Phys 311
Fall 2025

Electromagnetic Theory I: Homework 6

Due: 9 September 2025

This assignment will be graded immediately after the due date. If you get all problems correct, then you will receive 100%. If you have made any errors, then I will deduct 10%, point the errors out and you must submit a corrected assignment by 16 September 2025. If there are still errors, then I will deduct another 10% and you must submit the corrected assignment by 23 September 2025. This will continue until you **have solved every problem correctly.**

1 Divergence theorem

Let

$$\mathbf{v} = 5xy^3\hat{\mathbf{x}} + 5yx^3\hat{\mathbf{y}}.$$

Consider the region enclosed by the rectangular box for which $0 \leqslant x \leqslant 2$, $0 \leqslant y \leqslant 2$, and $0 \leqslant z \leqslant 1$.

a) Determine $\oint \mathbf{v} \cdot \mathrm{d}\mathbf{a}$ for the entire surface.

b) Determine $\int \boldsymbol{\nabla} \cdot \mathbf{v} \, \mathrm{d}\tau$ for the region and verify that the divergence theorem is satisfied.

2 Uniform vector fields

a) Let $\mathbf{v} = v\hat{\mathbf{x}}$, where $v > 0$ and consider the sphere centered at the origin. Is $\oint \mathbf{v} \cdot \mathrm{d}\mathbf{a}$ positive, negative or zero for the spherical surface? Explain your answer.

b) Consider an arbitrary uniform vector field and *any* closed surface. Is $\oint \mathbf{v} \cdot \mathrm{d}\mathbf{a}$ positive, negative or zero for the spherical surface? Explain your answer.

3 Stokes' theorem

Let

$$\mathbf{v} = \frac{y}{2}\,\hat{\mathbf{x}} - \frac{1}{2x^3}\,\hat{\mathbf{y}}$$

and consider the path with straight line segments $(1,1,0) \rightarrow (2,1,0) \rightarrow (2,2,0) \rightarrow (1,2,0) \rightarrow (1,1,0)$. Verify that Stokes' theorem is true for this loop using the flat surface that it encloses.

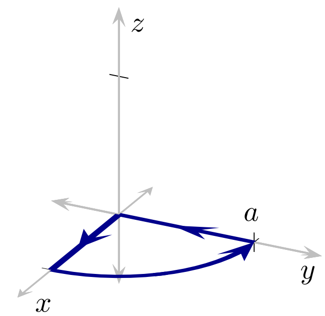
4 Surface integrals for uniform fields

Consider an arbitrary vector field $\mathbf{v}$. The surface integral $\oint \boldsymbol{\nabla} \times \mathbf{v} \cdot \mathrm{d}\mathbf{a}$ is computed over two surfaces: 1) a disk in the xy plane centered at the origin (normal points along $\hat{\mathbf{z}}$) and 2) a hemisphere whose base is the same as the disk and is above the base (normal is outward). How are the two surface integrals related? *Hint: Don't try to actually evaluate an integral. Think about Stokes' theorem.*

5 Cylindrical unit vectors

Consider two points in the $z = 0$ plane. Denote the point $(1,1,0)$ by P_1 and the point $(1,-1,0)$ by P_2. Is $\hat{\mathbf{s}}$ the same at P_1 as P_2? Is $\hat{\boldsymbol{\phi}}$ the same at P_1 as P_2? Explain your answers.

6 Stokes' theorem: cylindrical coordinates, 5



Consider the vector field, in cylindrical coordinates,

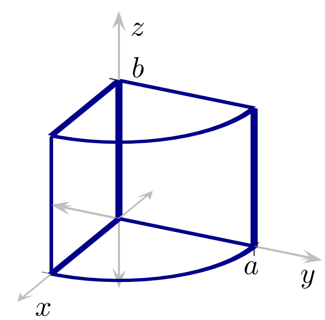
$$\mathbf{v} = s\cos\phi\,\hat{\mathbf{s}} - s\sin\phi\,\hat{\boldsymbol{\phi}}.$$

a) Sketch the vector field in the xy plane.

b) Determine the line integral along the illustrated path (the curved path is a section of a circle).

c) Verify that Stokes' theorem is true in this case.

7 Divergence theorem: cylindrical coordinates, 2



Consider the vector field, in cylindrical coordinates,

$$\mathbf{v} = s\sin\phi\,\hat{\mathbf{s}} + s\cos\phi\hat{\boldsymbol{\phi}} + z\hat{\mathbf{z}}$$

and the illustrated surface which is a quarter cylinder of radius a and height b.

a) Determine the surface integral integral over the entire closed surface.

b) Verify that the divergence theorem is true in this case.