Fundamental Mechanics: Class Exam 1

20 February 2025

Name: SOLUTION **Total:** /70

Instructions

- There are 8 questions on 6 pages.
- Show your reasoning and calculations and always explain your answers.

Physical constants and useful formulae

$$g = 9.80\,\text{m/s}^2$$

Question 1

A ball falls toward a table and bounces back. At an instant 0.10 s *before* it hits the table it moves down with speed 12 m/s and at an instant 0.10 s *after* it hits the table it moves up with speed 8.0 m/s. Determine the average acceleration of the ball between the two instants.

$a_{avg} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{\Delta t}$

$v_i = -12\,m/s$ ↓ ⊗ ⊗ ↑ $v_f = 8.0\,m/s$

$= \frac{8.0\,m/s - (-12\,m/s)}{0.20\,s} = \frac{20\,m/s}{0.20\,s}$

$\Rightarrow a_{avg} = 100\,m/s^2$

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Question 2

A ball rolls to the left with constantly decreasing speed. Assuming the usual *position variable* increasing rightwards, which of the following (choose one) is true?

i) The ball's acceleration is zero.
ii) The ball's acceleration is positive.
iii) The ball's acceleration is negative.

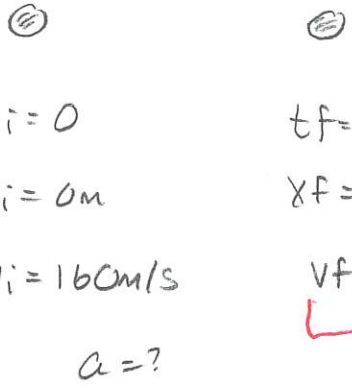
Briefly explain your choice.

$a = \frac{\Delta v}{\Delta t}$ Here v is increasing $(-3\,m/s \to -2\,m/s \to -1\,m/s)$ /6

Thus $\Delta v > 0 \Rightarrow a > 0$

Question 3

An molecule moving in a straight line passes a mark, moving right with speed 160 m/s at that instant. It subsequently moves with constant acceleration to the right in a straight line and stops a distance of 3.20 m from the mark. Determine the time taken for the molecule to reach a stop.



$t_i = 0$ $\quad t_f =$

$X_i = 0m$ $\quad X_f = 3.20m$

$V_i = 160m/s$ $\quad V_f = 0m/s$] +1

$a = ?$ +1

Need acceleration: $V_f^2 = V_i^2 + 2a(X_f - X_i)$] +1

$(0m/s)^2 = (160m/s)^2 + 2a\ 3.20m$

$\Rightarrow \quad \frac{-(160\ m/s)^2}{6.4\ m} = a$

$\Rightarrow \quad a = -4000m/s^2$] +4

Now time:

$V_f = V_i + a\,\Delta t$] +1

$\Rightarrow \quad 0m/s = 160m/s - 4000m/s^2\,\Delta t$

$\Rightarrow \quad \Delta t = \frac{160m/s}{4000m/s^2} = 0.040s$] +4

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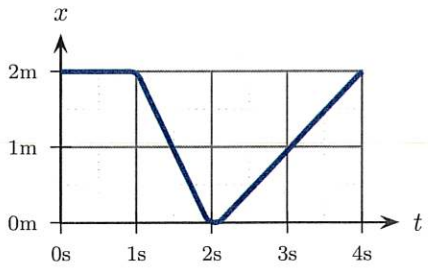
Question 4

An ant walks back and forth along a perfectly straight stick. The position vs. time graph for the ant during a period of its motion is illustrated. In the following, consider right as positive.



a) Is the ant's speed at 1.5 s the same, larger, or smaller than that at 3.0 s? Explain your answer.

speed = $|v|$ and v = slope of x vs t.

From 1-02s slope = $\frac{-2m}{1s} = -2m/s$ From 2-04s slope = $\frac{2m}{2s} = 1m/s$

⇒ speed at 1.5s larger

b) Is the average acceleration of the ant from 1.5 s to 3.0 s zero, positive or negative? Explain your answer.

$a_{avg} = \frac{\Delta v}{\Delta t} = \frac{1m/s - (-2m/s)}{1.5s} = 2m/s^2$ positive

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Question 5

Two balls, one heavier and the other lighter, roll off the edge a horizontal table. Ignore air resistance.

a) Suppose that the balls are launched with the same speed. Which of the following (choose one) is true?

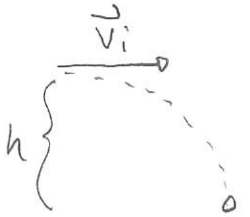
+4

i) The heavier ball travels the same distance as the lighter ball.
ii) The heavier ball travels further than the lighter ball.
iii) The lighter ball travels further than the heavier ball.

$X_f = X_i + V_{ix}\Delta t$

$X_f = V_i \Delta t$ same

b) Does the time taken to hit the floor depend on the launch speed or the mass? Explain your answer.



$V_{ix} = V_i$

$V_{iy} = 0$

$y_f = y_i + \cancel{V_{iy}\Delta t}^{0} + \frac{1}{2}a_y \Delta t^2$

$\Rightarrow 0 = +h - g/2\,\Delta t^2 \Rightarrow \Delta t = \sqrt{\frac{2h}{g}}$

This is independent of launch speed or mass. So No, time does not depend on mass nor launch speed

+1 +1

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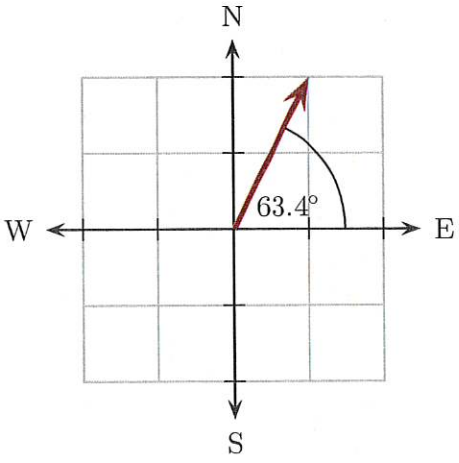
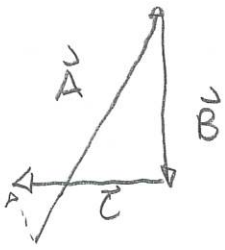
need $V_{iy}=0$ to complete explanation.

Question 6

A crab scuttles around a horizontal piece of sand. First the crab moves at an angle 63.4° North of East (direction indicated in the diagram) for 25 cm. Then the crab moves directly south (S) for a distance of 35 cm and finally directly west (W) for a distance of 20 cm. Determine how far the crab is from its starting point at the end of these three stages. The diagram indicates the directions N, S, E, W.





$\vec{B}$ = 35cm South

$\vec{C}$ = 20cm West

Overall displacement $\vec{D} = \vec{A} + \vec{B} + \vec{C}$.

Then $D_x = A_x + B_x + C_x$

$D_y = A_y + B_y + C_y$

$A_x = A\cos 63.4^\circ = 25\text{cm} \cos 63.4^\circ = 11.2\text{cm}$

$A_y = A \sin 63.4^\circ = 25\text{cm} \sin 63.4^\circ = 22.4\text{cm}$

$B_x = 0\text{cm}$

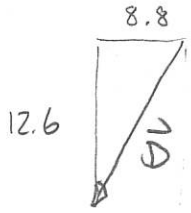
$B_y = -35\text{cm}$

$C_x = -20\text{cm}$

$C_y = 0\text{cm}$

$\Rightarrow D_x = 11.2\text{cm} + 0\text{cm} - 20\text{cm} = -8.8\text{cm}$

$\Rightarrow D_y = 22.4\text{cm} - 35\text{cm} + 0\text{cm} = -12.6\text{cm}$



$D = \sqrt{D_x^2 + D_y^2} = \sqrt{(-8.8\text{cm})^2 + (-12.6\text{cm})^2}$

$= 15.4\text{cm}$ $\Rightarrow$ travels 15.4 cm

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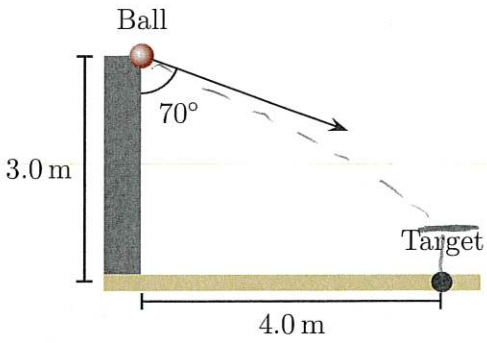
Question 7

A person on a wall attempts to throw a ball at a target on the ground below. The ball leaves the person's hand at a height of 3.0 m from the ground. The target is 4.0 m from the base of the wall. The ball is aimed so that it leaves the wall initially traveling with speed 10 m/s at an angle of 70° from the wall. Determine whether the ball passes above the target and, if so, how high it is when it is directly above the target or, if not, how far it falls short of the target.



Find vertical position when $x = 4.0\,m$

$t_i = 0\,s$ $\quad t_f = ?$

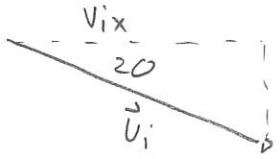
$x_i = 0m$ $\quad x_f = 4.0m$

$y_i = 3.0m$ $\quad y_f = ???$ $\quad$ +1

$v_{ix} = 9.4\,m/s$ $\quad v_{fx} =$

$v_{iy} = -3.4\,m/s$ $\quad v_{fy} =$

$a_x = 0\,m/s^2$ $\quad a_y = -9.8\,m/s^2$



$v_{ix} = v_i \cos 20^\circ = 10\,m/s \cos 20^\circ$

$= 9.4\,m/s$ $\quad$ +3

$v_{iy} = -v_i \sin 20^\circ = -10\,m/s \sin 20^\circ$

$= -3.4\,m/s$ $\quad$ +1

Now we need

+1 $\quad y_f = y_i + v_{iy}\Delta t + \frac{1}{2} a_y \Delta t^2$

$y_f = 3.0m - 3.4\,m/s\,\Delta t - \frac{9.8\,m/s^2}{2}\Delta t^2$

This requires Δt. We get this from

+1 $\quad x_f = x_i + v_{ix}\Delta t + \frac{1}{2} a_x \Delta t^2$

+3 $\quad 4.0m = 9.4\,m/s\,\Delta t$

$\Rightarrow \Delta t = \frac{4.0m}{9.4\,m/s} = 0.43s$

$y_f = 3.0m - 3.4\,m/s \times 0.43s - 4.9\,m/s^2 (0.43s)^2$

$= 3.0m - 2.36m$ $\quad$ +4

$\Rightarrow y_f = 0.64m$

+1 This is above the ground

It passes 0.64m above the target.

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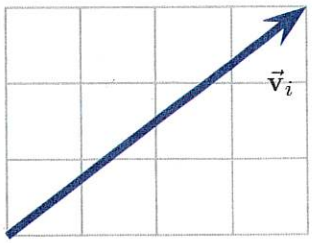
Question 8

A ball is launched at an angle with speed 5.0 m/s. Its velocity at the moment of launch, $\vec{v}_i$ is illustrated. The blocks have units of 1.0 m/s. Which of the following (choose one) best represents its velocity when it reaches its highest point?



i) 0 m/s.
ii) 3.0 m/s pointing right.
iii) 3.0 m/s pointing down.
iv) 4.0 m/s pointing right.
v) 4.0 m/s pointing down.
vi) 9.8 m/s down.

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$V_{ix} = 4.0 m/s$. This stays constant $\Rightarrow V_{fx} = 4.0 m/s$

At highest point $V_{fy} = 0 \Rightarrow V_f \longrightarrow$ 4.0 m/s