Fundamental Mechanics: Final Exam (Version 2)

17 May 2023

Name: Solution **Total:** /150

Instructions

- There are 17 questions on 11 pages.
- Show your reasoning and calculations and always explain your answers.

Physical constants and useful formulae

$g = 9.80\,\text{m/s}^2$ $G = 6.67 \times 10^{-11}\,\text{Nm}^2/\text{kg}^2$ Disk/solid cylinder: $I = \frac{1}{2}MR^2$

Hoop/hollow cylinder: $I = MR^2$ Hollow sphere: $I = \frac{2}{3}MR^2$ Solid sphere: $I = \frac{2}{5}MR^2$

Question 1 ✓✓

A ball falls toward a table and bounces back. At an instant 0.10 s *before* it hits the table it moves down with speed 12 m/s and at an instant 0.10 s *after* it hits the table it moves up with speed 8.0 m/s. Determine the average acceleration of the ball between the two instants.

$a_{avg} = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1}$

$v_1 = -12\,\text{m/s}$

$v_2 = 8\,\text{m/s}$

$= \frac{8.0\,\text{m/s} - (-12\,\text{m/s})}{0.20\,\text{s}}$

$= 100\,\text{m/s}^2$ up

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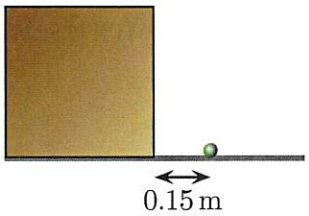
Question 2 ✓/

A cat bats a ball, which then rolls along the horizontal surface of a box, 0.85 m above the ground. The ball launches off the top lands at the illustrated spot, 0.15 m from the bottom edge of the box. Determine the speed with which the ball left the top of the box. Ignore air resistance.



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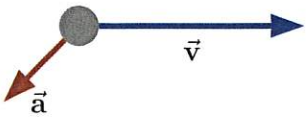
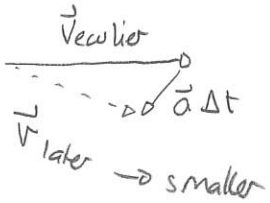
Question 3 ✓✓

An object has the illustrated acceleration and velocity vectors at one instant. Describe whether the object is speeding up, slowing down or moving at constant speed. Describe whether the object moves in a straight line or curves up or down. Explain your answers using vectors.



$\vec{a} = \frac{\vec{v}_{later} - \vec{v}_{earlier}}{\Delta t} \Rightarrow \vec{v}_{later} - \vec{v}_{earlier} = \vec{a}\,\Delta t$

$\Rightarrow \vec{v}_{later} = \vec{v}_{earlier} + \vec{a}\,\Delta t$



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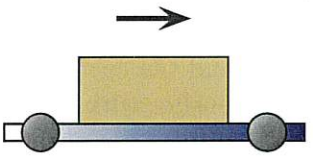
⇒ slows down

⇒ curves down

Question 4 ✓✓

A box lies a rough horizontal surface of a cart. The cart and box both move at the same constant speed in a straight line to the right. During the period while everything moves at constant speed to the right, which of the following (choose one) is true?



i) The cart exerts no friction force on the box.

ii) The cart exerts a friction force on the box. This force points right.

iii) The cart exerts a friction force on the box. This force points left.

iv) Whether there is a friction force or not depends on the speed of the cart.

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Question 5 ✓✓

A 5.0 kg book sits on a horizontal board on the ground. Starting at rest, the board is lifted with a constant force and the book reaches a speed of 3.0 m/s in 1.5 s. Determine the normal force exerted by the board on the book.

$\sum F_y = ma_y$

$\Rightarrow n - mg = ma_y \Rightarrow n = mg + ma_y \Rightarrow n = m(g + a_y)$

So we require a_y from kinematics

final ⊖

inital ⊖

$V_{yi} = 0 m/s$ $V_{yf} = 3.0 m/s$

$t_0 = 0s$ $t = 1.5s$

$V_{yf} = V_{y0} + a_y \Delta t$

$3.0 m/s = a_y\, 1.5s \Rightarrow a_y = 2.0 m/s^2$

$n = 5.0\,kg\,(9.8 m/s^2 + 2.0 m/s^2)$

$\Rightarrow n = 59N$

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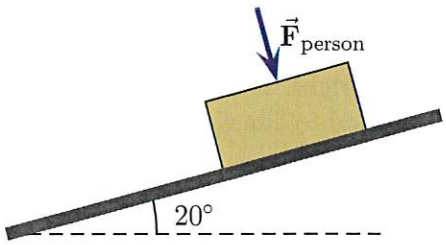
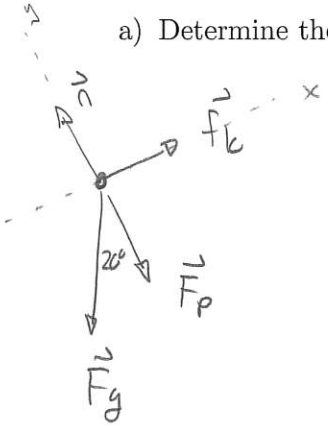
Question 6 ✓✓

+13 A 50 kg box is on an inclined ramp. The coefficient of kinetic friction between the block and ramp is 0.10 and the coefficient of static friction is 0.25. A person pushes with a 300 N force on the block and perpendicular to the ramp. The block slides down the ramp.



a) Determine the acceleration of the block.



$\Sigma F_x = ma_x$

$\Sigma F_y = ma_y = 0$

$F_g = mg \qquad f_k = \mu_k n$

	x	y
$\vec{F}_g$	$-mg\sin 20^\circ$	$-mg\cos 20^\circ$
$\vec{n}$	0	n
$\vec{F}_p$	0	$-F_p$
$\vec{f}_k$	$\mu_k n$	0

$\Sigma F_y = 0 \Rightarrow -mg\cos 20^\circ + n - F_p = 0 \Rightarrow n = F_p + mg\cos 20^\circ = 300\,N + 460\,N$

$= 760\,N$

$\Sigma F_x = ma_x \Rightarrow -mg\sin 20^\circ + \mu_k n = ma_x$

$\Rightarrow -mg\sin 20^\circ + \mu_k(mg\cos 20^\circ + F_p) = ma_x$

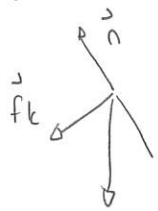
$\Rightarrow a_x = g(\mu_k \cos 20^\circ - \sin 20^\circ) + \frac{F_p \mu_k}{m}$

$= -2.43\,m/s^2 + 0.60\,m/s^2$

$\Rightarrow a_x = -1.83\,m/s^2$

b) Suppose that the block were to slide up the ramp. Explain whether the magnitude of the acceleration be larger than, smaller than or the same as that when it slides down the ramp? Explain your choice.

+3 Larger. The friction force reverses and its component along the slope does not subtract (but adds to) gravity.



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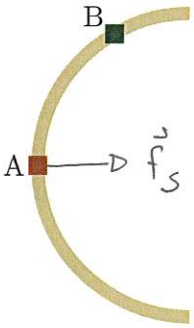
Question 7

Two identical cars travel on a rough horizontal surface. Both follow the same circular curve. The diagram illustrates this viewed from above, with the cars going counterclockwise. Car B travels faster than car A. Which of the following (choose one) is true?



i) The friction force on car A is the same as the friction force on car B.
ii) The friction force on car A is larger than the friction force on car B.
iii) The friction force on car A is smaller than the friction force on car B.

Briefly explain your choice.

In each case the static friction force points inward. Then

$\sum \vec{F} = m\vec{a} \Rightarrow f_s = m\frac{v^2}{r}$

r is same, $v_A < v_B$.

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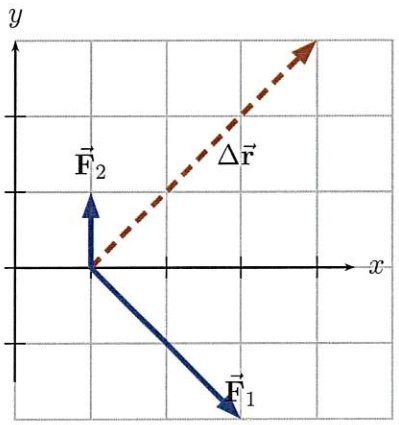
Question 8

A particle moves along the straight line illustrated by $\Delta\vec{r}$. During this time, two constant forces $\vec{F}_1$ and $\vec{F}_2$, illustrated to scale, act on the particle. Let W_1 be the work done by $\vec{F}_1$ and W_2 that done by $\vec{F}_2$. Which of the following (choose one) is true?



i) $W_1 = W_2 = 0$
ii) $W_1 = W_2 \neq 0$
iii) $W_1 > W_2$
iv) $W_1 < W_2$

Briefly explain your choice.

$W = \vec{F} \cdot \Delta\vec{r}$

Then $\vec{F}_1$ is perpendicular to $\Delta\vec{r} \Rightarrow W_1 = 0$

$\vec{F}_2$ is not perpendicular to $\Delta\vec{r}$ $\quad \Delta\vec{r} = 3\hat{i} + 3\hat{j}$

$\vec{F} = 1\hat{j}$

$\Rightarrow W_2 = 3 > 0$

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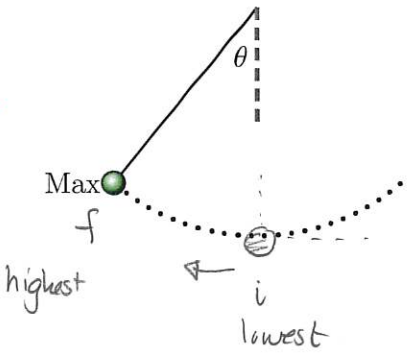
Question 9

A 0.150 kg ball is at the end of a 1.60 m string and swings in a circular arc as illustrated. The maximum speed that the ball attains as it swings is 3.00 m/s. Ignore air resistance.



a) Determine the maximum **height** and **angle from the vertical** (θ as illustrated) that the pendulum reaches.

→ above lowest point

Energy is conserved.

$E_f = E_i$

$K_f + U_{gf} = K_i + U_{gi}$

$\frac{1}{2} m v_f^2 + mgy_f = \frac{1}{2} m v_i^2 + mgy_i$

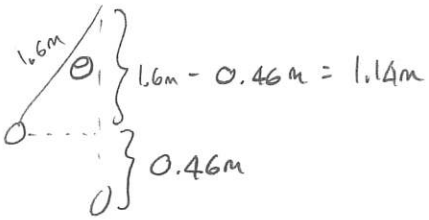
$\Rightarrow \frac{1}{2} v_f^2 + gy_f = \frac{1}{2} v_i^2 + gy_i$

$\Rightarrow gy_f = \frac{1}{2} v_i^2$

$\Rightarrow y_f = \frac{v_i^2}{2g} = \frac{(3.00\,m/s)^2}{2 \times 9.8\,m/s^2} = 0.46\,m$

$v_i = 3.00\,m/s$ $\quad v_f = 0\,m/s$

$y_i = 0\,m$ $\quad y_f = ?$



$\cos\theta = \frac{1.14}{1.6}$

$\Rightarrow \theta = \arccos(0.7125)$

$= 45^\circ$

(+2)

b) Explain whether your answers depend on the mass of the ball.

(+2) No, mass cancels as shown above.

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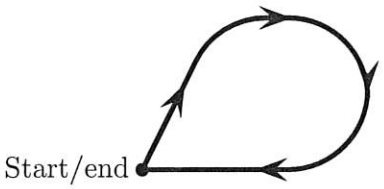
Question 10

A heavy wooden block follows the illustrated path (viewed from above) on a rough wooden horizontal table. The end point of the path is the same as the start point. Which of the following (choose one) is true regarding, W_f, the work done by friction?



i) $W_f < 0$.

ii) $W_f > 0$.

iii) $W_f = 0$.

iv) One would need more information about the path to decide between the previous options.

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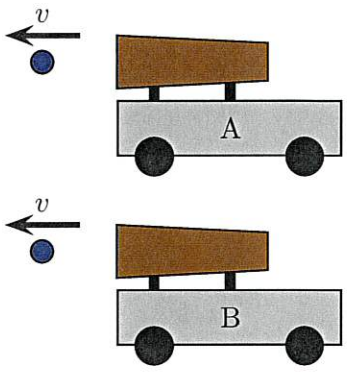
Question 11

Two carts are equipped with devices which can launch identical balls horizontally. The mass of cart A is twice the mass of cart B. Each is initially at rest and later launches a ball to the right; the speeds of the balls after launch are the same. Which of the following is true (choose one) regarding the speeds of the carts (v_A for cart A and v_B for cart B) after the balls have been launched? Ignore friction and air resistance.



i) $v_B > 2v_A$

ii) $v_B = 2v_A$

iii) $2v_A > v_B > v_A$

iv) $v_B = v_A$

$P_{tot\,f} = P_{tot\,i} = 0$

$M_{cart} V_{cartf} + M_{ball} V_{ballf} = 0$

$V_{cartf} = -\frac{M_{ball}}{M_{cart}} V_{ballf}$

$V_A = \frac{1}{2} V_B$

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Question 12

A disk can pivot about the origin. A force $\vec{\mathbf{F}} = 20\,\mathrm{N}\hat{\mathbf{j}}$ is applied to the disk at the point $\vec{\mathbf{r}} = 2\,\mathrm{m}\hat{\mathbf{i}} + 2\,\mathrm{m}\hat{\mathbf{j}}$. Determine the torque exerted by the force.

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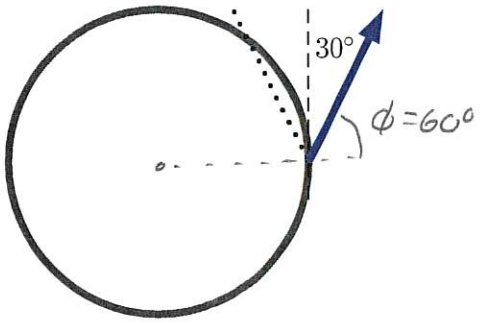
Question 13

A small rocket engine is attached to the rim of a bicycle wheel in such a way that, when activated, it exerts a force of magnitude 16.0 N at an angle of 30° to the rim. The radius of the wheel, which can be assumed to be a hoop, is 0.40 m and its mass is 0.450 kg.



a) Determine the angular acceleration of the wheel.

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$\tau_{net} = I\alpha$

For the hoop $I = MR^2$ $\Rightarrow$ $\tau_{net} = MR^2\alpha$

$\tau_{net} = \tau_{jet} + \tau_{grav}$ 0 since $r = 0$

$\tau = rF\sin\phi$ $\Rightarrow$ $\tau_{jet} = RF\sin 60°$

$\Rightarrow$ $\tau_{net} = RF\sin 60°$ $= 0.40\,m \times 16.0\,N \times \sin 60° = 5.54\,Nm$

Thus $RF\sin 60° = MR^2\alpha$

$\Rightarrow \alpha = \dfrac{F\sin 60°}{MR} = \dfrac{16.0N \sin 60°}{0.450kg \times 0.40m} = 77\ rad/s^2$

2

b) Suppose that the jet were rotated so that it exerts a force at an angle of 30° up and left of the vertical (along the dotted line). Explain whether the angular acceleration of the disk would be any different to the original situation considered in the previous part.

The angle would now be $\phi = 120°$ But $\sin 120° = \sin 60°$.

So the net torque would be the same. $\Rightarrow$ α same

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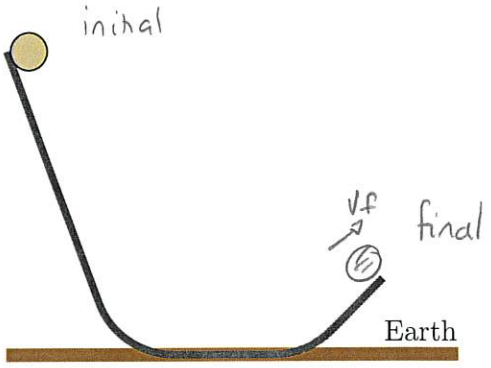
Question 14

A 0.250 kg solid sphere with radius 0.10 m is held at rest on a track as illustrated. At this instant, the bottom of the sphere is exactly 2.0 m above the Earth. The sphere is released and rolls without slipping along the track. The end of the track is at a height of 0.40 m from the Earth. Determine the speed of the sphere at the moment that it reaches the end of the track.



$E_f = E_i$

$K_{rot\,f} + K_{trans\,f} + mgy_f = K_{rot\,i} + K_{trans\,i} + mgy_i$

$y_i = 2.0\,m$ $\quad y_f = 0.40\,m$

$v_i = 0\,m/s$ $\quad v_f = ?$

$\omega_i = 0\,rad/s$ $\quad \omega_f =$

$\frac{1}{2}mv_f^2 + \frac{1}{2}I\omega_f^2 = mg(y_i - y_f)$

$I = \frac{2}{5}mR^2$

$\omega = v/R$

$\Rightarrow \frac{1}{2}mv_f^2 + \frac{1}{2}\frac{2}{5}mR^2\frac{v_f^2}{R^2} = mg(y_i - y_f)$

$\Rightarrow \frac{1}{2}\left(1+\frac{2}{5}\right)v_f^2 = g(y_i - y_f)$

$\Rightarrow \frac{7}{10}v_f^2 = g(y_i - y_f)$

$\Rightarrow v_f^2 = \frac{10g(y_i - y_f)}{7}$

$\Rightarrow v_f = \sqrt{\frac{10 \times 9.8\,m/s^2 \times 1.60\,m}{7}} = 4.7\,m/s$

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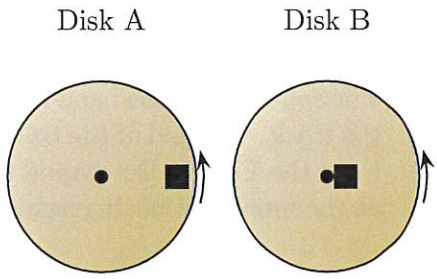
Question 15

Two identical disks initially rotate (about their centers) at the same rate. Identical blocks are lowered gently onto each disk. For disk A the block is placed near the rim while for disk B it is placed near the center. Each block sticks to its disk. Which of the following (choose one) best describes the rates at which the disks rotate **after the blocks have settled on them**?



i) Disk A rotates faster than it did previously while disk B rotates more slowly.
ii) Disk A rotates at the same rate as disk B.
iii) Disk A rotates at a slower rate than disk B.
iv) Disk A rotates at a faster rate than disk B.

Briefly explain your answer.

$L_f = L_i$ $\quad I_f = I_{disk} + I_{block}$

$I_f \omega_f = I_i \omega_i$ $\quad \omega_f = \dfrac{I_i \omega_i}{I_{disk} + I_{block}}$

same

larger (MR^2) when block is further from axle

/7

Question 16

Two satellites, called A and B, each orbit Earth in circles with constant speeds. The mass of satellite A is three times the mass of satellite B. The distance from the center of Earth to satellite B is the same as that for satellite A.

a) Which of the following (choose one) is true regarding the accelerations of the satellites?
   i) Accelerations are same.
   ii) Acceleration of A is three times that of B.
   iii) Acceleration of A is one third that of B.

b) Which of the following (choose one) is true regarding the speeds of the satellites?
   i) Speeds are same.
   ii) Speed of A is three times that of B.
   iii) Speed of A is one third that of B.
   iv) Speed of A is nine times that of B.
   v) Speed of A is one ninth that of B.

/6

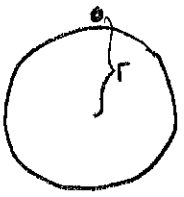
$F = ma$

$G\frac{Mm}{r^2} = ma \Rightarrow a = \frac{GM_{Earth}}{r^2}$ $\quad a = \frac{v^2}{r} \Rightarrow \frac{v^2}{r} = \frac{GM}{r^2}$

$\Rightarrow a$ same $\quad \Rightarrow v = \sqrt{\frac{2M}{r}}$

Question 17 ✓✓

An object with mass m is released from rest a height 1.65 m above the Moon's surface (mass $M_E = 7.342 \times 10^{22}$ kg and radius 1.78×10^6 m). Determine the acceleration of the object the moment after it is released. **Note: to receive full credit for this problem, your solution must with Newton's second law and use this to derive the answer.**



$\downarrow \vec{F}_G$ $\quad \Sigma \vec{F} = m\vec{a}$

$\Rightarrow F_G = ma$

$\Rightarrow G\frac{mM}{r^2} = ma$

$\Rightarrow \quad a = \frac{GM}{r^2}$

Here $r \approx$ radius of Moon

$$a = \frac{6.67\times10^{-11}\,Nm^2/kg^2 \times 7.342\times10^{22}kg}{(1.78\times10^6 m)^2}$$

$$= 1.55 m/s^2$$

/10