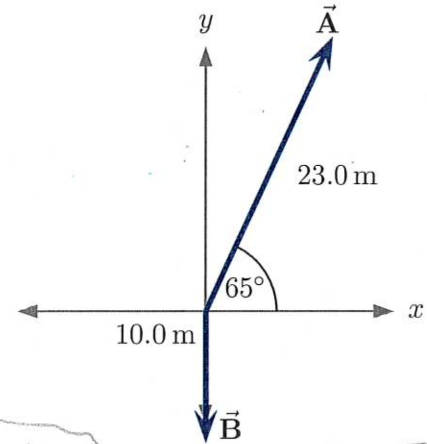
$$\phi = \arctan\left(D_y / D_x\right)$$

87 Vector addition: algebraic method, 4

Two displacement vectors, $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ are illustrated. *(131Sp2025)*

a) Determine the components of $\vec{\mathbf{C}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$.

b) Determine the magnitude of $\vec{\mathbf{C}}$.



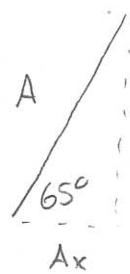
Answer: The scheme is:

Determine components of $\vec{A}, \vec{B}$ $\rightarrow$ Use components to determine components of $\vec{C}$ $\rightarrow$ Reconstruct $\vec{C}$ and find its magnitude

a) $C_x = A_x + B_x$

$C_y = A_y + B_y$

So we need trig $\rightarrow$



$\frac{A_x}{A} = \cos 65^\circ \Rightarrow A_x = A\cos 65^\circ = 23.0\text{m} \cos 65^\circ = 9.7\text{m}$

$\frac{A_y}{A} = \sin 65^\circ \Rightarrow A_y = A\sin 65^\circ = 23.0\text{m} \sin 65^\circ = 20.8\text{m}$

$B_x = 0\text{m}$

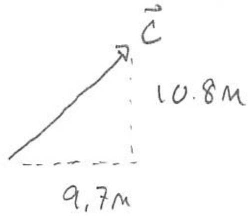
$B_y = -10.0\text{m}$

	x comp	y comp
$\vec{A}$	9.7m	20.8m
$\vec{B}$	0m	-10.0m
$\vec{C}$	9.7m	10.8m

$\Rightarrow C_x = 9.7\text{m} + 0\text{m} = 9.7\text{m}$

$C_y = 20.8\text{m} - 10\text{m} = 10.8\text{m}$

b)



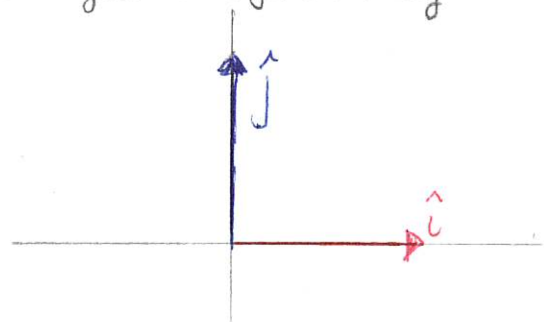
$C = \sqrt{C_x^2 + C_y^2} = \sqrt{(9.7\text{m})^2 + (10.8\text{m})^2} \Rightarrow \boxed{C = 14.5\text{m}}$

Unit vectors

The strategy of using components allows for accurate vector algebra. However it requires one chain of algebra for horizontal components and another for vertical components. These can be combined into a single algebraic system using special unit vectors. These are

$\hat{\imath} \equiv$ unit vector along x-axis

$\hat{\jmath} \equiv$ " " " y-axis



Then:

For any vector $\vec{A}$ in two dimensions,

$$\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath}$$

where $\left.\begin{array}{l} A_x = \text{x-component} \\ A_y = \text{y-component} \end{array}\right\}$ A pair of numbers each could be positive or negative

Slide1
Slide2
Slide3

Consider the exercise 87. Here

$$\vec{A} = 9.7\text{m}\,\hat{\imath} + 20.8\text{m}\,\hat{\jmath}$$

$$\vec{B} = 0\text{m}\,\hat{\imath} - 10.0\text{m}\,\hat{\jmath} = -10.0\text{m}\,\hat{\jmath}$$

$$\vec{C} = \vec{A} + \vec{B} = 9.7\text{m}\,\hat{\imath} + 20.8\text{m}\,\hat{\jmath} - 10.0\text{m}\,\hat{\jmath} = 9.7\text{m}\,\hat{\imath} + [20.8\text{m} - 10.0\text{m}]\,\hat{\jmath}$$

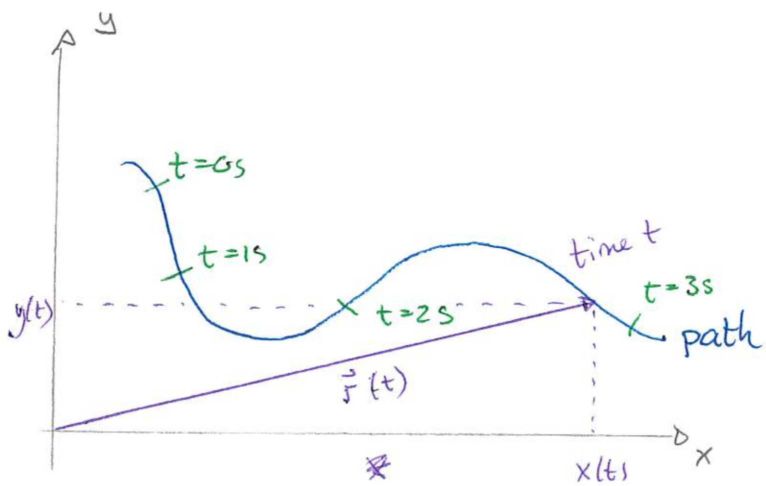
$$\Rightarrow \vec{C} = 9.7\text{m}\,\hat{\imath} + 10.8\text{m}\,\hat{\jmath}$$

Quiz1 typo $\vec{A}+\vec{B}$

~80%

Kinematics in Two Dimensions

The language of vectors will describe kinematics in two dimensions. One could graph the path followed in the xy plane, marking instants along the path. This allows us to represent the position as:

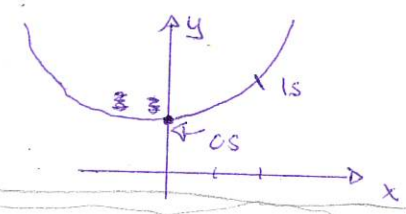


$x(t)$ = horizontal position

$y(t)$ = vertical position

Example:

$$\begin{cases} x(t) = 2\,\text{m/s}\ t \\ y(t) = 6\,\text{m/s}^2\ t^2 + 3\,\text{m} \end{cases}$$ position functions



Position vector

$$\vec{r}(t) = x(t)\,\hat{i} + y(t)\,\hat{j}$$

$$\vec{r}(t) = (2\,\text{m/s}\ t)\,\hat{i} + (6\,\text{m/s}^2\ t^2 + 3\,\text{m})\,\hat{j}$$

The displacement vector from time t_i to t_f is

$$\Delta\vec{r} = \Delta x\,\hat{i} + \Delta y\,\hat{j}$$

where $\Delta x = x_f - x_i$

$\Delta y = y_f - y_i$

e.g. from $t_i = 0\,\text{s}$ to $t_f = 1\,\text{s}$

$\Delta x = x_f - x_i$

$x_f = 2\,\text{m}$ $\quad x_i = 0$

$\Rightarrow \Delta x = 2\,\text{m}$

$\Delta y = y_f - y_i$

$y_f = 27\,\text{m}$ $\quad y_i = 3\,\text{m}$

$\Delta y = 24\,\text{m}$

$\Delta\vec{r} = 24\,\text{m}\,\hat{i} + 2\,\text{m}\,\hat{j}$

The average velocity vector is:

$$\vec{v}_{avg} = \frac{\Delta x}{\Delta t}\,\hat{i} + \frac{\Delta y}{\Delta t}\,\hat{j} \equiv \frac{\Delta\vec{r}}{\Delta t}$$

from $t_i = 0\,\text{s}$ to $t_f = 1\,\text{s}$

$\frac{\Delta x}{\Delta t} = 2\,\text{m/s}$ $\quad \frac{\Delta y}{\Delta t} = 24\,\text{m/s}$

$\Rightarrow \vec{v}_{avg} = 2\,\text{m/s}\,\hat{i} + 24\,\text{m/s}\,\hat{j}$

Quiz 2 30% - 70%

Then instantaneous velocity is a vector

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}\hat{\imath} + \lim_{\Delta t \to 0} \frac{\Delta y}{\Delta t}\hat{\jmath}$$

$$= \frac{dx}{dt}\hat{\imath} + \frac{dy}{dt}\hat{\jmath}$$

$x = 2\,m/s\; t$

$y = 6\,m/s^2\, t^2 + 3\,m$

$\frac{dx}{dt} = 2\,m/s \qquad \frac{dy}{dt} = 12\,m/s^2\, t$

$\Rightarrow \vec{v} = 2\,m/s\,\hat{\imath} + 12\,m/s^2\, t\,\hat{\jmath}$

Note that for velocity

$$\vec{v} = v_x\hat{\imath} + v_y\hat{\jmath}$$

$$v_x = \frac{dx}{dt} \qquad v_y = \frac{dy}{dt}$$

The velocity vector in two dimensions has two components.

* magnitude of velocity = speed = $v = \sqrt{v_x^2 + v_y^2}$
* direction of velocity ≡ tangent to trajectory along direction of motion