Lecture 3

Fri: HW due by 5pm

Mon: Warm Up 2 (D2L)

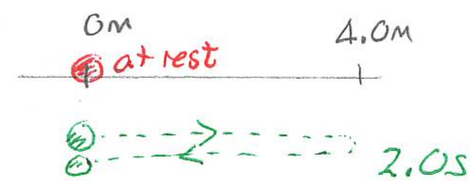
Group Exercise (not graded)

Tues: Discussion / quiz Ex 25, 29, 30, 33, 34, 35, 38

* bring problems to class / not collected / not graded
* quiz 5 pts

Velocity

Average velocity quantifies the rate at which position changes over some (possibly long) time interval. It is an imperfect way to describe motion. For example consider the situations of the illustrated objects. Both have an average velocity of zero. But their motions are very different.



In order to distinguish between these, we need to develop a concept of velocity at each instant in time. We illustrate this with an animation

DEMO: PhET Moving Man → Charts

$x_0 = +10m$

$V_0 = -6$

$a = 2$

What would the velocity at 4.0s be? We can use average velocity over smaller and smaller intervals

4.0s → 5.0s → get V_{avg}

4.0s → 4.5s → get V_{avg} improves accuracy of velocity

DEMO: Slide of data.

It appears that, in this example, the average velocity approaches a fixed value (2.00m/s) as the time interval decreases ($\Delta t \to 0$). This will be the instantaneous velocity at 4.0s.

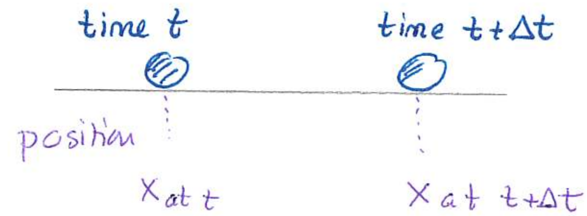
This gives an idea of the quantity we need.

concept (Instantaneous) velocity ~ rate at which position changes at one instant

definition

The (instantaneous) velocity of an object at time t is the limiting value of the average velocity over the interval

$t \to t+\Delta t$ as $\Delta t \to 0$



$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \lim \frac{x_{\text{at } t+\Delta t} - x_{\text{at } t}}{\Delta t}$$

Units: m/s

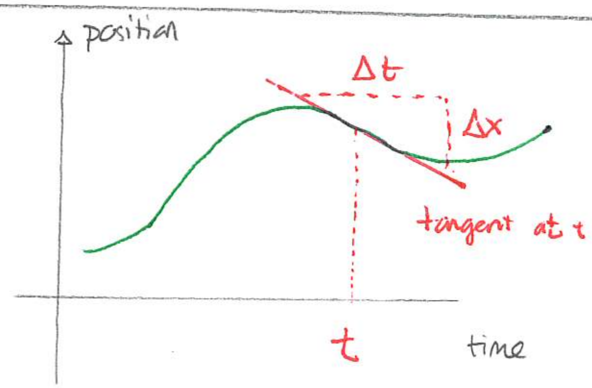
We now need methods for calculating velocity. Possibilities are:

1) approximately using position versus time data
2) exactly using calculus, given a function of x versus t.

Additionally velocity can be related to a graph of position versus time

The velocity of an object at time t is the

v = slope of the tangent line to the graph of x vs t at time t



A further definition is

$$\boxed{\text{(Instantaneous) speed} = s = \text{magnitude of velocity}}$$

Quiz1 : 90%

Quiz#2: 80%

Note that there are two aspects to velocity:

<u>magnitude</u> $\rightsquigarrow$ speed

<u>sign</u> $\rightsquigarrow$ direction of travel

$v > 0 \Rightarrow$ moves right $\bullet \longrightarrow$ $v>0$

$v < 0 \Rightarrow$ moves left $\longleftarrow \bullet$ $v<0$

<u>Position from velocity</u>

If we are given velocity information about an object, how can we determine its position. In general we cannot determine its position but we can determine change in position.

FOR UNIFORM MOTION

CONSTANT VELOCITY ONLY

Given velocity v $\longrightarrow$ Change in position is $\Delta x = v\Delta t$ in time interval Δt

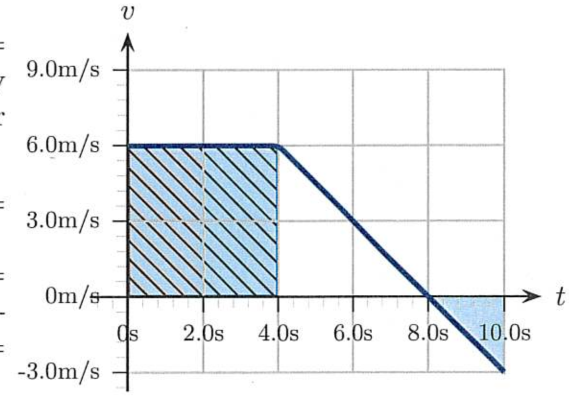
So we can get <u>displacement</u>. In order to get absolute position, we would need to know the position at the start of the interval. Note that

IF VELOCITY IS NOT CONSTANT $\Delta x \neq v\Delta t$

For non-constant velocity, we can use areas to get displacement.

31 Crawling slug

A slug crawls along a straight wire, starting at $x = 0.0\,\text{m}$ at $t = 0.0\,\text{s}$. A graph of the slug's velocity versus time is illustrated. Use the graph to answer the following. *(131Sp2025)*



a) Determine the displacement of the slug from $t = 0.0\,\text{s}$ to $t = 4.0\,\text{s}$.

b) How is the displacement of the slug from $t = 0.0\,\text{s}$ to $t = 4.0\,\text{s}$ related to the shaded area *between the graph and the horizontal axis* ($v = 0.0\,m/s$)?

c) Assuming that the answer to the previous question is true in general, determine the displacement of the slug from $t = 4.0\,\text{s}$ to $t = 8.0\,\text{s}$.

d) Is the displacement of the slug from $t = 8.0\,\text{s}$ to $t = 10.0\,\text{s}$ positive or negative? How might this relate to the shaded area from $t = 8.0\,\text{s}$ to $t = 10.0\,\text{s}$?

Answers:

a) v is constant so

$\Delta x = v\,\Delta t = 6.0\,\text{m/s} \times 4.0\,\text{s} = 24\,\text{m}$

b) area $= 6.0\,\text{m/s} \times 4.0\,\text{s} = 24\,\text{m}$. It's the same.

c) area $= \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times 4.0\,\text{s} \times 6.0\,\text{s} = 12\,\text{m}$.

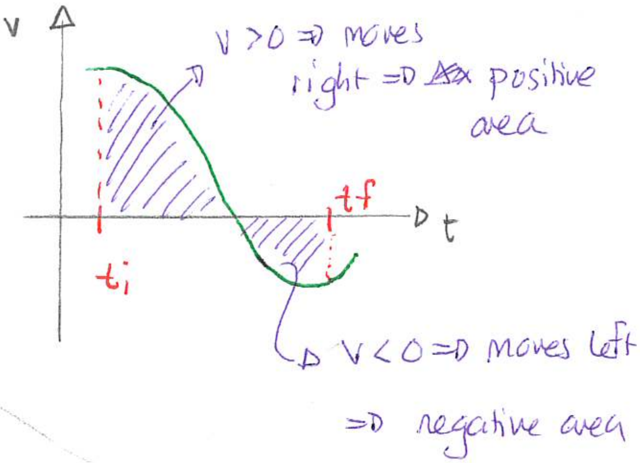
d) It moves left since $v < 0$. So $\Delta x < 0$

area $= \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times 2.0\,\text{s} \times +3.0\,\text{m/s} = 3\,\text{m}$

thus $\Delta x = -3\,\text{m}$

In general

Given a graph of velocity versus time



$\Rightarrow$ Displacement from t_i to t_f is

$$\boxed{\Delta x = \text{area between graph and } t \text{ axis from } t_i \text{ to } t_f}$$

Calculating velocity from position

We can calculate velocity precisely given an equation (or function) for position versus time: Calculus gives

$$\boxed{v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} \equiv \text{"derivative of } x \text{ with respect to } t\text{"}}$$

Calculus also provides rules to determine derivatives. For polynomial functions:

$$\boxed{\text{If } \quad x = a t^n \quad \text{where} \quad a, n \text{ are constants then } \frac{dx}{dt} = n a t^{n-1}}$$

37 Velocity as a derivative, 2

Suppose that the position of an object is

$$x = \left(0.25\,\mathrm{m/s^{3}}\right)t^3 + \left(6\,\mathrm{m/s^{2}}\right)t$$

Determine the velocity of the object at $t = 4\,\mathrm{s}$. *(131Sp2025)*

Answer:

$$V = \text{deriv of } x \text{ w.r.t. } t$$

$$= \text{deriv of}\left[(0.25\,\mathrm{m/s^3})\,t^3\right] + \text{deriv of}\left[6\,\mathrm{m/s^2} \times t\right]$$

$$= 3 \times (0.25\,\mathrm{m/s^3})\,t^2 + 6\,\mathrm{m/s}\ 1 \times t^0$$

$$= 0.75\,\mathrm{m/s^3}\,t^2 + 6\,\mathrm{m/s}$$

At $t = 4\,\mathrm{s}$

$$V = 0.75\,\mathrm{m/s^2}\,(4\,\mathrm{s})^2 + 6\,\mathrm{m/s} \quad = 18\,\mathrm{m/s}$$