Fundamental Mechanics: Final Exam (Version 1)

15 May 2023

Name: Solution **Total:** /150

Instructions

- There are 17 questions on 10 pages.
- Show your reasoning and calculations and always explain your answers.

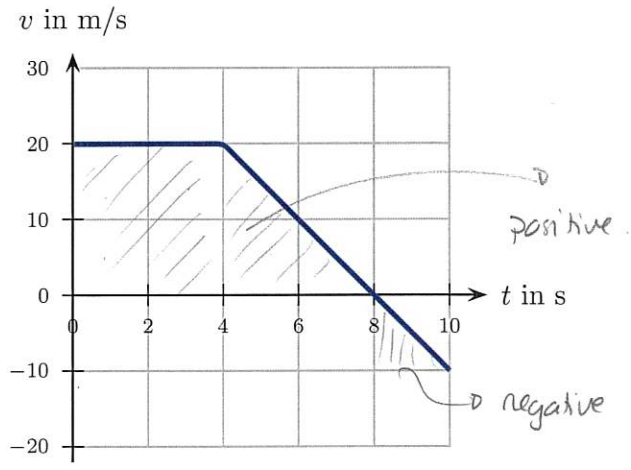
Physical constants and useful formulae

$g = 9.80\,\mathrm{m/s^2}$ $G = 6.67 \times 10^{-11}\,\mathrm{Nm^2/kg^2}$ Disk/solid cylinder: $I = \frac{1}{2}MR^2$

Hoop/hollow cylinder: $I = MR^2$ Hollow sphere: $I = \frac{2}{3}MR^2$ Solid sphere: $I = \frac{2}{5}MR^2$

Question 1 ✓

The velocity vs time graph for an asteroid is as illustrated. At $t = 0\,\mathrm{s}$ the asteroid is at $x = 0\,\mathrm{m}$. Determine the asteroid's position at $t = 10\,\mathrm{s}$.



Displacement = area under v vs t graph

Here each block has area $10\,\mathrm{m/s} \times 2\,\mathrm{s} = 20\,\mathrm{m}$

There are 6 positive blocks
½ negative block

⇒ $5.5 \times 20\,\mathrm{m} = 110\,\mathrm{m}$ = displacement

Initial position = 0m ⇒ final position = 110m

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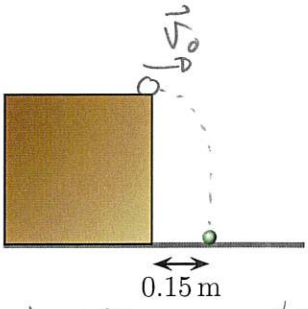
Question 2

A cat bats a ball, which then rolls along the horizontal surface of a box, 0.85 m above the ground. The ball launches off the top lands at the illustrated spot, 0.15 m from the bottom edge of the box. Determine the speed with which the ball left the top of the box. Ignore air resistance.



$t_0 = 0s$ $\quad t = ?$

$X_0 = 0$ $\quad x = 0.15m$

$Y_0 = 0.85m$ $\quad y = 0m$

$V_{ox} = V_0$ $\quad V_x =$

$V_{oy} = 0$ $\quad V_y =$

$a_x = 0 m/s^2$ $\quad a_y = -9.8 m/s^2$

+6

$X = X_0 + V_{ox} t + \frac{1}{2} a_x t^2$

$0.15m = 0m + V_0 t$

$\Rightarrow$ $0.15m = V_0 t$

We need t, from vertical motion

$y = y_0 + V_{oy} t + \frac{1}{2} a_y t^2$

$0m = 0.85m - \frac{1}{2} 9.8 m/s^2 \, t^2 \quad \Rightarrow \quad 4.9 m/s^2 \, t^2 = 0.85m$

$\Rightarrow \quad t^2 = \frac{0.85m}{4.9 m/s^2} = 0.17 s^2$ +8

$\Rightarrow \quad t = 0.42 s$

Combine $\quad V_0 = \frac{0.15m}{t} = \frac{0.15m}{0.42s} = 0.36 m/s$

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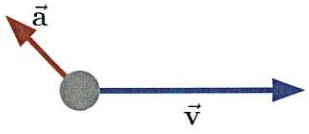
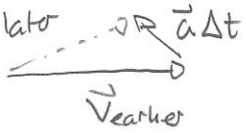
Question 3

An object has the illustrated acceleration and velocity vectors at one instant. Describe whether the object is speeding up, slowing down or moving at constant speed. Describe whether the object moves in a straight line or curves up or down. Explain your answers using vectors.



$\vec{V}_{later} = \vec{V}_{earlier} + \vec{a} \, \Delta t$



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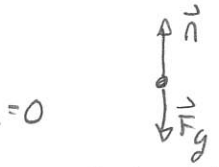
V_{later} smaller = slows

V_{later} → = curves up

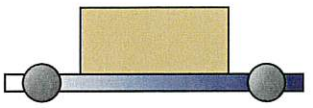
$\vec{a} = 0$ for box $\Rightarrow \Sigma F_x = 0$



$\Rightarrow$ no horizontal forces $\Rightarrow$ no friction

Question 4 ✓✓

A box lies a rough horizontal surface of a cart. The cart and box both move at the same constant speed in a straight line to the right. During the period while everything moves at constant speed to the right, which of the following (choose one) is true?



i) The cart exerts no friction force on the box.
ii) The cart exerts a friction force on the box. This force points right.
iii) The cart exerts a friction force on the box. This force points left.
iv) Whether there is a friction force or not depends on the speed of the cart.

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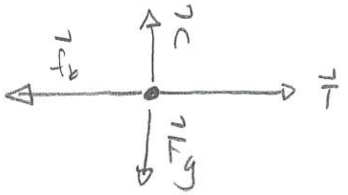
Question 5 ✓✓

A 150 kg crate is pulled by a horizontal rope. It moves in a straight line along a rough horizontal floor at a constant speed of 5.0 m/s. The coefficient of kinetic friction between these surfaces is 0.40 and the coefficient of static friction is 0.60. Determine the tension in the rope.



Speed is constant $\Rightarrow \vec{a} = 0$

$\Sigma F_x = ma_x = 0$

$\Sigma F_y = ma_y = 0$

$\therefore T - f_k = 0 \Rightarrow T = f_k = \mu_k n$

$n - F_g = 0 \Rightarrow n = mg$

Thus $T = \mu_k mg = 0.40 \times 150\,kg \times 9.8\,m/s^2$

$= 588\,N$

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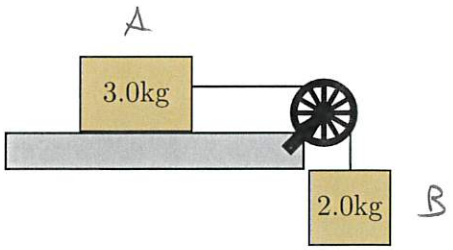
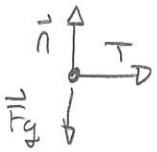
Question 6

Two blocks are connected by a string, which runs over a massless pulley. One block is suspended and the other block can move along a frictionless horizontal surface. The string connected to the block on the surface runs horizontally. Ignore air resistance and friction in this problem.



a) Determine the acceleration of the blocks.

3.0kg block



2.0kg block.

Let a = magnitude of acce

$\sum F_x = m_A a_{Ax}$

$\Rightarrow T = m_A a$

$\sum F_y = m_B a_{By}$

$\Rightarrow T - m_B g = m_B(-a)$

$m_A a - m_B g = -m_B a \Rightarrow (m_A + m_B) a = m_B g$

$\Rightarrow a = \frac{m_B}{m_A + m_B} g = \frac{2.0\,kg}{5.0\,kg} \times 9.8\,m/s^2$

$= 3.92\,m/s^2$

b) Determine the tension in the string.

$T = m_A a = 3.0\,kg \times 3.92\,m/s^2$

$= 11.8\,N$

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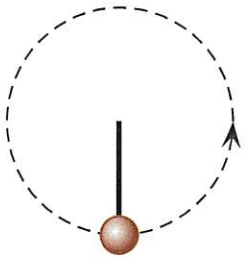
Question 7

A ball with mass m swings at the end of a string in a vertical circle. Which of the following (choose one) is true regarding the tension, T, in the string when the ball is at the *lowest point* in the circle?



i) $T = mg$ regardless of speed.

ii) $T > mg$ regardless of speed.

iii) $T < mg$ regardless of speed.

iv) $T > mg$ when the speed is large enough and $T < mg$ when the speed is small enough.

Briefly explain your choice.

At lowest point

$\Sigma F_y = ma_y$ But a_y is ↑

$\Rightarrow T > F_g \Rightarrow T > mg$

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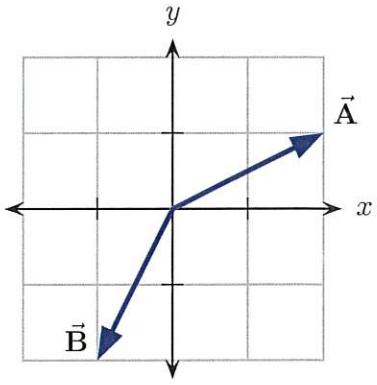
Question 8

Determine the dot product, $\vec{\mathbf{A}} \cdot \vec{\mathbf{B}}$, of the two illustrated vectors. The axis units are in meters.



$\vec{A} = 2m\,\hat{i} + 1m\,\hat{j}$

$\vec{B} = -1m\,\hat{i} - 2m\,\hat{j}$

$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y$

$= -2m^2 - 2m^2 = -4m^2$

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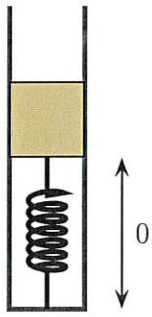
find highest pt

Question 9

A 2.0 kg block is held at rest against a spring compressing it by 0.40 m. At this point, the base of the block is 0.50 m above the ground. The spring constant of this spring is 800 N/m. These are contained in a cylinder with frictionless walls. The block is released and launches vertically, leaving the spring. Determine the maximum height above the ground that it reaches.



$E_f = E_i$ (since $W_{nc} = 0$)

$K_f + U_{gf} + U_{spf} = K_i + U_{gi} + U_{spi}$

$\cancel{\frac{1}{2}mv_f^2} + mgy_f + \cancel{\frac{1}{2}kx_f^2} = \cancel{\frac{1}{2}mv_i^2} + mgy_i + \frac{1}{2}kx_i^2$

$mgy_f = mgy_i + \frac{1}{2}kx_i^2$

spring compression

$x_i = 0.40\text{m}$ $x_f = 0$

$y_i = 0.50\text{m}$ $y_f = ?$

$v_i = 0\text{m/s}$ $v_f = 0\text{m/s}$

$2.0\text{kg} \times 9.8\text{m/s}^2\, y_f = 2.0\text{kg} \times 9.8\text{m/s}^2 \times 0.50\text{m} + \frac{1}{2}800\text{N/m}\,(0.40\text{m})^2$

$19.6\text{kgm/s}^2\, y_f = 74\text{J}$

$$y_f = \frac{74\text{J}}{19.6\text{kgm/s}^2} = 3.8\text{m}$$

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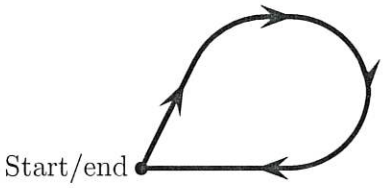
Question 10

A heavy wooden block follows the illustrated path (viewed from above) on a rough wooden horizontal table. The end point of the path is the same as the start point. Which of the following (choose one) is true regarding, W_f, the work done by friction?



i) $W_f < 0$.

ii) $W_f > 0$.

iii) $W_f = 0$.

iv) One would need more information about the path to decide between the previous options.

Friction is always 180° from displacement

$W_f = \vec{f}_k \cdot \Delta\vec{r}$

$= f_k \Delta r \cos 180^\circ$

negative

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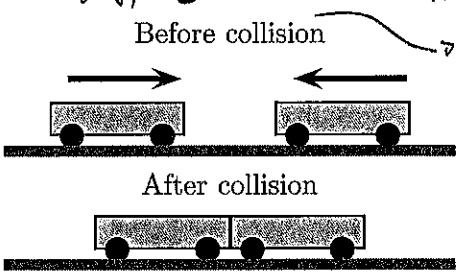
Momentum conserved $\Rightarrow p_{tot\,f} = p_{tot\,i}$ ($=0$ since equal opposite) $\Rightarrow v_f = 0$

Question 11

Two identical carts travel on a frictionless surface toward each other with identical speeds and collide and stick together as illustrated. Which of the following (choose one) is true after the collision?



$\Rightarrow K_f = 0 \neq K_i$

i) The carts are at rest and total kinetic energy is the same as before the collision.

ii) The carts are at rest and total kinetic energy is **not** the same as before the collision.

iii) The carts are **not** at rest and the total kinetic energy is the same as before the collision.

iv) The carts are **not** at rest and the total kinetic energy is **not** the same as before the collision.

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Question 12

A solid disk and a hoop with the same mass and radius each approach the same ramp with the same velocity as illustrated. They both roll without slipping. Let $h_{\text{max disk}}$ be the maximum height reached by the disk and $h_{\text{max hoop}}$ the maximum height reached by the hoop. Which of the following is true?

$E_f = E_i$

$U_{gf} = K_{rot\,i} + K_{trans\,i}$

i) $h_{\text{max disk}} = h_{\text{max hoop}}$

ii) $h_{\text{max disk}} > h_{\text{max hoop}}$

iii) $h_{\text{max disk}} < h_{\text{max hoop}}$

$mg\,h_{max} = \frac{1}{2} I\omega^2 + \frac{1}{2} mv^2$

$= \frac{1}{2}\frac{I}{r^2} v^2 + \frac{1}{2} mv^2$

I is larger for hoop

$\Rightarrow h_{max}$ larger hoop $= \frac{1}{2}\left(\frac{I}{r^2} + m\right) v^2$

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Question 13

A disk can pivot about the origin. A force $\vec{\mathbf{F}} = 20\,\text{N}\hat{\mathbf{j}}$ is applied to the disk at the point $\vec{\mathbf{r}} = 2\,\text{m}\hat{\mathbf{i}} + 2\,\text{m}\hat{\mathbf{j}}$. Determine the torque exerted by the force.

$\vec{\tau} = \vec{r} \times \vec{F}$

$= (2m\,\hat{i} + 2m\,\hat{j}) \times 20N\,\hat{j}$

$= 40\,Nm\,\hat{i} \times \hat{j} + 40\,Nm\,\hat{j} \times \hat{j}$

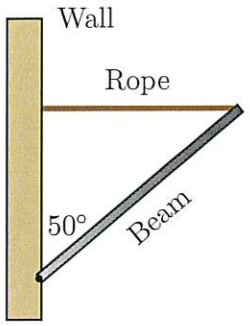
$\hat{i} \times \hat{j} = \hat{k}$, $\hat{j} \times \hat{j} = 0$

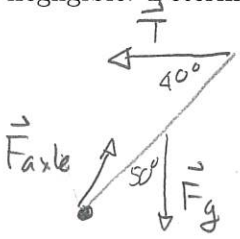
$\Rightarrow \tau = 40\,Nm\,\hat{k}$

/6

Question 14

A 3.0 m long, 250 kg uniform steel beam is anchored to a wall. A horizontal rope is attached to the end of the beam and holds it at rest at the illustrated angle. The thickness of the beam is negligible. Determine the tension in the rope.





$\tau_{net} = 0$

$\Rightarrow \tau_{axle} + \tau_g + \tau_{rope} = 0$

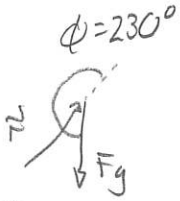
$\tau = rF\sin\phi$

Axle: $\tau = 0$ since $r = 0$

Gravity $\tau_g = r\,mg\sin 230^\circ$

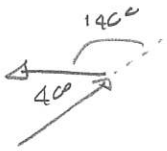
$= 1.5\text{m} \times 250\text{kg} \times 9.8\text{m/s}^2 \sin 230^\circ$

$= -2820\text{ Nm}$



Rope $\tau_{rope} = r\,T\sin 140^\circ$

$= 3.0\text{m}\,T\sin 140^\circ$



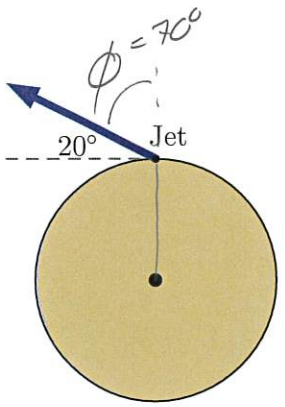
Adding: $0\text{ Nm} - 2820\text{Nm} + 3.0\text{m}\sin 140^\circ = T = 0$

$\underbrace{3.0\text{m}\sin 140^\circ}_{1.93} T = 2820\text{Nm}$

$\Rightarrow T = \dfrac{2820\text{Nm}}{1.93} = 1460\text{ Nm}$ /12

Question 15

A 4.0 kg solid disk with radius 0.25 m can rotate horizontally about a frictionless axle through its center. A small jet is mounted at the edge of the disk and exerts a constant 20 N force at the indicated angle.



10 a) Determine the angular acceleration of the wheel.

$\tau_{net} = I\alpha$

For a solid disk $I = \frac{1}{2}MR^2 = \frac{1}{2}\,4.0\text{kg} \times (0.25\text{m})^2 = 0.125\ \text{kg m}^2$

$\tau_{net} = \tau_{axle} + \tau_g + \tau_{jet}$

$\tau_{axle} = 0$ since $r = 0$

$\tau_{jet} = rF\sin\phi = RF\sin 70° = 0.25\text{m} \times 20\text{N}\sin 70°$

$= 4.7\ \text{Nm}$

$\tau_{net} = 4.7\ \text{Nm} = I\alpha$

$\Rightarrow \alpha = \frac{4.7\ \text{Nm}}{0.125\ \text{kgm}^2} = 37.6\ \text{rad/s}^2$

6 b) If the wheel is initially at rest determine its angular velocity after the first complete revolution.

$\theta_0 = 0$ rad

$\theta = 2\pi$ rad

$\omega_0 = 0$ rad/s

$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$

$= (0\text{rad/s})^2 + 2(37.6\ \text{rad/s}^2)\,2\pi\text{rad}$

$= 472\ \text{rad}^2/\text{s}^2$

$\Rightarrow \omega = 21.7\ \text{rad/s}$

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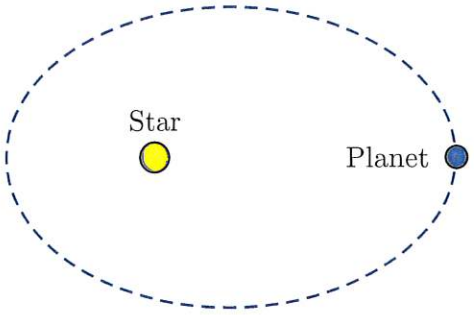
Question 16

A planet moves around a star in an elliptical orbit as illustrated. There are no other planets or stars nearby which have any noticeable affects on the planet depicted here. Which of the following (choose one) is true regarding the planet's speed?



i) The speed of the planet remains constant throughout its orbit.

ii) The speed of the planet increases as it gets nearer to the star.

iii) The speed of the planet decreases as it gets nearer to the star.

Briefly explain your answer. cannot use accel and tie to speed

E = constant

$K + U_G = \text{const}$

$K - G\frac{mM}{r} = \text{const} \Rightarrow \frac{1}{2}mv^2 = \text{const} + G\frac{mM}{r}$

r is distance to star. As r decreases v increases

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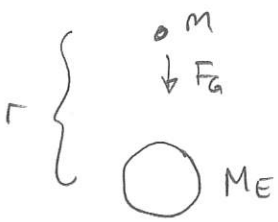
Question 17

An object with mass m is released from rest at a distance r from the center of Earth (mass $M_E = 5.98 \times 10^{24}$ kg and radius 6.37×10^6 m). Determine an expression for the acceleration of the object the moment after it is released. **Note: to receive full credit for this problem, your solution must with Newton's second law and use this to derive the answer.**



$\vec{F} = m\vec{a}$

$F_G = ma$

$G\frac{mM_E}{r^2} = ma$

$a = G\frac{M_E}{r^2}$

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