Phys 311 2025

Lecture 1

Intro: * Syllabus

* Course website - main page
  - lectures page

* Assignments - typically due Tues/Friday
  - first due Fri, Aug 22

* Exams - two in-class - see syllabus
  - final

Next (Thurs):

1.1.1 cross prod.

1.1.2

1.1.3

1.2.1, 1.2.2

Electromagnetism Overview

Electromagnetism considers how charged particles interact with each other. Electromagnetic theory provides a framework for describing all electrical and magnetic interactions involving charged particles. It does this via intermediaries called electric and magnetic fields and electromagnetic theory provides:

* complete methods for determining the electric and magnetic fields produced by source charges and currents.
* complete methods for determining the forces exerted by electric and magnetic fields on charges and currents.

DEMO: PhET Radio Waves + EM Fields

* hide fields and oscillate electron
* do same but observe fields

The electromagnetic theory presented here will be done within the framework of classical physics and will use ideas such as force and acceleration. The theory will be developed to the point where it reaches a compact, complete formalism

* Maxwell's equations
* Lorentz force law

This can eventually be adapted to fit within quantum theory.

Electromagnetic theory is important because:

* it is essential for describing any situation involving charged particles e.g.
  - atomic structure
  - trapped ions - show Ion Trap QI page
  - radio wave production and detection.
* it is extremely useful for describing optical phenomena and essential for describing interaction of light and matter.

Phys 311 aims to:

1) provide a complete description of electric and magnetic fields and their roles.
2) " various techniques for determining electric and magnetic fields.
3) arrive at Maxwell's equations.

This course will use and develop:

1) basic classical mechanics - Newton's Laws, energy...
2) vector algebra, vector calculus
3) calculus in three dimensions.

Vector algebra 1.6.1

In electromagnetic theory vectors will

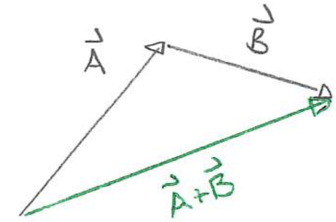
* describe locations of charges and currents
* directions of currents
* electric and magnetic fields

These will require:

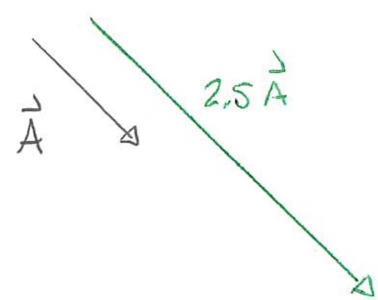
1) vector algebra ~ algebraic operations between vectors (addition, multiplication)
2) vector calculus ~ differentiation and integration of vectors and vector functions

We will illustrate these with displacement or position vectors but the basic operations exist for any type of vector. The two basic operations that exist for all types of vectors:

1) <u>vector addition</u> ~ for displacement vectors combine successive displacements



2) <u>scalar multiplication</u> ~ rescaled displacement



We can abstract the properties of these operations from intuitive ideas about displacement vectors. When these are abstracted, they can be formed into a rigorous definition of vectors and their operations.

The basic ideas is that vectors form a set (called a vector space) with two operations that satisfy a specific set of properties.

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See
Halmos
"Finite
dimensional
vector
spaces."

For any set of vectors there are two operations:

i) addition $\Rightarrow$ for $\vec{A}, \vec{B}$ the sum, $\vec{A} + \vec{B}$ is also a vector

ii) scalar multiplication $\Rightarrow$ for $\vec{A}$ and a scalar λ, $\lambda \vec{A}$ is a vector

These satisfy

1) addition is commutative: $\vec{A} + \vec{B} = \vec{B} + \vec{A}$

2) addition is associative: $\vec{A} + (\vec{B} + \vec{C}) = (\vec{A} + \vec{B}) + \vec{C}$

3) there is a zero vector, $\vec{0}$, $\vec{A} + \vec{0} = \vec{A}$

4) for any vector, $\vec{A}$, there is an additive inverse, $-\vec{A}$ s.t. $\vec{A} + (-\vec{A}) = \vec{0}$

5) for any vector $\vec{A}$ and scalars, α, β $\alpha(\beta \vec{A}) = (\alpha \beta) \vec{A}$

6) for the scalar 1 and any vector $\vec{A}$ $1 \vec{A} = \vec{A}$

7) for any vectors $\vec{A}, \vec{B}$ and scalar α: $\alpha(\vec{A} + \vec{B}) = \alpha \vec{A} + \alpha \vec{B}$

8) for any vector $\vec{A}$ and scalars α, β $(\alpha + \beta) \vec{A} = \alpha \vec{A} + \beta \vec{A}$

There are many examples of vector spaces:

1) column vectors $\vec{v} = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$

2) matrices $M = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix}$

3) continuous functions.

The theory of linear algebra then provides a framework that applies to all types of vectors.

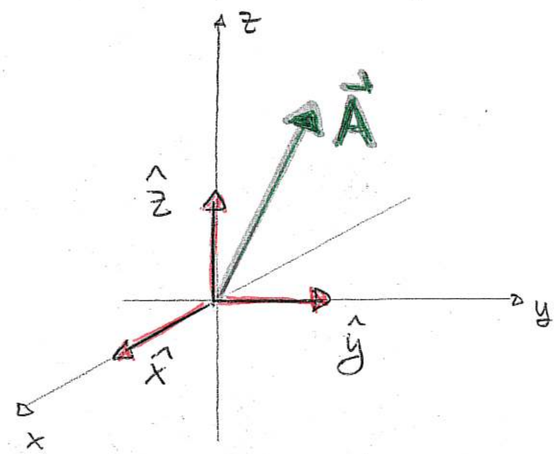
Vector bases and vector components 1.1.2

Consider a displacement vector in three dimensions, $\vec{A}$. It is intuitive that it can be expressed in terms of three special unit vectors along the Cartesion x, y, z axes. Specifically

vector

$$\vec{A} = A_x \hat{x} + A_y \hat{y} + A_z \hat{z}$$

scalar



where A_x, A_y, A_z are scalars.

This possibility is asswed via various theorems of linear algebra

> For any three dimensional vector, $\vec{A}$, there exist
> 1) three basis vectors $\hat{x}, \hat{y}, \hat{z}$
> 2) three unique scalars A_x, A_y, A_z
>
> such that
>
> $$\vec{A} = A_x \hat{x} + A_y \hat{y} + A_z \hat{z}$$

The three scalars are called the components of $\vec{A}$ (in the basis $\{\hat{x}, \hat{y}, \hat{z}\}$).
If we use one particular fixed basis then the vectors can be expressed uniquely (in a 1-1 correspondence) with column vectors

$$\vec{A} = A_x \hat{x} + A_y \hat{y} + A_z \hat{z} \quad \Rightarrow \quad \vec{A} \leftrightarrow \begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix}$$

$$\vec{B} = B_x \hat{x} + B_y \hat{y} + B_z \hat{z} \quad \Rightarrow \quad \vec{B} \leftrightarrow \begin{pmatrix} B_x \\ B_y \\ B_z \end{pmatrix}$$

and

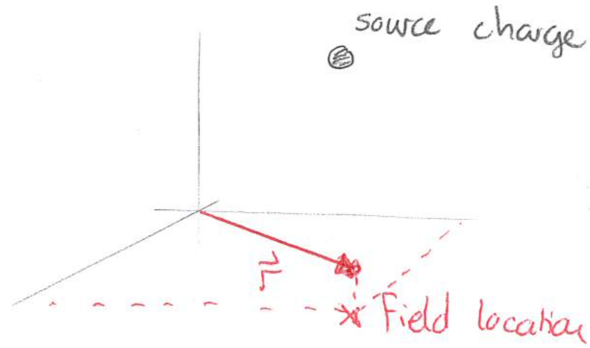
$$\vec{A} + \vec{B} \leftrightarrow \begin{pmatrix} A_x + B_x \\ A_y + B_y \\ A_z + B_z \end{pmatrix} \quad \text{and} \quad \alpha \vec{A} \leftrightarrow \alpha \begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix} = \begin{pmatrix} \alpha A_x \\ \alpha A_y \\ \alpha A_z \end{pmatrix}$$

Position vectors

1.1.4

In electromagnetic theory we use position vectors to describe

- locations at which fields are to be calculated
- locations of sources



Consider the location of a point where a field is to be calculated. We can describe this by:

$$\text{position vector} \equiv \text{displacement vector from the origin to field point}$$

We denote position vectors for field points by

vector

$$\vec{r} = \underbrace{x}_{\text{components}}\overbrace{\hat{x}}^{\text{basis}} + y\hat{y} + z\hat{z} \quad \longleftrightarrow \quad \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

vector   components / basis   column

We can show that given two vectors $\vec{r}_1$ and $\vec{r}_2$ we can combine them as follows

$$\begin{array}{ccccc} \vec{r}_1 & & \vec{r}_2 & & \vec{r}_1 + \vec{r}_2 \\ \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} & + & \begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix} & = & \begin{pmatrix} x_1 + x_2 \\ y_1 + y_2 \\ z_1 + z_2 \end{pmatrix} \end{array}$$

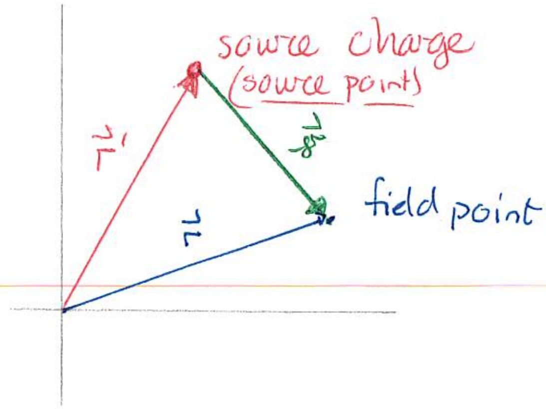
and for any number α

$$\begin{array}{cccc} & \vec{r} & & \alpha\vec{r} \\ \alpha & \begin{pmatrix} x \\ y \\ z \end{pmatrix} & = & \begin{pmatrix} \alpha x \\ \alpha y \\ \alpha z \end{pmatrix} \end{array}$$

We will denote position vectors as follows:

field locations ~ unprimed $\vec{r}$

source locations ~ primed $\vec{r}\,'$



Separation vectors

We will frequently have to determine the field at the location, $\vec{r}$, produced by a source at $\vec{r}\,'$. The crucial quantity will be the separation from the source point to the field point. This is described by:

specify field point $\vec{r}$

specify source point $\vec{r}\,'$

$\longrightarrow$ **separation vector**

$$\boxed{\vec{\mathscr{r}} = \vec{r} - \vec{r}\,'}$$

In terms of components:

$$\vec{r} = x\hat{x} + y\hat{y} + z\hat{z}$$

$$\vec{r}\,' = x'\hat{x} + y'\hat{y} + z'\hat{z}$$

Then

$$\vec{\mathscr{r}} = \vec{r} - \vec{r}\,' = (x-x')\,\hat{x} + (y-y')\,\hat{y} + (z-z')\,\hat{z}$$

1 Pyramid

A pyramid has a square base with sides of length L. The apex of the pyramid is a height h above the center of the base. The base lies in the first quadrant of the xy plane. Let A be the corner at the origin, B that along the x axis, C that away from either axis and D that along the y axis. Let E be the apex.

a) Determine expressions in terms of the standard basis vectors for the position vector of each corner.

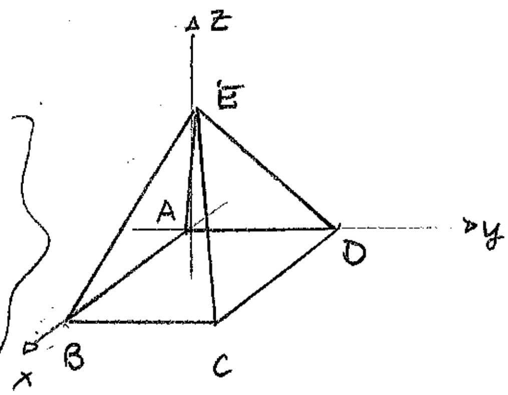
b) Determine an expressions in terms of the standard basis vectors for the separation vector from B to C.

c) Determine an expressions in terms of the standard basis vectors for the separation vector from B to E.

d) Determine an expressions in terms of the standard basis vectors for the separation vector from C to E.

Answer:

a)
$$\vec{r}_A = 0\hat{x} + 0\hat{y} + 0\hat{z} = \vec{0}$$
$$\vec{r}_B = L\hat{x} + 0\hat{y} + 0\hat{z} = L\hat{x}$$
$$\vec{r}_C = L\hat{x} + L\hat{y} + 0\hat{z} = L\hat{x} + L\hat{y}$$
$$\vec{r}_D = 0\hat{x} + L\hat{y} + 0\hat{z} = L\hat{y}$$
$$\vec{r}_E = \frac{L}{2}\hat{x} + \frac{L}{2}\hat{y} + h\hat{z}$$



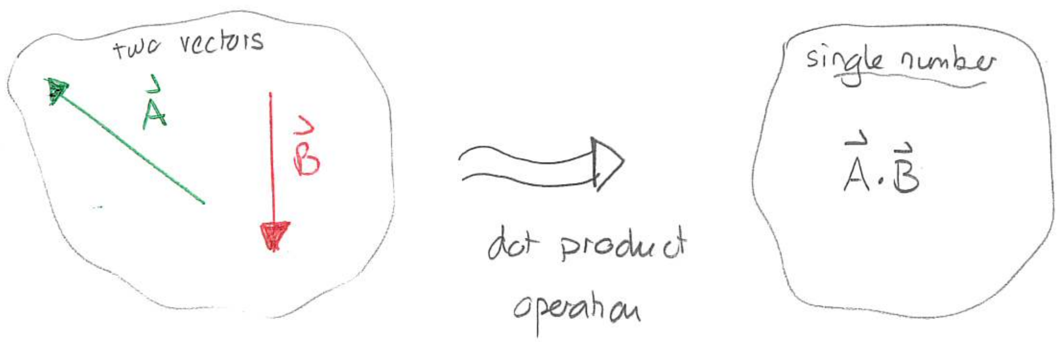
b) $$\vec{r}_{B \text{ to } C} = \vec{r}_C - \vec{r}_B = L\hat{x} + L\hat{y} - L\hat{x} = L\hat{y}$$

c) $$\vec{r}_{B \text{ to } E} = \vec{r}_E - \vec{r}_B = \frac{L}{2}\hat{x} + \frac{L}{2}\hat{y} + h\hat{z} - L\hat{x} = -\frac{L}{2}\hat{x} + \frac{L}{2}\hat{y} + h\hat{z}$$

d) $$\vec{r}_{C \text{ to } E} = \vec{r}_E - \vec{r}_C = \frac{L}{2}\hat{x} + \frac{L}{2}\hat{y} + h\hat{z} - L\hat{x} - L\hat{y} = -\frac{L}{2}\hat{x} - \frac{L}{2}\hat{y} + h\hat{z}.$$

Dot product 1.1.1

Some vector spaces have additional operations beyond addition and scalar multiplication. One example is the dot product (inner product) that maps two vectors to a scalar



One way to define the dot product is:

> Suppose $\vec{A}$ and $\vec{B}$ are two vectors represented in a Cartesian basis as
>
> $$\vec{A} = A_x\hat{x} + A_y\hat{y} + A_z\hat{z}$$
>
> $$\vec{B} = B_x\hat{x} + B_y\hat{y} + B_z\hat{z}$$
>
> Then their dot product is
>
> $$\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z$$

Note that this implies:

$$\hat{x}\cdot\hat{x} = (1\hat{x} + 0\hat{y} + 0\hat{z})\cdot(1\hat{x} + 0\hat{y} + 0\hat{z}) = 1$$

$$\hat{x}\cdot\hat{y} = \quad \ldots \quad = 0$$

and thus

$$\hat{x}\cdot\hat{x} = 1 \qquad \hat{y}\cdot\hat{y} = 1 \qquad \hat{z}\cdot\hat{z} = 1$$

$$\hat{x}\cdot\hat{y} = 0 \qquad \hat{x}\cdot\hat{z} = 0 \qquad \hat{y}\cdot\hat{z} = 0$$

etc..

We can also see the following abstract properties:

The dot product satisfies:

1) $\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$

2) $\vec{A}\cdot(\vec{B}+\vec{C}) = \vec{A}\cdot\vec{B} + \vec{A}\cdot\vec{C}$

3) $(\lambda\vec{A})\cdot\vec{B} = \lambda(\vec{A}\cdot\vec{B})$

4) $\vec{A}\cdot\vec{A} \geq 0$ with $\vec{A}\cdot\vec{A} = 0 \Leftrightarrow A = 0$

We can use the dot product to define the magnitude of a vector

The magnitude of $\vec{A}$ is

$$A = |\vec{A}| = \sqrt{\vec{A}\cdot\vec{A}}$$

Then the Cauchy-Schwarz inequality is

$$-1 \leq \frac{\vec{A}\cdot\vec{B}}{|\vec{A}||\vec{B}|} \leq 1$$

and this means that one can define the angle between two vectors via:

If $\vec{A}, \vec{B}$ are two vectors then the angle, θ, between them satisfies

$$\cos\theta = \frac{\vec{A}\cdot\vec{B}}{|\vec{A}||\vec{B}|} \quad \text{or} \quad \vec{A}\cdot\vec{B} = AB\cos\theta$$

Then

$\vec{A}$ and $\vec{B}$ are orthogonal $\Leftrightarrow$ the angle between them is $90°$

$$\Leftrightarrow \vec{A}\cdot\vec{B} = 0$$

2 Pyramid face angles

A pyramid has a square base with sides of length L. The apex of the pyramid is a height h above the center of the base. The base lies in the first quadrant of the xy plane. Let A be the corner at the origin, B that along the x axis, C that away from either axis and D that along the y axis. Let E be the apex.

a) Determine an expression for the length of the side from B to E.

b) Determine the angle on the EBC face at point B.

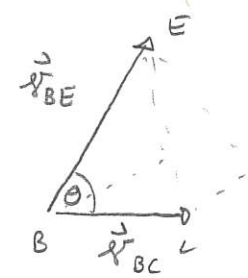
c) Determine the angle on the EBC face at point E.

Answers: a) this requires $\vec{r}_{B\text{ to }E}$ and

$$r_{B\text{ to }E} = \sqrt{\vec{r}_{B\text{ to }E} \cdot \vec{r}_{B\text{ to }E}} = \left[\left(-\frac{L}{2}\right)^2 + \left(\frac{L}{2}\right)^2 + h^2\right]^{1/2}$$

$$= \sqrt{L^2/2 + h^2}$$

b) $$\cos\theta = \frac{\vec{r}_{BE} \cdot \vec{r}_{BC}}{|\vec{r}_{BE}||\vec{r}_{BC}|}$$



Then

$$r_{BC} = \sqrt{\vec{r}_{BC} \cdot \vec{r}_{BC}}$$

$$= \left[L^2\right]^{1/2} = L.$$

$$\vec{r}_{BE} \cdot \vec{r}_{BC} = \left(-\frac{L}{2}\hat{x} + \frac{L}{2}\hat{y} + h\hat{z}\right) \cdot L\hat{y}$$

$$= -\frac{L}{2}L\cancel{\hat{x}\cdot\hat{y}}^{\,0} + \frac{L}{2}L\underbrace{\cancel{\hat{y}\cdot\hat{y}}}_{1} + hL\cancel{\hat{z}\cdot\hat{y}}^{\,0} = L^2/2$$

$$\Rightarrow \cos\theta = \frac{L^2/2}{\sqrt{L^2/2 + h^2}\,L} = \frac{L^2}{2\sqrt{L^2\left(\frac{1}{2} + h^2/L^2\right)}\,L} = \frac{1}{2\sqrt{\frac{1}{2} + h^2/L^2}}$$

Note that this ranges from

$$h=0 \Rightarrow \cos\theta = \frac{1}{\sqrt{2}} \Rightarrow \theta = 45^\circ$$

$$h=\infty \Rightarrow \cos\theta = 0 \Rightarrow \theta = 90^\circ$$

c) This is

$$\cos\theta = \frac{\vec{r}_{EB} \cdot \vec{r}_{EC}}{|\vec{r}_{EB}|\,|\vec{r}_{EC}|}$$

Now

$$\vec{r}_{EB} = -\vec{r}_{BE} \qquad \vec{r}_{EC} = -\vec{r}_{CE} \Rightarrow \cos\theta = \frac{\vec{r}_{BE} \cdot \vec{r}_{CE}}{|\vec{r}_{BE}|\,|\vec{r}_{CE}|}$$

Then

$$|\vec{r}_{EB}| = |\vec{r}_{BE}| = \sqrt{L^2/2 + h^2}$$

$$|\vec{r}_{EC}| = |\vec{r}_{EB}| = \sqrt{L^2/2 + h^2}$$

and

$$\vec{r}_{BE} \cdot \vec{r}_{CE} = \left(-\tfrac{L}{2}\hat{x} + \tfrac{L}{2}\hat{y} + h\hat{z}\right)\left(-\tfrac{L}{2}\hat{x} - \tfrac{L}{2}\hat{y} + h\hat{z}\right)$$

$$= L^2/4 - L^2/4 + h^2$$

Thus

$$\cos\theta = \frac{h^2}{L^2/2 + h^2} \Rightarrow \cos\theta = \frac{1}{1 + L^2/2h^2}$$

So if $h=0$ $\cos\theta = 0 \Rightarrow \theta = 90^\circ$ (the two lines intersect in the plane)

if $h \to \infty$ $\cos\theta \to 1 \Rightarrow \theta = 0^\circ$ (the lines are parallel)