Electromagnetic Theory I: Homework 2

Due: 26 August 2025

This assignment will be graded immediately after the due date. If you get all problems correct, then you will receive 100%. If you have made any errors, then I will deduct 10%, point the errors out and you must submit a corrected assignment by 28 August 2025. If there are still errors, then I will deduct another 10% and you must submit the corrected assignment by 4 September 2025. This will continue until you **have solved every problem correctly.**

1 Vector Triple Product

Let

$$\begin{aligned}\mathbf{A} &= 4\hat{\mathbf{x}}\\ \mathbf{B} &= 2\hat{\mathbf{x}} + 3\hat{\mathbf{y}}\\ \mathbf{C} &= 2\hat{\mathbf{x}} - 3\hat{\mathbf{y}}\end{aligned}$$

Verify that these satisfy the rule

$$\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = \mathbf{B}(\mathbf{A} \cdot \mathbf{C}) - \mathbf{C}(\mathbf{A} \cdot \mathbf{B})$$

by explicitly calculating all the terms on both sides.

2 Vector Triple Associativity

Consider the vector triple product $\mathbf{A} \times (\mathbf{B} \times \mathbf{C})$. This would be associative if

$$\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = (\mathbf{A} \times \mathbf{B}) \times \mathbf{C}.$$

a) Show, by particular choices of three distinct vectors, that there are some vectors such that the triple product is associative.

b) Show, by particular choices of three distinct vectors, that there are some vectors such that the triple product is not associative.

3 Functions and Vector Fields

Consider a fluid that can flow through a three dimensional region of space. Any location in this region can be specified using coordinates x, y, z. Assume that the fluid is continuous (does not consist of individual molecules) and that the temperature, pressure and velocity of the the fluid can vary from one location to another. Describe which of the following are scalar functions (of the location x, y, z) and which are vector fields.

a) Volume of the entire fluid.

b) Mass of the entire fluid.

c) Temperature.

d) Pressure.

e) Velocity.

4 Gradient of a function

Let $V(x, y) = x^2 + y^2$.

a) Sketch several contours of $V(x, y)$ in the xy-plane. Indicate which provide larger values and which provide smaller values.

b) Determine $\boldsymbol{\nabla} V$ and sketch the resulting vector field on your contour plot.

c) Verify, using the contour sketch, that $\boldsymbol{\nabla} V$ is perpendicular to the contours.

5 Gradients of distances

Consider a conventional coordinate system and suppose that $\mathbf{r}$ is a position vector for the locations with coordinates (x, y, z).

a) Using the general rule that the unit vector along $\mathbf{A}$ is $\hat{\mathbf{A}} = \mathbf{A}/A$, determine an expression for $\hat{\mathbf{r}}$ in terms of x, y, z and the standard basis. Determine an expression for r in terms of x, y and z.

b) For any integer n show that $\nabla(r^n) = nr^{n-1}\hat{\mathbf{r}}$. (Note that $\hat{\mathbf{r}}$ is the unit vector along $\mathbf{r}$ and $\hat{\mathbf{r}} = \mathbf{r}/r$.)

6 Gradients of separations

Consider the standard separation vector $\boldsymbol{\mathscr{r}}$. Denote the magnitude of this by $\mathscr{r}$.

a) Determine an expression for $\mathscr{r}$ in terms of x, y, z, x', y', z'.

b) Consider the operator ∇ where the derivatives in this operator refer to unprimed coordinates x, y, y. The primed coordinates can be regarded as constants. Show that:

$$
\begin{aligned}
\nabla(\mathscr{r}) &= \hat{\boldsymbol{\mathscr{r}}} \\
\nabla(\mathscr{r}^2) &= 2\boldsymbol{\mathscr{r}} \\
\nabla\left(\frac{1}{\mathscr{r}}\right) &= -\frac{\hat{\boldsymbol{\mathscr{r}}}}{\mathscr{r}^2}.
\end{aligned}
$$