Phys 131 2025

Lecture 7

Tues: Discussion Ex: 58, 59, 60, 62, 63, 66, 67

Thurs: –

Vector Algebra

Displacement vectors are arrows with both magnitude and velocity. We aim to develop a system of algebra and mathematics for manipulating vectors. The first issue is vector equality

Two vectors $\vec{A}$ and $\vec{B}$ are equal, i.e. $\vec{A}=\vec{B}$	$\Longleftrightarrow$	1) $\vec{A},\vec{B}$ have the same magnitude AND 2) $\vec{A},\vec{B}$ have the same direction

Quiz! 60% – 90%

Vector addition

We now define vector addition. The idea is

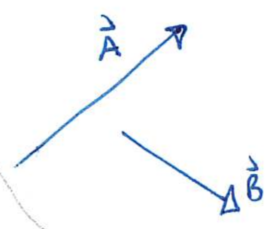
Given: Two vectors $\vec{A},\vec{B}$ ↳ Sum of vectors: single vector attained by successive displacements.

PhET Vector addition

→ Explore 2D → Select two vectors → Show sum

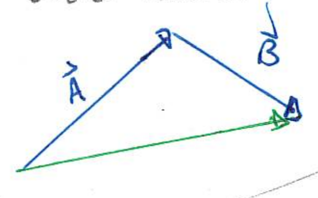
The procedure is

Given: Two vectors



→ Process

Move $\vec{B}$ parallel to itself until tail of $\vec{B}$ touches head of $\vec{A}$



→ Sum

Single displacement vector from tail of $\vec{A}$ to head of $\vec{B}$ after rearrangement

Quiz 1 60-80%

This example shows that

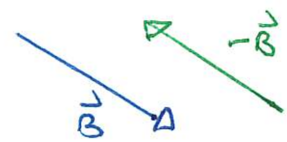
If $\vec{C} = \vec{A} + \vec{B}$ then it is <u>not</u> generally true that $C = A + B$

vectors magnitudes

Vector subtraction

Subtraction is an inverse operation to addition in the sense that, given vector $\vec{B}$, the vector $-\vec{B}$ is a vector so that $\vec{B} + (-\vec{B}) = 0$. Thus

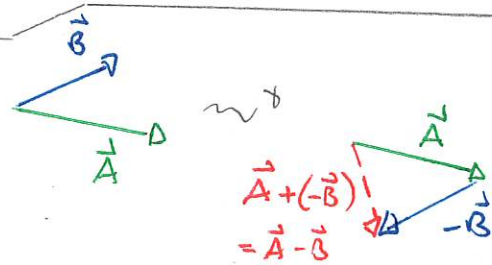
If $\vec{B}$ is a vector then $-\vec{B}$ is a vector with the same magnitude and opposite direction



Then subtraction is done via

For any vectors $\vec{A}, \vec{B}$

$$\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$$



Warm Up 1

Scalar multiplication

We can multiply vectors by numbers. To illustrate consider repeated addition. $\vec{A} + \vec{A} + \vec{A} \overset{??}{=} 3\vec{A}$. This motivates

Let $\vec{A}$ be any vector and c any number. Then $c\vec{A}$ is a vector with

1) magnitude $|c|A$
2) direction $= \begin{cases} \text{same as } \vec{A} & \text{if } c > 0 \\ \text{opposite to } \vec{A} & \text{if } c < 0 \end{cases}$

Vector components

We need to do vector algebra without using diagrams and eventually using numbers. The strategy to do this requires representing vectors in terms of components:

Given any vector $\vec{A}$ in two dimensions $\longrightarrow$ Equivalent to 2 real numbers A_x, A_y

DEMO: PhET Vector addition – 2D Explore Tab.

→ Single vector → component vectors

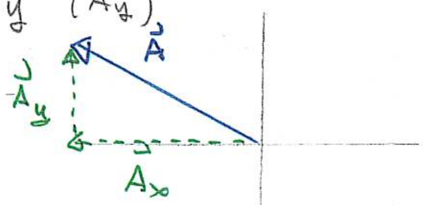
→ component numbers.

The process to do this.

ESSENTIALLY ONE PAIR ~~A_x, $\vec{A}_y$ for any $\vec{A}$~~.

A_x, A_y for any $\vec{A}$

Given vector $\vec{A}$ decompose into two vectors, one along x $(\vec{A}_x)$ and the other along y $(\vec{A}_y)$.



→ The components are two numbers:

Horizontal component A_x

$= \pm$ magnitude of A_x

+ if $\vec{A}_x$ right

− if $\vec{A}_x$ left

Vertical component A_y

$= \pm$ magnitude of A_y

+ if $\vec{A}_y$ up

− if $\vec{A}_y$ down

Warm Up 2

We can then use these for vector algebra via:

If $\vec{D} = \vec{A} + \vec{B} + \vec{C} + \ldots$ then

$$D_x = A_x + B_x + C_x + \ldots$$

$$D_y = A_y + B_y + C_y + \ldots$$

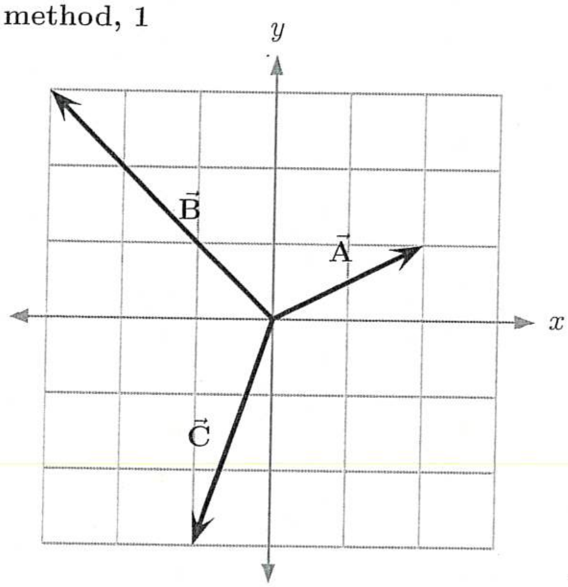
If $\vec{D} = c\vec{A}$ where c is a number then

$$D_x = cA_x$$

$$D_y = cA_y$$

82 Vector addition: graphical and algebraic method, 1

Displacement vectors, $\vec{\mathbf{A}}, \vec{\mathbf{B}}$, and $\vec{\mathbf{C}}$ are illustrated. Let $\vec{\mathbf{D}} = \vec{\mathbf{A}} + \vec{\mathbf{B}} + \vec{\mathbf{C}}$. *(131F2024)*



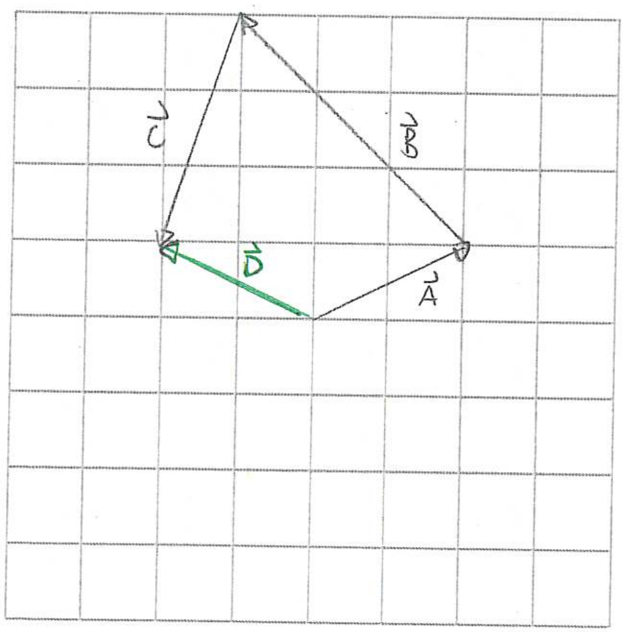
a) Using the graph sheet below, determine $\vec{\mathbf{D}}$ graphically via the head–to–tail method. Use the result to determine the magnitude of $\vec{\mathbf{D}}$.

b) List the horizontal and vertical components of each of $\vec{\mathbf{A}}, \vec{\mathbf{B}}$, and $\vec{\mathbf{C}}$ and use these components to determine the components of $\vec{\mathbf{D}}$. Use the result to determine the magnitude of $\vec{\mathbf{D}}$.

a)

$D = \sqrt{2^2+1^2}$

$= \sqrt{5} = 2.24$



b)

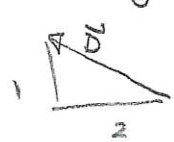
$A_x = 2$ $\quad A_y = 1$

$B_x = -3$ $\quad B_y = 3$

$C_x = -1$ $\quad C_y = -3$

$D_x = A_x + B_x + C_x = 2-3-1 = -2$

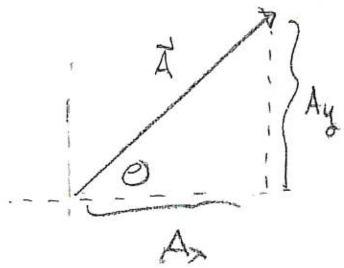
$D_y = A_y + B_y + C_y = 1+3-3 = 1$



$D = \sqrt{2^2+1^2} = \sqrt{5} = 2.24$

Calculating vector components

We can calculate vector components using trigonometry



Then A = magnitude of A (hypotenuse)

$\Rightarrow A_x = A\cos\theta$

$A_y = A\sin\theta$

Thus

Given $\vec{A}, \vec{B}$ want $\vec{D} = \alpha\vec{A} + \beta\vec{B}$

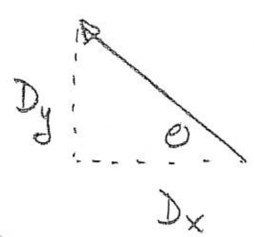
α, β real

$\rightarrow$ $D_x = \alpha A_x + \beta B_x$

$D_y = \alpha A_y + \beta B_y$

gives components of $\vec{D}$

reconstruct $\vec{D}$ from components



$\rightarrow$ magnitude $D = \sqrt{D_x^2 + D_y^2}$

direction via angle, e.g

$\theta = \arctan D_y / D_x$