Phys 311
Fall 2025

Electromagnetic Theory I: Homework 3

Due: 29 August 2025

This assignment will be graded immediately after the due date. If you get all problems correct, then you will receive 100%. If you have made any errors, then I will deduct 10%, point the errors out and you must submit a corrected assignment by 4 September 2025. If there are still errors, then I will deduct another 10% and you must submit the corrected assignment by 11 September 2025. This will continue until you **have solved every problem correctly.**

1 Divergence of a vector field

Consider vector fields in a two dimensional plane. These have the form:

$$\mathbf{v} = v_x(x, y)\hat{\mathbf{x}} + v_y(x, y)\hat{\mathbf{y}}$$

where $v_x(x, y)$ and $v_y(x, y)$ are two functions of x and y. Construct a vector field by choosing two such functions, which must satisfy the following: they were not used previously in class, they are not just constant functions and they are relatively simple (don't try complicated expressions with trigonometric and other special functions).

a) Sketch several vectors for the vector field that you chose. Based on the appearance of the vector field, will the divergence be zero or not? Explain your answer.

b) Compute the divergence of the vector field. Was your prediction correct?

2 Divergence and curl of a vector field with three components, 2

Determine the divergence and curl of

$$\mathbf{v} = xy\hat{\mathbf{x}} + yz\hat{\mathbf{y}} + xz\hat{\mathbf{z}}.$$

3 Radial vector field

Let

$$\mathbf{v} = \frac{\hat{\mathbf{r}}}{r^n}$$

where $\mathbf{r} = x\hat{\mathbf{x}} + y\hat{\mathbf{y}} + z\hat{\mathbf{z}}$ and n is an integer.

a) Sketch this vector field. Use the sketch describe whether you expect $\boldsymbol{\nabla} \cdot \mathbf{v}$ to be positive, negative or zero. Use the sketch to describe whether you expect $\boldsymbol{\nabla} \times \mathbf{v}$ to be zero or not.

b) Show that

$$\boldsymbol{\nabla} \cdot \mathbf{v} = \frac{2 - n}{r^{n+1}}$$

For which values of n is this positive, negative or zero? Do the results result agree your predictions? *Hint: first rewrite* $\mathbf{v}$ *in terms of* $\mathbf{r}$.

c) Determine $\boldsymbol{\nabla} \times \mathbf{v}$. Does the result agree with your prediction?

4 Differentiating products

Consider

$$\begin{aligned}\mathbf{A} &= \hat{\mathbf{x}} + \hat{\mathbf{y}} \quad \text{and} \\ \mathbf{B} &= y\hat{\mathbf{x}} - x\hat{\mathbf{y}}.\end{aligned}$$

Show, by direct substitution into either side that

$$\boldsymbol{\nabla}\left(\mathbf{A}\cdot\mathbf{B}\right) = \mathbf{A}\times\left(\boldsymbol{\nabla}\times\mathbf{B}\right) + \mathbf{B}\times\left(\boldsymbol{\nabla}\times\mathbf{A}\right) + \left(\mathbf{A}\cdot\boldsymbol{\nabla}\right)\mathbf{B} + \left(\mathbf{B}\cdot\boldsymbol{\nabla}\right)\mathbf{A}.$$

for these vector fields.

5 Gradient and vector fields

Consider the vector fields

$$\begin{aligned}\mathbf{A} &= x\hat{\mathbf{x}} \\ \mathbf{B} &= y\hat{\mathbf{x}}\end{aligned}$$

a) Based on sketches of these vectors fields would you say that either is the gradient of a function? That is, is there some function f so that $\mathbf{A} = \nabla(f)$ and similarly for $\mathbf{B}$. Explain your answer.

b) How could you check precisely if either vector is the gradient of some function?