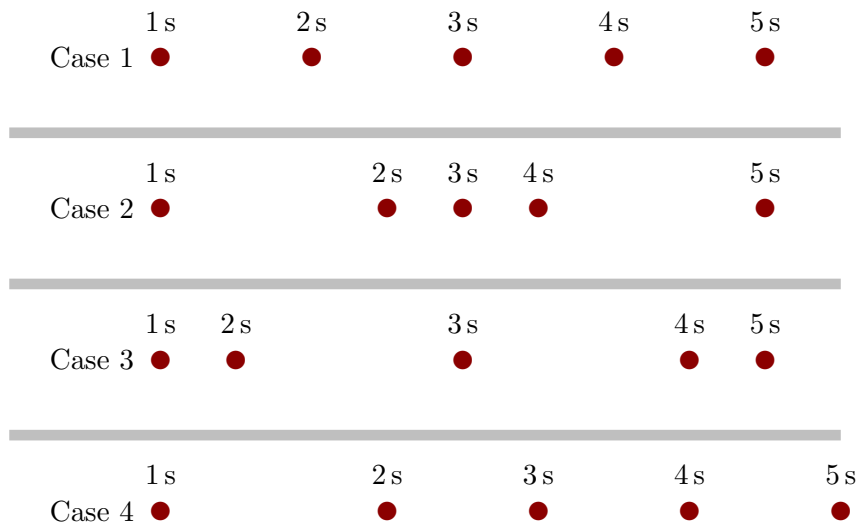


## Phys 131: Exercises

# One-Dimensional Kinematics

## 1 Motion diagrams: car slowing and accelerating

A car moves to the right. Earlier in its motion it slows down and later it speeds up. Which of the following (choose one) best represents its location as time passes? (131F2024)



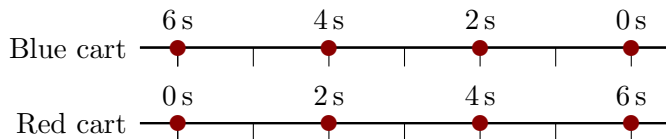
Briefly explain your choice.

## 2 Drone motion diagram

A drone (unmanned aircraft) hovers at one height above the ground. The drone is subsequently allowed to descend in a straight line toward the ground. It initially descends at an increasing rate, then at a steady rate and, a few feet before the ground, the descent rate decreases. Draw a motion diagram for the drone while it descends. (131F2024)

## 3 Motion diagrams: racing carts

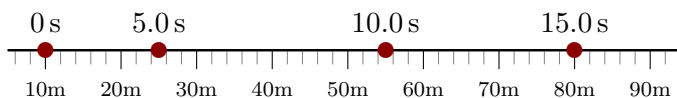
Motion diagrams for two carts are as illustrated. (131F2024)



- Is the speed of the red cart the same as or different to that of the blue cart? Explain your answer.
- Is the velocity of the red cart the same as or different to that of the blue cart? Explain your answer.

#### 4 Motion diagrams and average velocity

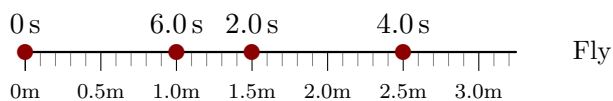
A car moves from left to right and its position is recorded every 5.0 s. The resulting motion diagram is illustrated. (131Sp2025)



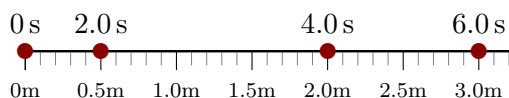
- Determine the average velocity from 0 s to 5.0 s.
- Determine the average velocity from 5.0 s to 10.0 s.
- Determine the average velocity from 5.0 s to 15.0 s.
- Determine the average velocity from 0 s to 15.0 s. Is this the average of the two velocities in the interval  $0\text{ s} \rightarrow 5.0\text{ s}$  and  $5.0\text{ s} \rightarrow 15.0\text{ s}$ ?

#### 5 Motion diagrams, speed and average velocity

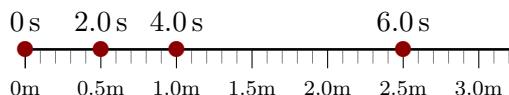
Several insects move along a straight line and their positions are recorded every 2.0 s. The resulting motion diagrams are illustrated. (131Sp2025)



Fly



Beetle



Cricket

- Rank the insects in order of average speed from 0.0 s to 6.0 s.
- Rank the insects in order of average velocity from 0.0 s to 6.0 s.

#### 6 Katie Ledecky splits

Katie Ledecky, one of the greatest female swimmers, won the women's 800 m freestyle race at the 2023 World Aquatics Championships. In the final, she covered the first 200 m in 1:58.29 (means 1 min and 58.29 s), the second 200 m in 2:03.63, the third 200 m in 2:04.00 and the final 200 m in 2:01.50. Determine her average speed for each 200 m section and also the entire race. Which section most resembled her speed for the entire race? (131F2024)

#### 7 Bouncing hockey puck

A hockey puck can slide along a horizontal surface. Starting at an initial location, it slides right and takes 3.0 s to hit a board 18 m away. It bounces back to the left and reaches a final point 4.0 m from the board 2.0 s from the time it first hit the board. Determine the average velocity of the puck from the initial to final moments. (131F2024)

## 8 Red car, blue car, green car

Determine the average speed and the average velocity for cars moving in a straight line as follows:

- a) Red car moves right for 50 m in 10.0 s.
- b) Blue car moves right for 16 m in 8.0 s and then continues right for 34 m in 2.0 s.
- c) Red car moves right for 30 m in 5.0 s reverses and moves left for 20 m in 5.0 s. (131F2024)

## 9 Man versus dog

The following objects lie along a straight line: a bicycle, a coffee cup and a soccer ball. The distance from the coffee cup to the bicycle is 400 m and from the cup to the ball is 500 m. A man starts at the cup and travels in a straight line to the ball. This takes 200 s. A dog is initially at the cup and runs at constant speed to the bicycle, taking 50 s to do so. The dog immediately turns around and runs to ball; this takes the dog an additional 150 s. Consider the entire trip from the cup to the ball for each. Who has the larger average velocity for this entire trip? Explain your answer. (131F2024)



## 10 Displacement and average velocity

Various people move as described below over a total interval of 40 s. In each case determine the total displacement and the average velocity over the 40 s period. (131F2024)

- a) Anna takes a trip in two stages. In the first stage, lasting 20 s, she moves 30 m to the right. In the second stage, lasting 20 s, she moves 10 m to the left.
- b) Bill takes a trip in two stages. In the first stage, lasting 10 s, he moves 10 m to the left. In the second stage, lasting 30 s, he moves 30 m to the right.
- c) Explain whether displacement and average velocity capture all the information about the motion of each. If not, how might one modify the description of the motion in terms of displacement and velocity to describe the motion more accurately?

## 11 Average velocity, speed for motion with changing directions

- a) A person takes a trip, first walking 30 m to the right in 5.0 s and then returning to his starting point in another 10 s. Determine the average velocity of the person for the entire trip.
- b) Anna takes a trip in two stages. First she moves 100 m right in 40 s. She briefly stops and then she moves 120 m to the left in another 60 s. Determine her average velocity for each stage of the trip and also for the entire trip.

- c) Bill takes a trip in two stages. First he moves 200 m left in 100 s. He briefly stops and then he moves 150 m to the right in another 100 s. Determine his average velocity for each stage of the trip and also for the entire trip. (131F2024)

## 12 Average velocity and direction of motion

- a) An object moves in such a way that during a certain period the average velocity of the object is negative. Is it possible that at the end of the period the object is to the right of the origin of the coordinate system/axis? Explain your answer.
- b) An object moves in such a way that over a certain period the average velocity of the object is positive. Is it possible that at some point during the period the object moves to the left? Explain your answer. (131F2024)

## 13 Average velocity and displacement down a field

Three people, Alice, Bob and Charlie leave one end of a 100 m long field at the same instant and travel in the same direction toward the other end. (131F2024)

- a) Alice travels at a constant speed of 8.0 m/s for half the distance and then at 4.0 m/s for the remaining half. Determine Alice's average velocity for the entire trip.
- b) Bob travels at a constant speed of 10.0 m/s for half the distance and then at 2.0 m/s for the remaining half. Determine Bob's average velocity for the entire trip.
- c) Charlie travels for half of the time at a constant speed of 8.0 m/s for the remaining half of the time at a constant speed of 4.0 m/s. Determine Charlie's average velocity for the entire trip.
- d) Determine the order in which they arrive at the other end of the field.

## 14 El Capitan climb

The rock climbing route called "The Nose" ascends the El Capitan formation in Yosemite National Park. The elevation gain on the climb is 2900 ft and the quickest ascent took 1 hr, 58 min, 7 s. Determine the average speed for this ascent in m/s. (131F2024)

## 15 Average speed of the Moon

The Moon orbits the Earth in an ellipse with average radius 239,000 miles. It completes one orbit in 27 days, 7 hrs, 43 min, 11.5 s. Suppose that it orbits in a perfect circle with radius 239,000 miles. (131F2024)

- a) Determine the average speed of the Moon in mph.
- b) Determine the average speed of the Moon in m/s.

## 16 Running speed

Estimate the speed with which you could run a distance of about 100 yards. Convert your answer to standard metric units. (131F2024)

## 17 Grass growing

Estimate the speed with typical lawn grass that one might find on campus grows. Express your answer in standard metric units. Describe all of the steps that give your estimate. *There is no one correct numerical answer to this. Answers will depend on the numbers that you estimate. It's more important to develop a **correct method** for answering this question and to get a ballpark idea of the speed.* (131F2024)

## 18 Moving clock hand

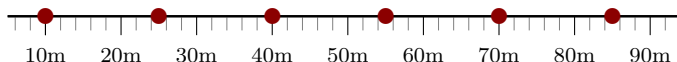
There is a clock outside on the tower of the CMU University Center. We would like an idea of how fast the tip of the minute hand (the longer of the two hands) moves in standard metric units. Assume that the hand moves continuously.

- Without any calculation guess how fast the tip of the minute hand moves.
- Estimate as accurately as you can how fast the tip of the minute hand moves in standard metric units. Explain the steps in the process of estimating and any other quantities that you may have estimated to reach your result.

*The answers will vary amongst students in the class because some of the estimated quantities will vary. The important thing is to develop a **correct method** for answering this question and to get a ballpark idea of the speed.* (131F2025)

## 19 Motion diagrams and position vs. time graphs, 1

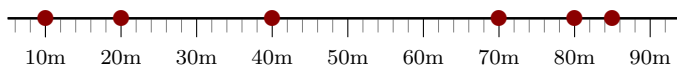
A car moves from left to right and its position, measured in meters, is recorded every 5.0 s. The resulting motion diagram is illustrated. (131Sp2025)



- Produce a table of numerical data for position versus time for the car for the duration of the motion.
- Produce a position versus time graph for the car for the duration of the motion. This graph must be drawn by hand using axes that are clearly labeled.

## 20 Motion diagrams and position vs. time graphs, 2

A car moves from left to right and its position, measured in meters, is recorded every 5.0 s. The resulting motion diagram is illustrated. (131Sp2025)



- Produce a table of numerical data for position versus time for the car for the duration of the motion.
- Produce a position versus time graph for the car for the duration of the motion. This graph must be drawn by hand using axes that are clearly labeled.

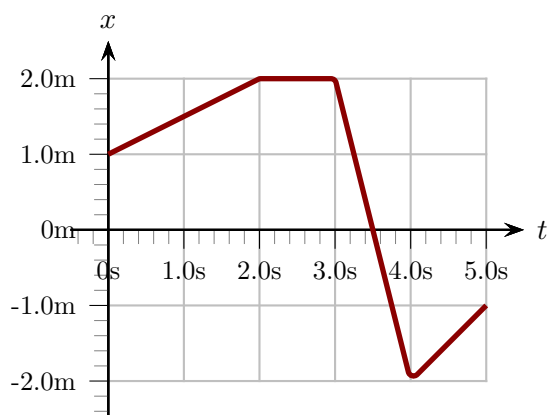
### 21 Hockey puck trip time

A hockey puck travels along a horizontal surface to the right for a distance of 12 m at speed 5.0 m/s. It hits a board and bounces to the left traveling a distance of 20 m at speed 4.0 m/s. Determine the total time for this trip. (131F2024)

### 22 Angry ant on a stick

An angry ant walks along a straight stick. The graph illustrates the ant's position vs. time. (131Sp2025)

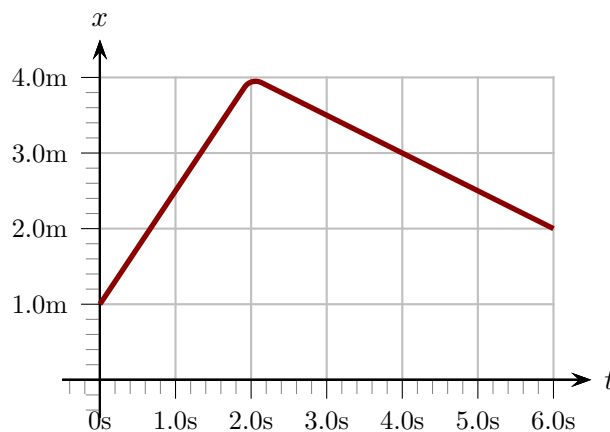
- Describe in words qualitatively how the ant moves during the period from 0 s to 5 s. Include as much detail without using any numbers.
- During which period is the ant's speed largest? During which period is it smallest? Explain your answers.



### 23 Slug on a stick

A slippery slug crawls along a straight stick. The graph illustrates the slug's position vs. time. Answer the following, giving explanations for each answer. (131Sp2025)

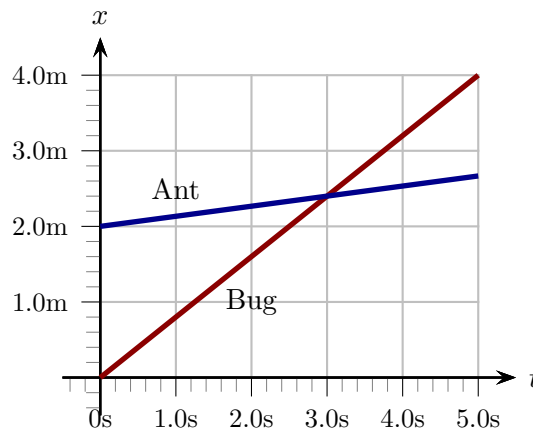
- Determine the velocity of the slug at 1.0 s.
- Determine the velocity of the slug at 4.0 s.



## 24 Ant and bug on a stick

An ant and a bug walk along a straight stick. The graph illustrates their position vs. time. Answer the following, explaining each answer. (131Sp2025)

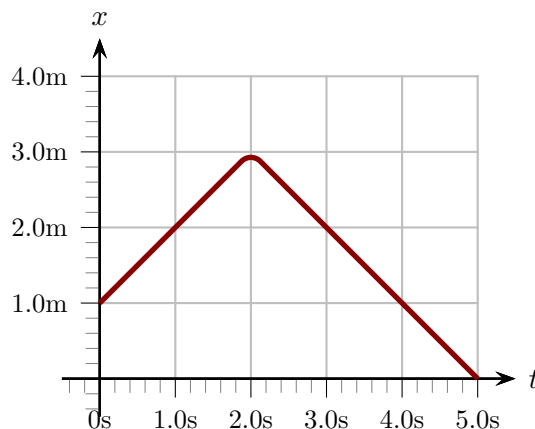
- When and where, if ever, are the ant and the bug at the same location?
- When, if ever, do the ant and the bug have the same speed?
- When, if ever, does the ant move faster than the bug?



## 25 Red ant on a stick

A red ant walks along a straight stick. The graph illustrates the ant's position vs. time. Answer the following, giving explanations for each answer. (131Sp2025)

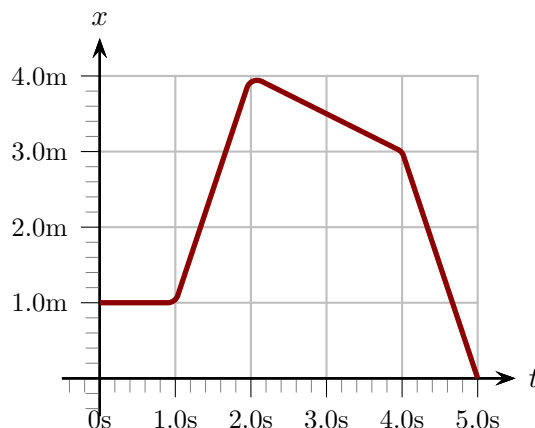
- During which times is the ant moving right? During which times is it moving left?
- When, if ever, is the velocity of the ant 0 m/s?
- How does the speed of the ant at 1.0 s compare to its speed at 4.0 s?
- How does the velocity of the ant at 1.0 s compare to its velocity at 4.0 s?



## 26 Timid tick on a stick

A timid tick walks along a straight stick. The graph illustrates the tick's position vs. time. In each of the following explain your answers. (131Sp2025)

- Determine the velocity of the tick at 0.5 s.
- Determine the velocity of the tick at 1.5 s.
- Determine the velocity of the tick at 3.0 s.
- Determine the velocity of the tick at 4.5 s.

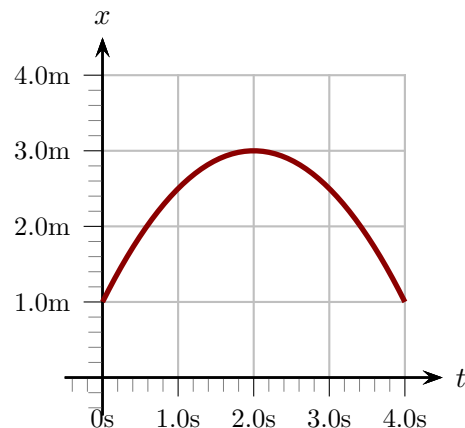




### 27 Bug on a stick

A bug walks along a straight stick. The graph illustrates the bug's position vs. time. In each of the following, explain your answers. (131Sp2025)

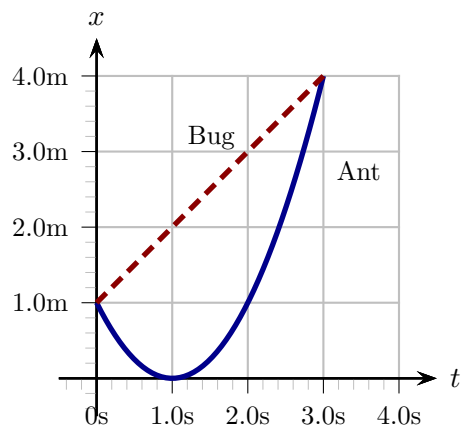
- Does the bug ever return to the position that it occupied initially ( $t = 0$  s)? If so, when?
- Does the bug ever reverse direction? If so when?
- Does the bug ever have zero velocity? If so, when?



### 28 Ant and bug on a stick: speeds

An ant and a bug walk along straight sticks. The solid graph illustrates the ant's position vs. time. The dashed graph indicates the bug's position vs. time. Answer the following, giving explanations for each answer. (131Sp2025)

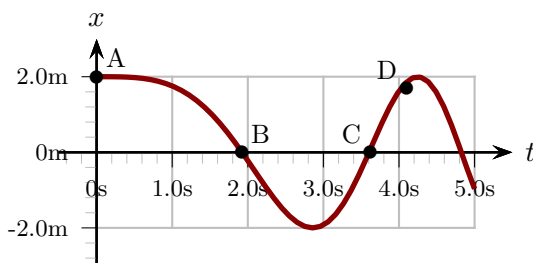
- At what time(s) are the ant and bug at the same location?
- Which is moving faster at 2 s?
- Do the ant and bug ever have the same velocity? If so when?



### 29 Graceful ladybug on a stick

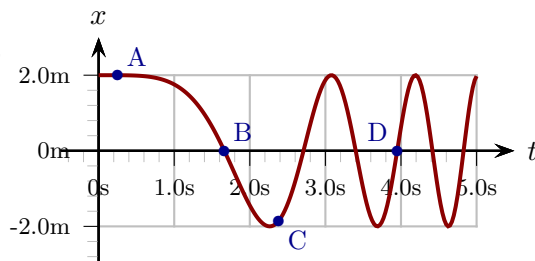
A ladybug insect walks gracefully back and forth along a straight stick. A graph of position vs. time for its motion is illustrated with several instants labeled A, B, C and D. Explain your answers for the following. (131Sp2025)

- At which of these instants is the ladybug moving right?
- At which of these instants is the ladybug slowing down?
- Rank the moments in order of increasing instantaneous speed.
- Sketch a graph of velocity versus time for the ladybug.



### 30 Agitated cat on a wall

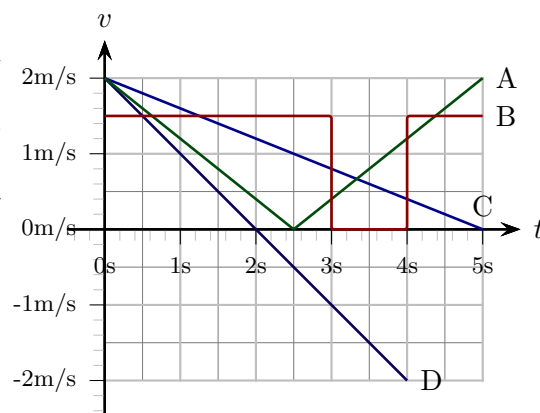
An increasingly agitated cat walks back and forth along a straight wall. A graph of position vs. time for its motion is illustrated with several instants labeled A, B, C and D. Explain your answers for the following. (131Sp2025)



- At which of these instants is the cat moving right?
- Rank the instants in order of increasing instantaneous speed.
- Describe an interval during which the cat slows down. Provide approximate initial and final times for this interval.
- Sketch a graph of velocity versus time for the cat from 0 s to 3 s.

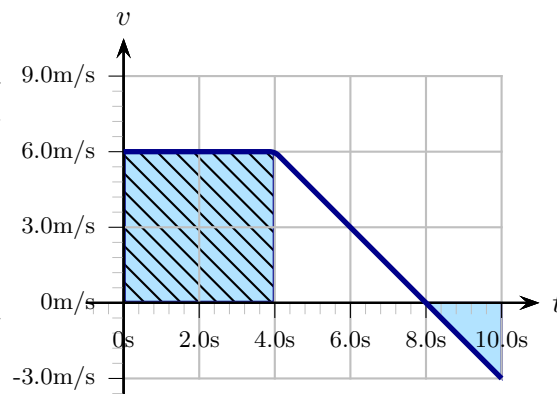
### 31 Bead sliding along a wire

A bead slides along a straight piece of wire. After an initial moment it slides right with decreasing speed, coming to a brief stop. Immediately after this it slides right with increasing speed. Taking right as positive, which of the following represents a graph of *velocity versus time* for the bead? Explain your choice. (131F2024)



### 32 Crawling slug

A slug crawls along a straight wire, starting at  $x = 0.0\text{ m}$  at  $t = 0.0\text{ s}$ . A graph of the slug's velocity versus time is illustrated. Use the graph to answer the following. (131Sp2025)

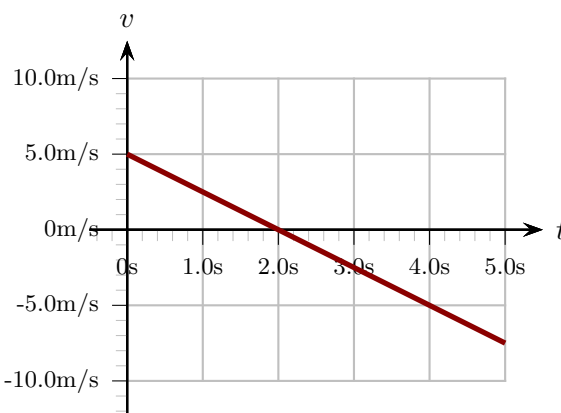


- Determine the displacement of the slug from  $t = 0.0\text{ s}$  to  $t = 4.0\text{ s}$ .
- How is the displacement of the slug from  $t = 0.0\text{ s}$  to  $t = 4.0\text{ s}$  related to the shaded area *between the graph and the horizontal axis* ( $v = 0.0\text{ m/s}$ )?
- Assuming that the answer to the previous question is true in general, determine the displacement of the slug from  $t = 4.0\text{ s}$  to  $t = 8.0\text{ s}$ .
- Is the displacement of the slug from  $t = 8.0\text{ s}$  to  $t = 10.0\text{ s}$  positive or negative? How might this relate to the shaded area from  $t = 8.0\text{ s}$  to  $t = 10.0\text{ s}$ ?

### 33 Stick insect on a stick

A stick insect walks back and forth along a straight stick. A graph of velocity vs. time for its motion is illustrated. (131Sp2025)

- Determine the displacement of the stick insect from  $t = 0$  s to  $t = 2$  s.
- Determine the displacement of the stick insect from  $t = 2$  s to  $t = 4$  s.
- Determine the displacement of the stick insect from  $t = 0$  s to  $t = 4$  s.



### 34 Ladybug on a stick

A ladybug insect walks back and forth along a straight stick. A graph of velocity vs. time for its motion is illustrated. The ladybug is initially at  $x = -1.0$  m. (131Sp2025)

- Determine the position of the ladybug at  $t = 1$  s.
- Determine the position of the ladybug at  $t = 3$  s.
- Determine the position of the ladybug at  $t = 4$  s.
- Determine the position of the ladybug at  $t = 5$  s.
- Describe in words how the position of the ladybug evolves with time during the illustrated interval.



### 35 Slithering worm

A worm is initially at rest and subsequently slithers along a straight wire. The worm's velocity increases steadily from 0.000 m/s to 0.015 m/s over a period of 120 s. Determine the distance traveled by the worm during this period. *Hint: A graph will help!* (131F2023)

### 36 Instantaneous velocity as a limit

An atom is trapped in such a way that it can move back and forth along one straight line. Its position is tracked as time passes and is represented by the function  $x = t^2 - 2t - 1$  (the coefficients all have units such that  $x$  is in units of meters and  $t$  in seconds). (131F2024)

- a) Produce a list of positions at every 0.5 s from  $t = 0.0$  s to  $t = 4.0$  s. Use this data to plot an accurate graph of position versus time for  $t = 0.0$  s  $\leq t \leq 4.0$  s. *The graph must be drawn accurately enough to draw and calculate slopes of tangent lines.*
- b) Determine the velocity of the atom at 3.0 s by using a tangent line construction for the graph of position versus time.

The aim of the next parts of this problem is to determine the instantaneous velocity at 3.0 s.

- c) Use the function of position versus time to determine the average velocity over the time interval from  $t = 3.0$  s to  $t = 3.1$  s.
- d) Use the function of position versus time to determine the average velocity over the time interval from  $t = 3.00$  s to  $t = 3.01$  s.
- e) Does the value of average velocity at 3.0 s appear to approach a limit as the time interval decreases? If so what does the limit appear to be?
- f) Use the derivative of position to determine the exact instantaneous velocity at 3.0 s. Does the result agree with your answer to the previous part?
- g) At what time is the instantaneous velocity exactly zero? Explain your answer.

### 37 Velocity as a derivative, 1

Suppose that the position of an object is

$$x = (5 \text{ m/s}^2) t^2 + (3 \text{ m/s}) t$$

Determine the velocity of the object at  $t = 3$  s. (131Sp2025)

### 38 Velocity as a derivative, 2

Suppose that the position of an object is

$$x = (0.25 \text{ m/s}^3) t^3 + (6 \text{ m/s}) t$$

Determine the velocity of the object at  $t = 4$  s. (131Sp2025)

### 39 Velocity as a derivative, 3

Suppose that the position of an object is

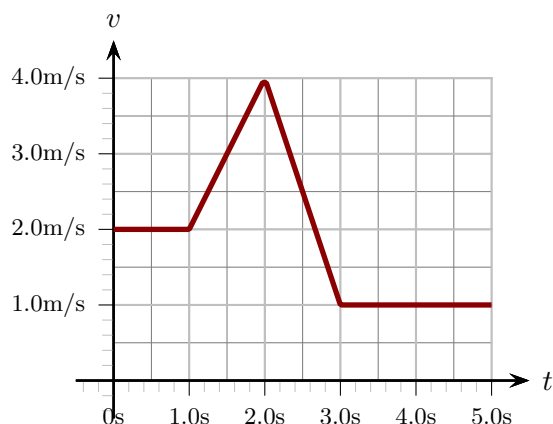
$$x = (2 \text{ m/s}^3) t^3 + (6 \text{ m/s}^2) t^2$$

Determine the velocity of the object at  $t = 2$  s. Is this the same as  $x/t$ ? (131Sp2025)

#### 40 Wandering ant

An ant walks along a straight stick. The graph illustrates the ant's velocity vs. time. Answer the following, giving explanations for each answer. (131Sp2025)

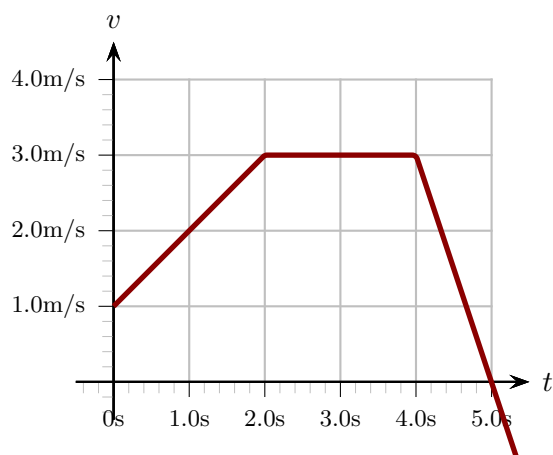
- a) How does the magnitude of the ant's acceleration at 1.5 s compare (larger, smaller, same) to its acceleration at 2.5 s?
- b) How does the magnitude of the ant's acceleration at 0.5 s compare (larger, smaller, same) to its acceleration at 2.5 s?
- c) How does the magnitude of the ant's acceleration at 0.5 s compare (larger, smaller, same) to its acceleration at 4.0 s?



#### 41 Accelerating bug on a stick

A bug walks along a straight stick. The graph illustrates the bug's velocity vs. time. At  $t = 0$  s the bug is at the  $x = 2.0$  m mark. (131Sp2025)

- a) Determine the position, velocity and acceleration of the bug at 1.0 s.
- b) Determine the position, velocity and acceleration of the bug at 3.0 s.
- c) Determine the position, velocity and acceleration of the bug at 5.0 s.

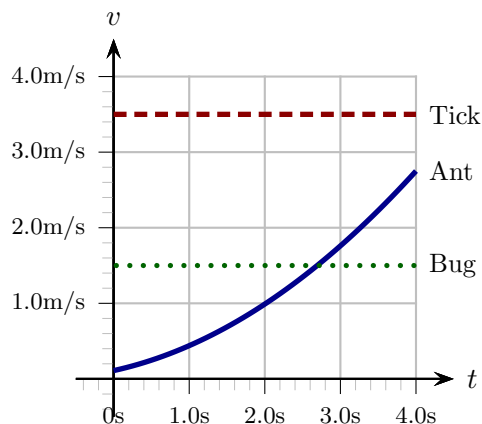


#### 42 Cyclist versus skateboarder

A skateboarder moves down a gentle slope. She passes a cyclist, who is at rest and at this moment the skater's speed is 4.0 m/s. At an instant 10 s later the skater's speed is 10.0 m/s and the cyclist's is 8.0 m/s. During this period, who has the greater acceleration? Explain your answer. (131F2024)

### 43 Insects on sticks

An ant, a tick and a bug walk along straight sticks. The solid graph illustrates the ant's velocity vs. time. The dashed graph indicates the tick's velocity vs. time. The dotted line indicates the bug's velocity versus time. (131Sp2025)



- a) Which of the following is true during the period from 0s to 4s? Explain your answer.
- i) The acceleration of the ant is the same as that of the tick.
  - ii) The acceleration of the tick is always larger than that of the ant.
  - iii) The acceleration of the tick is always smaller than that of the ant.
  - iv) The acceleration of the tick is sometimes smaller than that of the ant, sometimes larger.
- b) Which of the following is true during the period from 0s to 4s? Explain your answer.
- i) The acceleration of the ant is the same as that of the bug.
  - ii) The acceleration of the bug is always larger than that of the ant.
  - iii) The acceleration of the bug is always smaller than that of the ant.
  - iv) The acceleration of the bug is sometimes smaller than that of the ant, sometimes larger.

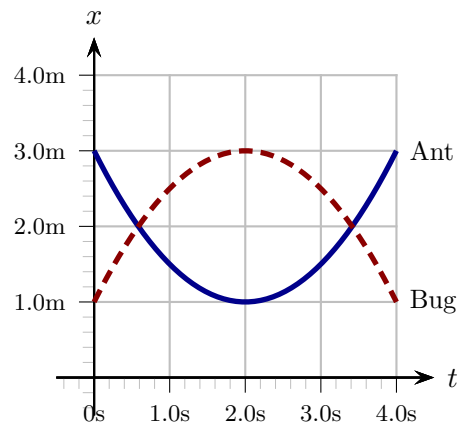
### 44 Acceleration sign

A bicycle can move east (positive) or west (negative). (111F2023)

- a) If the bicycle moves east can the acceleration be negative? Explain your answer.
- b) If the bicycle moves west can the acceleration be positive? Explain your answer.

#### 45 Ant and bug on sticks

An ant and a bug walk along straight sticks. The solid graph illustrates the ant's position vs. time. The dashed graph indicates the bug's position vs. time. (131Sp2025)



- a) For the bug and, separately, the ant, which of the following is true during the period from 0s to 4s? Explain your answers.
- i) Velocity is zero at all times.
  - ii) Velocity is positive at all times.
  - iii) Velocity is negative at all times.
  - iv) Velocity is first positive and later negative.
  - v) Velocity is first negative and later positive.
- b) For the bug and, separately, the ant, which of the following is true during the period from 0s to 4s? Explain your answers.
- i) Acceleration is zero at all times.
  - ii) Acceleration is positive at all times.
  - iii) Acceleration is negative at all times.
  - iv) Acceleration is first positive and later negative.
  - v) Acceleration is first negative and later positive.

#### 46 Moving man, 1

A man in an animation walks along a horizontal line. Positions are marked in meters along the line with positive numbers to the right of the origin and negative to the left. His positions and velocities are recorded at equally spaced intervals in time. The data is:

Time in s	Position in m	Velocity in m/s
0.0	-5.00	-4.00
2.0	-7.00	2.00
4.0	3.00	8.00

- a) Determine the man's average acceleration from 0.0s to 2.0s.
- b) Determine the man's average acceleration from 2.0s to 4.0s. (111F2023)

#### 47 Person walking left and right

A person walks along a horizontal line. Positions are marked in meters along the line with positive numbers to the right of the origin and negative to the left. His velocity is recorded at equally spaced intervals in time. The data is:

Time in s	Velocity in m/s
10.00	-1.00
10.10	-0.80
10.20	-0.60
10.30	-0.40
10.40	-0.20
10.50	0.00
10.60	0.20
10.70	0.40
10.80	0.60
10.90	0.80

- a) During which period is the person moving left? During which period is the person moving right?
- b) Determine the person's acceleration while moving left.
- c) Determine the person's acceleration while moving right.
- d) Does the person's acceleration appear to change during the period from 10.10 s to 10.90 s?
- e) At what moment does the person reverse direction? According to the data is the acceleration zero or not at this moment? (*111F2023*)

#### 48 Bungee jumper

A bungee jumper falls downward stretching the cord, reaching a low point, after which the cord pulls him up again. His velocity is recorded at equally spaced intervals in time. The data is:

Time in s	Velocity in m/s
10.0	-20.0
10.5	-15.0
11.0	-10.0
11.5	-5.0
12.0	0.0
12.5	5.0
13.0	10.0
13.5	15.0
14.0	20.0



- a) During which period is the man falling? When is he rising?
- b) By how much does the man's velocity change per second? Is this change constant throughout the recorded motion?
- c) Determine the man's acceleration while he is falling and also while he is rising. Are these accelerations the same or not?
- d) What is the man's acceleration (according to the data) at his low point? (111F2023)

#### 49 Moving man activity

Go to the moving man animation at:

<http://phet.colorado.edu/en/simulation/moving-man>

Run the moving man animation. Click on the charts tab. Set the position to 0.00 m, the velocity to  $-5.00\text{ m/s}$  and the acceleration to  $2.00\text{ m/s}^2$ . Run the animation, stopping it just before the man hits the wall. The animation will have recorded the motion. Check the playback button at the bottom. You can slide the light blue bar left and right to get data for the motion. Gray zoom icons at the right will let you rescale the charts. (111F2023)

- a) Consider the interval from 2.0 s to 3.0 s. Describe the motion verbally during this time.
- b) How does the speed of the man at 2.0 s compare to that at 3.0 s? Explain your answer.
- c) How does the velocity of the man at 2.0 s compare to that at 3.0 s? Explain your answer.
- d) Will the average acceleration over the interval from 2.0 s to 3.0 s be positive, negative or zero? Explain your answer.
- e) If the acceleration is not zero, does it vary during this interval? Explain your answer.
- f) Determine the average acceleration over the interval from 2.0 s to 3.0 s.

## 50 Person moving with constant acceleration

A person is initially at rest and subsequently moves right with a constant acceleration. The person's reaches speed 6.0 m/s at a point 9.0 m to the right of the starting location. The aim of this exercise will be to determine the time taken to reach this point. A first step will be to determine the acceleration of the person.

- a) Sketch the situation, illustrating the person at the two instants described above.

List all relevant variables for the two instants:

$t_i =$	$t_f =$
$x_i =$	$x_f =$
$v_i =$	$v_f =$

- b) Determine the acceleration by selecting one of the kinematic equations, substituting and solving for  $a$ .
- c) Using a different kinematic equation, find the time that it takes the person to reach speed 6.0 m/s.
- d) Suppose that you had tried to find the time taken to reach speed 6.0 m/s by using

$$v = \frac{\Delta x}{\Delta t} \Rightarrow 6.0 \text{ m/s} = \frac{9.0 \text{ m}}{\Delta t}.$$

What time does this give? Does it agree with the answer that you obtained to the previous part? Is it correct?

## 51 Avoid the wall!

A skateboarder slides toward a wall. Initially the skateboarder is 18 m left of the wall and moving with speed 6.0 m/s to the right. The aim of this exercise will be to determine the minimum acceleration to barely avoid hitting the wall. (111F2023)

- a) Sketch the situation, illustrating the skateboarder at the initial instant and the instant just before reaching the wall.

List all relevant variables for the two instants:

$$\begin{array}{ll} t_i = & t_f = \\ x_i = & x_f = \\ v_i = & v_f = \end{array}$$

- b) Determine the acceleration by selecting one of the kinematic equations, substituting and solving for  $a$ .
- c) Use one of the kinematic equations to determine the time that it takes for the skateboarder to reach the wall.
- d) Would the equation

$$v = \frac{\Delta x}{\Delta t} \Rightarrow 6.0 \text{ m/s} = \frac{18 \text{ m}}{\Delta t}$$

allow one to find the time taken to reach the wall correctly? Why or why not?

- e) Set up the moving man animation at:

<http://phet.colorado.edu/en/simulation/moving-man>

and run this to check your prediction. In order to verify that you have done this, use the animation to provide the times at which the man is 10 m to the left of the wall.

## 52 Braking car, 1

A car travels to the right with speed 30.0 m/s (about 67 mph). The car brakes and slows to a stop with constant acceleration. It does this in a distance of 80.0 m (about 260 ft). The aim of this exercise is to find the time taken to do this. (131F2024)

- a) Sketch the situation, illustrating the car at two key instants.

List all relevant variables for the two instants:

- b) Determine the acceleration using one of the kinematic equations. Write down the equation, substitute from your list of variables and solve for  $a$ . (is acceleration positive or negative?)

**Answer:**  $-5.63 \text{ m/s}^2$

- c) Using a different kinematic equation, find the time that it takes the car to stop. Write down the equation, substitute from your list of variables and solve for time.

**Answer:** 5.33 s

## 53 Braking car, 2

A car travels to the right with speed 24.6 m/s (about 55 mph). The car brakes, coming to a stop in 50 m (about 165 ft). The acceleration during the braking process is constant. (131F2024)

- a) Determine the acceleration of the car while it brakes.  
b) Determine the time taken for the car to stop.

## 54 Accelerating aircraft

An aircraft accelerates from rest to a speed of 120.0 m/s. It does this with a constant acceleration of  $4.00 \text{ m/s}^2$ . The aim of this exercise is to find the distance traveled by the aircraft during this process. (131F2024)

- a) Sketch the situation, illustrating the aircraft at two key instants. List all relevant variables for the two instants and list the acceleration.  
b) Determine the distance traveled by the aircraft using one of the kinematic equations. Write down the equation, substitute from your list of variables and solve for the relevant variable.

**Answer:** 1800 m

- c) Using a different kinematic equation, find the time that it takes for the aircraft to reach this speed. Write down the equation, substitute from your list of variables and solve for time.

**Answer:** 30 s

- d) Would it have worked to use  $v = \frac{\Delta x}{\Delta t} \Rightarrow 120.0 \text{ m/s} = \frac{1800 \text{ m}}{\Delta t}$ ?

### 55 Accelerating cart

A cart travels in one direction. At an initial instant it passes the 4.0 m mark while traveling to the right with speed 15 m/s. (131F2024)

- a) Determine the location of the cart at an instant 6.0 s later if its acceleration is  $3.0 \text{ m/s}^2$ .
- b) Determine the location of the cart at an instant 6.0 s later if its acceleration is  $-3.0 \text{ m/s}^2$ .

### 56 Accelerating shrew

A shrew (a small mammal) travels in one direction. At an initial instant it travels to the right with speed 3.0 m/s. It subsequently moves with constant acceleration and after traveling an additional 2.0 m to the right it reaches a speed of 7.0 m/s. Determine the time taken to do this. (131F2024)

### 57 Moving sloth

A sloth (a tree-climbing mammal) travels along a straight branch. At an initial instant it travels to the right with speed 0.030 m/s. It subsequently crawls with constant acceleration and after an additional 5.0 s has traveled an additional 0.40 m to the right. Determine its speed at the end of this period. (131F2024)

### 58 Car starting from rest

A car is initially at rest and subsequently moves in one direction with constant acceleration. After 12 s its speed is 50 mph. Determine the total distance that it has traveled in this time. (131F2024)

### 59 Feasible Acceleration?

Suppose that one aims to push a cart along a horizontal surface, starting from rest, with a constant acceleration. Could one sustain this over a period of several seconds? This problem will try to help you answer this. When doing this, assume that the cart is light enough that one can easily exert the necessary force. Explain your answers, preferably with calculations, and describe any assumptions you made. (131F2024)

- a) Your friend claims that they can push the cart with a constant acceleration of  $3.0 \text{ m/s}^2$  for a period of 10 s. Considering the state motion of the cart at the end of the 10 s period, does this seem reasonable?
- b) Suppose that you pushed the cart, starting from rest along a 100 m long straight track. What is the highest acceleration that you could feasibly attain? *Hint: Consider the highest speed you could attain at the end of the track.*

## 60 Reversing cart

At an initial instant a cart travels to the left with speed  $12\text{ m/s}$ . Subsequently the cart's acceleration is  $4.0\text{ m/s}^2$ . (131F2024)

- a) How long does it take for the cart to reverse its direction of travel?
- b) How far does the cart travel before it reverses its direction of travel?

## 61 Racing cyclists

Two cyclists can ride along a straight road. Juliette moves right with a constant speed of  $20\text{ m/s}$ . She passes Elisa, who is at rest. At the moment that Juliette passes, Elisa begins to move, accelerating at a constant rate of  $2.0\text{ m/s}^2$ . Elisa maintains this for  $12.0\text{ s}$  and after that she moves with constant velocity. (131F2024)

- a) Explain how you can be sure that Elisa eventually passes Juliette.
- b) Determine the time (after Elisa starts to move) that it takes Elisa to catch up to Juliette.
- c) Determine the distance traveled by each when Elisa catches up with Juliette.

## 62 Runway design, 1

The takeoff speed for an aircraft under certain conditions is  $260\text{ km/h}$ . During takeoff, one wants the acceleration to be as low as possible. (131F2024)

- a) Suppose that the runway has length  $1500\text{ m}$ . Determine the aircraft's minimum acceleration so that it can take off on this runway.
- b) Suppose that the acceleration is only  $0.20g = 1.96\text{ m/s}^2$ . Determine the minimum length of the runway so that the aircraft could take off successfully.

Note: It is useful to describe acceleration in terms of  $g$  as this is correlated to what humans can feel. An acceleration of  $g$  is what you feel when jumping off a diving board. An acceleration of around  $6g$  will cause a person to blackout.

### 63 Aircraft takeoff acceleration

The takeoff speed for an aircraft under certain conditions is 260 km/h. A typical runway has length 3500 m. (131F2024)

- a) Suppose that the aircraft starts at rest somewhere along the runway and uses  $2/3$  of the runway length to takeoff. Determine the acceleration of the aircraft, assuming that it is constant.
- b) It is useful to describe acceleration in terms of  $g$  as this is correlated to what humans can feel. An acceleration of  $g$  is what you feel when jumping off a diving board. An acceleration of around  $6g$  will cause a person to blackout. What multiple of  $g$  is the aircraft's acceleration?
- c) Determine the fraction of the runway that the aircraft would have to use so that the acceleration were  $g$ .

### 64 Jumping cat

A cat jumps, launching itself vertically. During its subsequent motion up and down, air resistance is negligible. In the following let the upward direction be positive. (131F2024)

- a) Which of the following is true in the period after the cat's feet have left the ground and before the cat reaches its highest point above the ground?
  - i) Acceleration is zero at all times.
  - ii) Acceleration is positive at all times.
  - iii) Acceleration is negative at all times.
- b) Which of the following is true in the period while the cat drops back to the ground?
  - i) Acceleration is zero at all times.
  - ii) Acceleration is positive at all times.
  - iii) Acceleration is negative at all times.
- c) Which of the following is true when the cat is at its highest point above the ground?
  - i) Acceleration is zero at all times.
  - ii) Acceleration is positive at all times.
  - iii) Acceleration is negative at all times.

### 65 Ball launched vertically

A ball is launched vertically from Earth's surface with speed  $10 \text{ m/s}$ . The aim of the first part of this exercise is to determine the maximum height reached by the ball and time taken to reach the maximum height. (131F2024)

- a) Sketch the situation, illustrating the ball at two key instants.

List all relevant variables, including the acceleration, for the two instants:

- b) Determine the maximum height reached by the ball by using one of the kinematic equations. Write down the equation, substitute from your list of variables and solve for the maximum height variable.

**Answer:**  $5.1 \text{ m}$

- c) Determine the time taken to reach the maximum height by using one of the kinematic equations. Write down the equation, substitute from your list of variables and solve for the maximum height variable.

**Answer:**  $1.0 \text{ s}$

The second part of this exercise aims to find the speed of the ball just before returning to the ground using the fall from its maximum height.

- d) Sketch the situation for the falling ball, illustrating the ball at two key instants.

List all relevant variables, including the acceleration, for the two instants:

- e) Determine the speed of the ball just before hitting the ground the ball by using one of the kinematic equations. Write down the equation, substitute from your list of variables and solve for the velocity variable.

**Answer:** *velocity:*  $-10 \text{ m/s}$ , *speed:*  $10 \text{ m/s}$ .

### 66 Ball maximum height

A ball is launched vertically upward from Earth's surface with speed  $25 \text{ m/s}$ . (131F2024)

- a) Determine the maximum height reached by the ball.

- b) Determine the speed of the ball when it is  $16 \text{ m}$  above Earth's surface.



### 67 Rock launch speed

A person, lying on their back, throws a rock vertically from the ground. The rock reaches a maximum height of 3.6 m above the ground (about 12 ft). We aim to determine the launch speed and the time of flight of the rock. (131Sp2025)

- a) First consider the launch speed. Sketch the situation, illustrating the rock at two key instants. List all relevant variables at these instants.
- b) Determine the launch speed of the rock.
- c) Now consider the time of flight. Again sketch the situation, illustrating the rock at two key instants. List all relevant variables at these instants. Find the time of flight.

### 68 Bullet return speed

A bullet is fired vertically upward from Earth's surface with various speeds. The bullet will return to Earth at its launch position. (131F2024)

- a) Suppose that the bullet is launched with speed 600 m/s. Ignoring air resistance, determine the speed of the bullet just before it hits the ground.
- b) Is the speed with which the bullet returns the same as that with which it is launched (regardless of the launch speed)? Explain your answer.

### 69 Thrown rock

A rock is thrown vertically down from a bridge. It leaves the hand with speed 20 m/s and hits the water 15 m below. Determine its speed at the instant before it hits the water. (131F2024)

**Answer:** 26 m/s

### 70 Ball thrown from above the ground, 1

A ball is thrown vertically upwards, leaving the hand at a height of 1.2 m above the ground. It hits the ground 2.5 s after leaving the hand. (131F2024)

- a) Determine the speed with which the ball left the hand.
- b) Determine the maximum height above the ground reached by the ball.

### 71 Ball thrown from above the ground, 2

A ball is thrown vertically upwards, leaving the hand with speed 4.0 m/s at a height of 1.2 m above the ground. Determine the speed of the ball just before it hits the ground. (131F2024)

## 72 Diver

A diver stands on a diving board that is 10 m above the surface of the water. The diver launches herself vertically up, leaving the board with a speed of 5.0 m/s. (131F2024)

- a) Determine the time taken for the diver to hit the water. **Answer:** 2.0 s
- b) Determine the diver's speed when she hits the water. **Answer:** 15 m/s

## 73 Jumping flea

A flea is at rest on a bed and launches itself vertically. A child watching this thinks that the flea is airborne for a total time of about 2 s. Determine the maximum height above the bed that the flea would reach if this were true. Ignore air resistance. Is it plausible that the flea could have been airborne for that long? What might be a more realistic range of times? (131F2024)

**Answer:** 4.9 m

## 74 Penny and a well

A penny is held at rest at the top of a well. The penny is released, falls freely and takes 1.75 s to hit the water below. Determine the depth (distance from the top to the water) of the well. (131F2024)

**Answer:** 15 m

## 75 Parachuting package

A package is released from rest at a height of 100 m above Earth's surface. It falls freely until it is 40 m above Earth's surface. At that instant it deploys a parachute and after this it falls with a constant speed. The aim of this exercise is to determine the time taken to reach Earth.

To do this we will calculate the time taken for the free fall motion,  $\Delta t_A$ , and separately that time taken to fall with the parachute open,  $\Delta t_B$ .

The aim of the first part of this exercise is to determine  $\Delta t_A$ . (131F2024)

- a) Sketch the situation, illustrating the ball at two key instants that will allow one to determine the time for the free fall portion of the motion.

List all relevant variables for the two instants and the acceleration.

- b) Determine  $\Delta t_A$  by using one of the kinematic equations. Write down the equation, substitute from your list of variables and solve for  $\Delta t_A$ .

**Answer:** 3.5 s.

The second part of this exercise aims to find  $\Delta t_B$ .

- c) Sketch the situation for the falling ball, illustrating the ball at two key instants that will allow one to determine the time for the parachuting portion of the motion.

List all relevant variables for the two instants and the acceleration.

- d) There is one quantity that one can obtain from the free fall part of the motion that will be needed to analyze the parachuting portion of the motion. Identify and compute this and insert it in to the list of variables from the previous part.

- e) Now determine  $\Delta t_B$  by using one of the kinematic equations. Write down the equation, substitute from your list of variables and solve for  $\Delta t_B$ .

**Answer:** 1.2 s.

- f) Determine the total time taken to fall.

**Answer:** 4.7 s.

## 76 Rocket flight

A rocket is at rest on the ground. Its engine then fires, producing a constant acceleration of  $40 \text{ m/s}^2$  for a period of 15 s and propelling the rocket vertically. The engine then stops and the rocket continues to move upward. (131F2024)

- a) Determine the maximum altitude (height) reached by the rocket.

- b) Determine the time taken by the rocket to reach its maximum altitude.

- c) Determine the time taken for the rocket to fall back to the ground after reaching its maximum height.

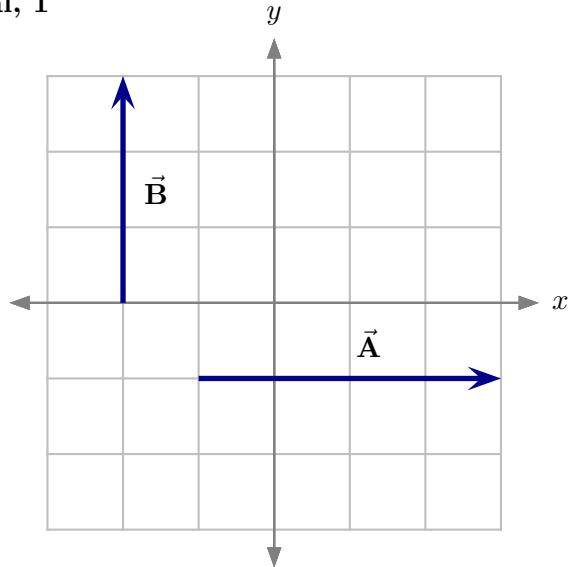
**Answer:** a) 23 km b) 76 s c) 69 s

## Vectors

### 77 Vector addition and subtraction: graphical, 1

Two displacement vectors,  $\vec{A}$  and  $\vec{B}$ , are illustrated. (131F2024)

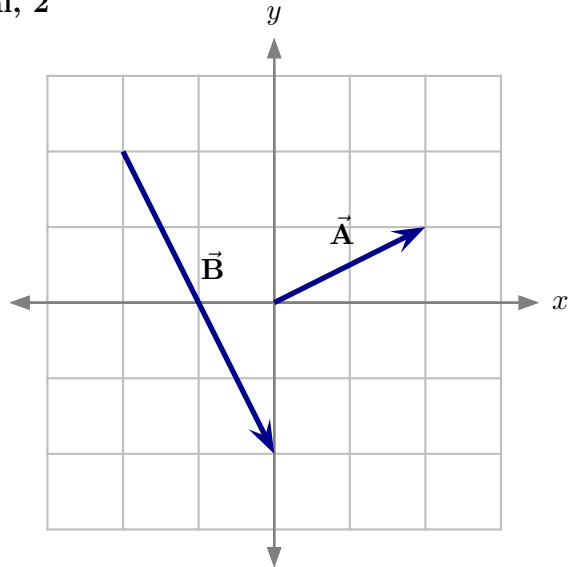
- Sketch  $\vec{C} = \vec{A} + \vec{B}$ . Determine the magnitude and direction of  $\vec{C}$ .
- Let  $A$  be the magnitude of  $\vec{A}$ , let  $B$  be the magnitude of  $\vec{B}$  and  $C$  be the magnitude of  $\vec{C}$ . In this case, do these magnitudes satisfy  $C = A + B$ ? Explain your answer.



### 78 Vector addition and subtraction: graphical, 2

Two displacement vectors,  $\vec{A}$  and  $\vec{B}$ , are illustrated. (131F2024)

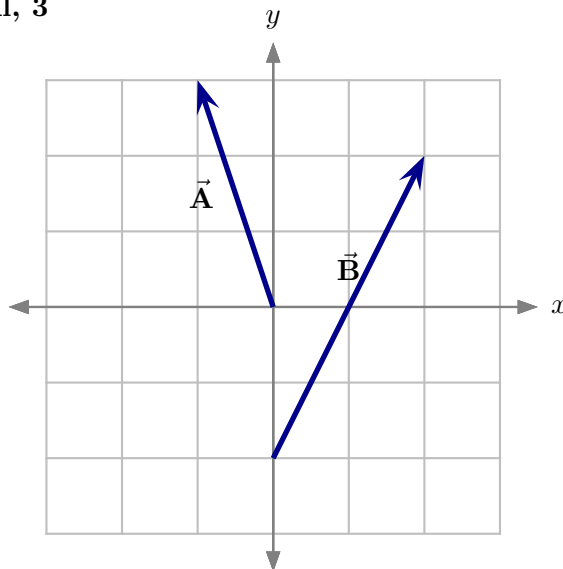
- Sketch  $\vec{C} = \vec{A} + \vec{B}$ . Determine the magnitude and direction of  $\vec{C}$ .
- Sketch  $\vec{C} = \vec{A} - \vec{B}$ . Determine the magnitude and direction of  $\vec{C}$ .
- Sketch  $\vec{C} = 2\vec{A} - \frac{1}{2}\vec{B}$ . Determine the magnitude and direction of  $\vec{C}$ .



### 79 Vector addition and subtraction: graphical, 3

Two displacement vectors,  $\vec{A}$  and  $\vec{B}$ , are illustrated. (131F2024)

- Sketch  $\vec{C} = \vec{A} + \vec{B}$ . Determine the magnitude and direction of  $\vec{C}$ .
- Sketch  $\vec{C} = \vec{A} - \vec{B}$ . Determine the magnitude and direction of  $\vec{C}$ .
- Sketch  $\vec{C} = 2\vec{A} - \frac{1}{2}\vec{B}$ . Determine the magnitude and direction of  $\vec{C}$ .



### 80 Vector algebra: conceptual

Consider two vectors labeled  $\vec{A}$  and  $\vec{B}$ . (131F2024)

- Suppose that  $\vec{A}$  and  $\vec{B}$  are perpendicular. Explain whether it is possible that  $\vec{A} - \vec{B} = 0$ .
- Suppose that the magnitudes satisfy  $B = 2A$ . Let  $\vec{C} = \vec{A} + \vec{B}$ . Explain whether it is possible that the magnitude of  $\vec{C}$  satisfies  $C = A$ . Explain whether it is possible that the magnitude of  $\vec{C}$  satisfies  $C = 2A$ .

### 81 Adding two vectors

Consider displacement vectors  $\vec{A}$ , whose magnitude is 4.0 m, and  $\vec{B}$ , whose magnitude is 3.0 m. Their directions are not specified. Let  $\vec{C} = \vec{A} + \vec{B}$ . (131F2024)

- What is the maximum possible magnitude for  $\vec{C}$ ? How must the vectors be arranged to give this?
- What is the minimum possible magnitude for  $\vec{C}$ ? How must the vectors be arranged to give this?

### 82 Vector components: conceptual

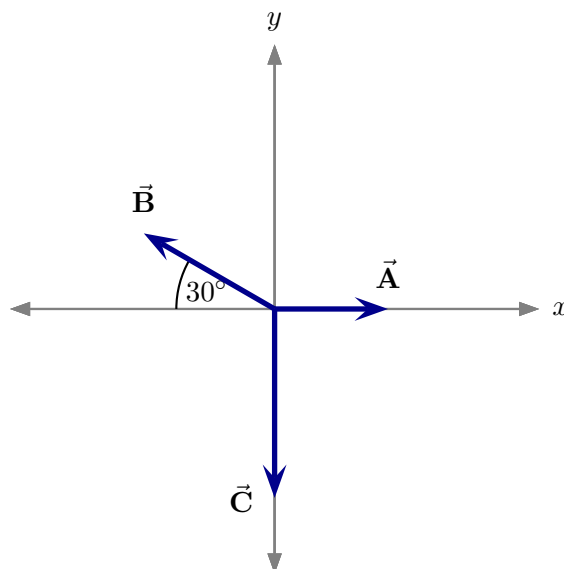
Consider a vector  $\vec{A}$  in two dimensions. (131F2024)

- Is it possible that either  $A_x$  or  $A_y$  is larger than the magnitude,  $A$ ? Explain your answer.
- Is it possible that both components of  $\vec{A}$  can be non-zero and the magnitude of  $\vec{A}$  is zero? Explain your answer.

### 83 Vector components: algebraic, 1

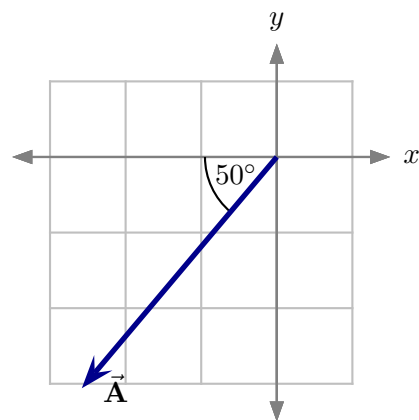
Displacement vectors,  $\vec{A}$ ,  $\vec{B}$ , and  $\vec{C}$  are illustrated. Their magnitudes are  $A = 15\text{ m}$ ,  $B = 20\text{ m}$  and  $C = 25\text{ m}$ . (131F2024)

- Determine the  $x$  and  $y$  components of each vector.
- Determine the components of  $\vec{D} = \vec{A} + \vec{B} + \vec{C}$ . Determine the magnitude of  $\vec{D}$ .



### 84 Vector components: algebraic, 2

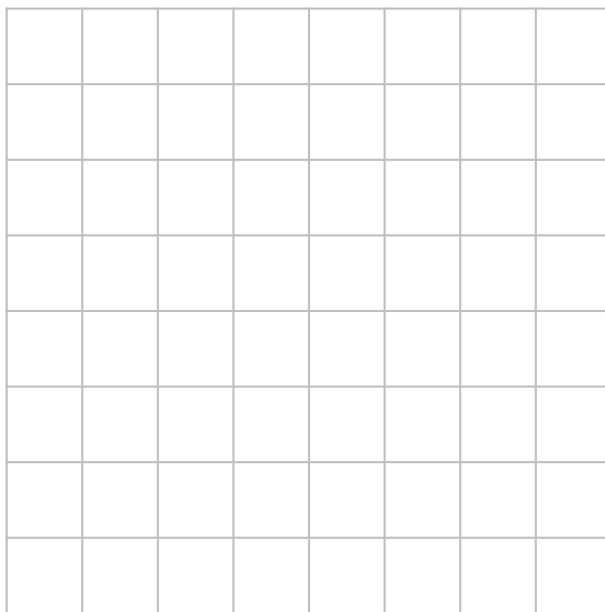
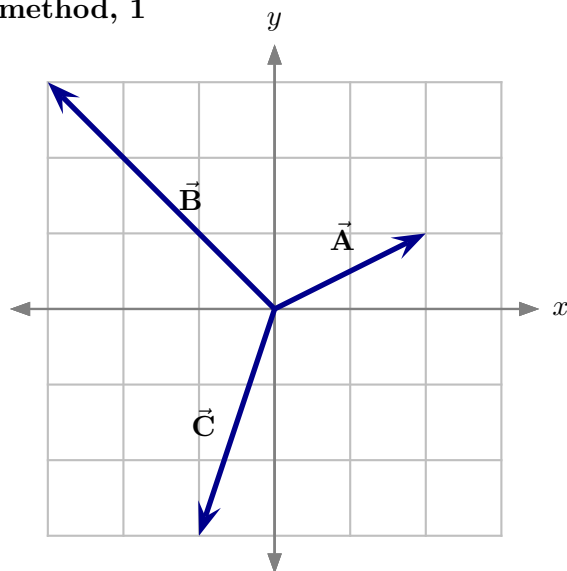
Determine the components of  $\vec{A}$ , whose magnitude is  $20.0\text{ m}$ . (131F2024)



### 85 Vector addition: graphical and algebraic method, 1

Displacement vectors,  $\vec{A}$ ,  $\vec{B}$ , and  $\vec{C}$  are illustrated. Let  $\vec{D} = \vec{A} + \vec{B} + \vec{C}$ . (131F2024)

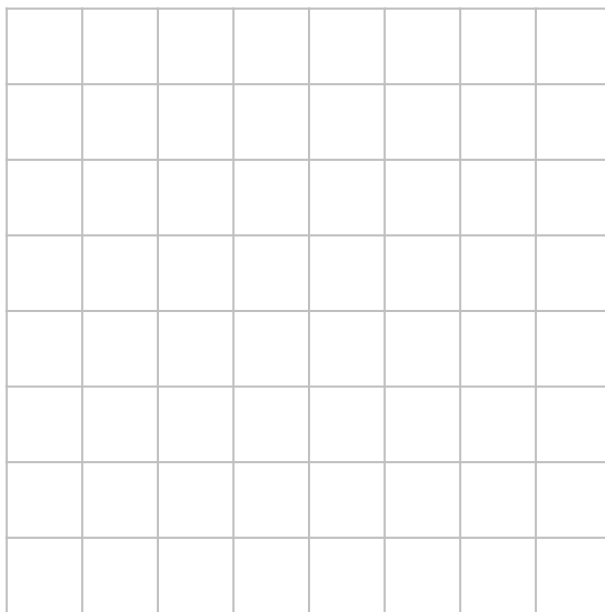
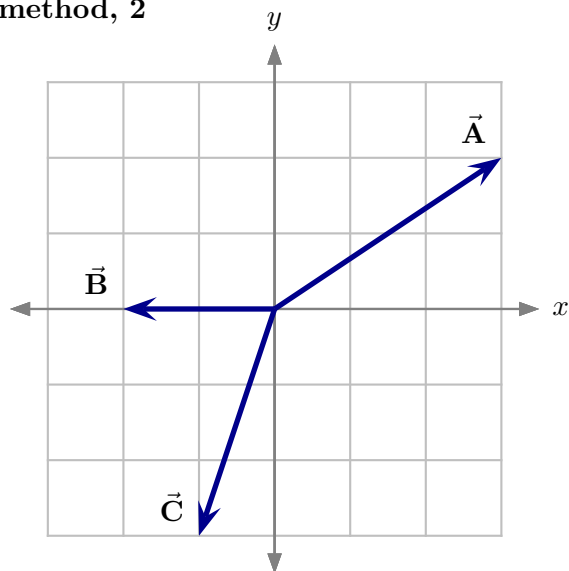
- Using the graph sheet below, determine  $\vec{D}$  graphically via the head-to-tail method. Use the result to determine the magnitude of  $\vec{D}$ .
- List the horizontal and vertical components of each of  $\vec{A}$ ,  $\vec{B}$ , and  $\vec{C}$  and use these components to determine the components of  $\vec{D}$ . Use the result to determine the magnitude of  $\vec{D}$ .



## 86 Vector addition: graphical and algebraic method, 2

Displacement vectors,  $\vec{A}$ ,  $\vec{B}$ , and  $\vec{C}$  are illustrated. Let  $\vec{D} = \vec{A} + \vec{B} + \vec{C}$ . (131F2025)

- Using the graph sheet below, determine  $\vec{D}$  graphically via the head-to-tail method. Use the result to determine the magnitude of  $\vec{D}$ .
- List the horizontal and vertical components of each of  $\vec{A}$ ,  $\vec{B}$ , and  $\vec{C}$  and use these components to determine the components of  $\vec{D}$ . Use the result to determine the magnitude of  $\vec{D}$ .

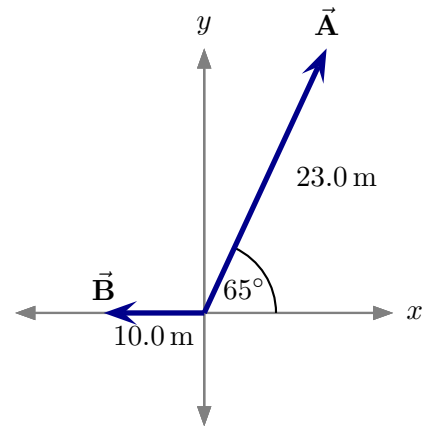




**87 Vector addition: algebraic method, 1**

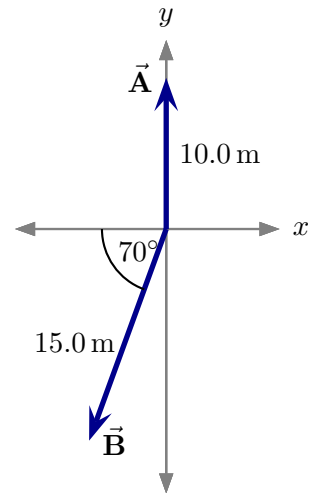
Two displacement vectors,  $\vec{A}$  and  $\vec{B}$  are illustrated.  
(131F2024)

- a) Determine the components of  $\vec{A}$ .
- b) Determine the components of  $\vec{B}$ .
- c) Determine the components of  $\vec{C} = \vec{A} + \vec{B}$ .
- d) Determine the magnitude of  $\vec{C}$ .

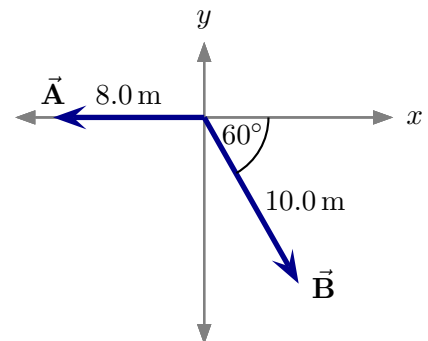
**88 Vector addition: algebraic method, 2**

Two displacement vectors,  $\vec{A}$  and  $\vec{B}$  are illustrated.  
(131F2024)

- a) Determine the components of  $\vec{A}$ .
- b) Determine the components of  $\vec{B}$ .
- c) Determine the components of  $\vec{C} = \vec{A} + \vec{B}$ .
- d) Determine the magnitude of  $\vec{C}$ .

**89 Vector addition: algebraic method, 3**

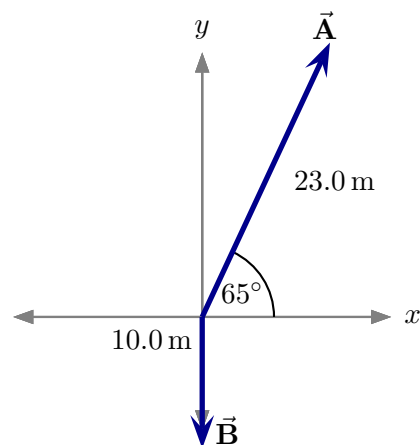
Two displacement vectors,  $\vec{A}$  and  $\vec{B}$  are illustrated. Determine the magnitude of  $\vec{C} = \vec{A} + \vec{B}$ . (131Sp2023)



### 90 Vector addition: algebraic method, 4

Two displacement vectors,  $\vec{A}$  and  $\vec{B}$  are illustrated.  
(111F2023)

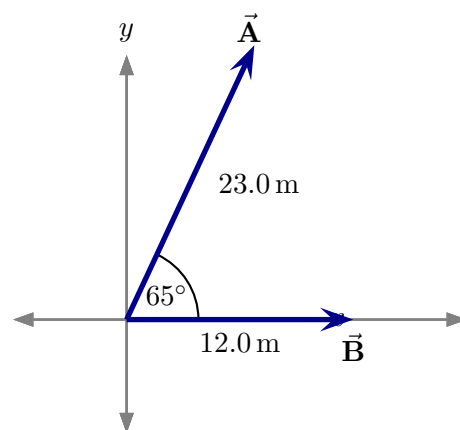
- Determine the components of  $\vec{C} = \vec{A} + \vec{B}$ .
- Determine the magnitude of  $\vec{C}$ .



### 91 Vector subtraction, 1

Two displacement vectors,  $\vec{A}$  and  $\vec{B}$  are illustrated.  
(131F2024)

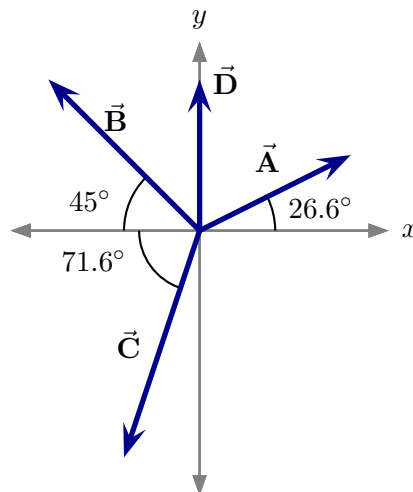
- Determine the components of  $\vec{C} = \vec{A} - \vec{B}$ .
- Determine the magnitude of  $\vec{C}$ .



### 92 Vector algebra using components

Displacement vectors,  $\vec{A}$ ,  $\vec{B}$ ,  $\vec{C}$ , and  $\vec{D}$  are illustrated. Their magnitudes are  $A = 2.0$  m,  $B = 2.5$  m,  $C = 3.0$  m and  $D = 1.5$  m. (131Sp2023)

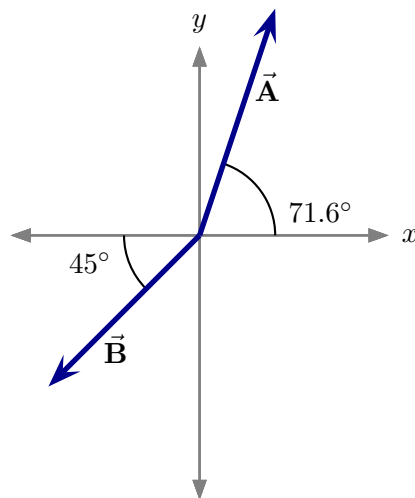
- Determine the  $x$  and  $y$  components for each vector.
- Determine the  $x$  and  $y$  components for  $\vec{E} = \vec{A} + \vec{B}$ . Determine the magnitude of  $\vec{E}$ .
- Determine the  $x$  and  $y$  components for  $\vec{E} = \vec{B} + \vec{C}$ . Determine the magnitude of  $\vec{E}$ .
- Determine the  $x$  and  $y$  components for  $\vec{E} = \vec{B} + \vec{D}$ . Determine the magnitude of  $\vec{E}$ .
- Determine the  $x$  and  $y$  components for  $\vec{E} = \vec{B} + \vec{C} + \vec{D}$ . Determine the magnitude of  $\vec{E}$ .



### 93 Vector subtraction using components

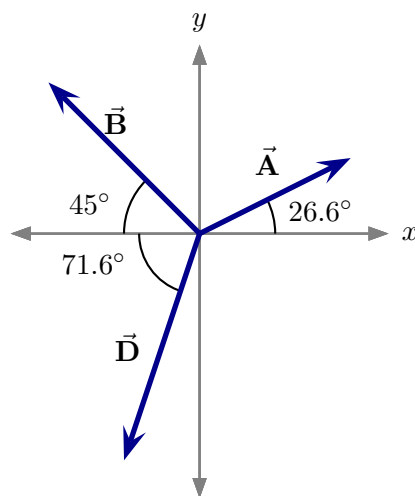
Displacement vectors,  $\vec{A}$ ,  $\vec{B}$  are illustrated. Their magnitudes are  $A = 3.16$  m and  $B = 2.83$  m. (131Sp2023)

- Determine the components of  $\vec{C} = \vec{A} - \vec{B}$ . Determine the magnitude of  $\vec{C}$ .
- Determine the components of  $\vec{C} = 4\vec{A} - 3\vec{B}$ . Determine the magnitude of  $\vec{C}$ .



### 94 Vector algebra using unit vectors

Displacement vectors,  $\vec{A}$ ,  $\vec{B}$ , and  $\vec{D}$  are illustrated. Let  $\vec{D} = 3\vec{A} + 2\vec{B} + \vec{C}$ . Their magnitudes are  $A = 2.0$  m,  $B = 3.5$  m and  $D = 6.0$  m. Express each of the vectors in terms of unit vectors and use these to determine  $\vec{C}$ . (131Sp2023)

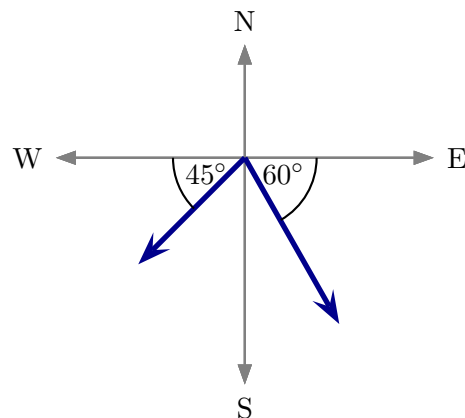


### 95 Marching soldier

A soldier marches around a playing field whose edges are along North-South (N-S) and East-West (E-W) lines. The soldier starts in the southwest corner, marches in a straight line in the direction  $30^\circ$  N of E for 40 m. He then marches straight south for 12 m. Finally he marches in a straight line in the direction  $50^\circ$  N of W for 30 m. After this, how far is the soldier from his starting point? (131Sp2023)

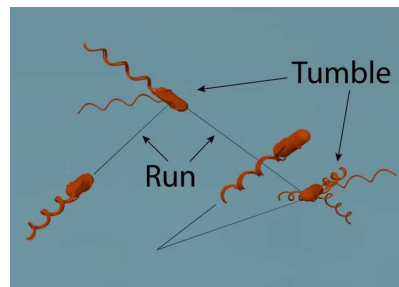
### 96 Successive displacements

An insect crawls along a horizontal two dimensional surface in two straight stages. First the insect walks 8.0 cm at an angle of  $45^\circ$  south of west. Second, the insect walks 10.0 cm in a straight line at an angle  $60^\circ$  south of east. Determine an expression for the displacement of the insect and how far the insect ends up from its starting point. (131Sp2023)



### 97 Bacterium Run-and-Tumble

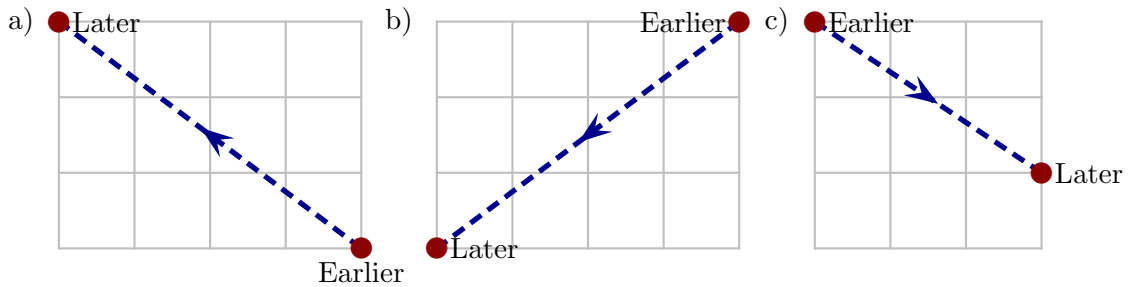
Some bacteria move through a fluid in a stop-start process called run-and-tumble. The figure (from the Figueroa Lab, Department of Physics, University of Colorado) illustrates this for a bacterium starting at the lower left and doing four runs and three tumbles. The lines indicate the runs. The typical distance in a run is in the range of  $1 - 10 \mu\text{m}$ . Assume that the first three runs have lengths  $3.5 \mu\text{m}$ ,  $4.5 \mu\text{m}$  and  $4.3 \mu\text{m}$ . Using vectors for each run, determine/estimate the displacement (distance and direction) from the start of the first to the end of the third run. *Note: Results will vary depending on additional estimates or measurements that you make. Analyzing such motion is often done by physicists.*



## Two dimensional kinematics

### 98 Velocity vectors from motion

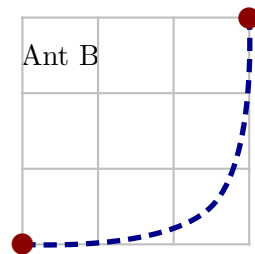
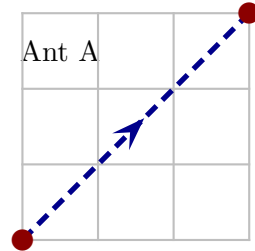
Particles move along the illustrated trajectories. Their locations are illustrated at instants 4.0 s apart. The grid units are meters. For each case determine the average velocity vector for the particle over the entire interval and express it in terms of the conventional unit vectors. (131F2024)



### 99 Ants on a table

Two ants move on a flat surface between two grains of sugar. Their trajectories are illustrated. The ants take the same time to move between the grains but follow different trajectories. Which of the following is true about the average velocities of the ants for the entire trip? Explain your answer. (131F2024)

- i) The average velocities are the same.
- ii) The average velocity of ant A is smaller than that for ant B.
- iii) The average velocity of ant A is larger than that for ant B.
- iv) The average velocities are different but one cannot say which is larger.



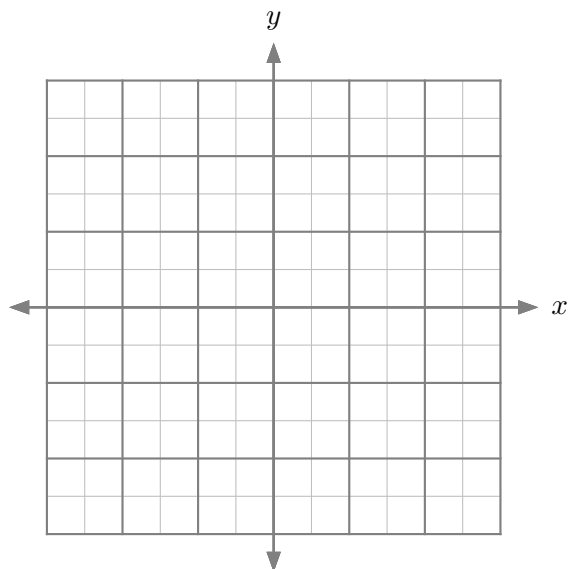
### 100 Model moon motion

A moon of a planet in a demonstration model of a planetary system travels a trajectory described by the position vector

$$\vec{r}(t) = 2.0 \text{ m} \cos(\omega t) \hat{i} + 2.0 \text{ m} \sin(\omega t) \hat{j}.$$

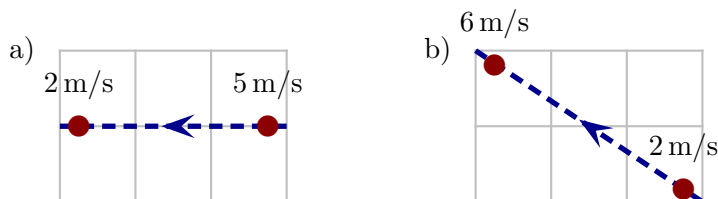
where  $\omega = 0.50\pi \text{ rad s}^{-1}$  (note that this expresses the arguments of the trigonometric functions in terms of radians). (131F2024)

- Determine expressions for the horizontal and vertical coordinates  $x(t)$  and  $y(t)$  for the moon from  $t = 0\text{s}$  to  $t = 2.0\text{s}$ .
- Sketch the trajectory of the moon as accurately as possible on the illustrated grid.
- Determine the displacement vector from  $t = 1.0\text{s}$  to  $t = 1.5\text{s}$  and use this to get the average velocity vector from  $t = 1.0\text{s}$  to  $t = 1.5\text{s}$ .
- Determine the displacement vector from  $t = 1.0\text{s}$  to  $t = 1.1\text{s}$  and use this to get the average velocity vector from  $t = 1.0\text{s}$  to  $t = 1.1\text{s}$ .
- Determine the displacement vector from  $t = 1.0\text{s}$  to  $t = 1.01\text{s}$  and use this to get the average velocity vector from  $t = 1.0\text{s}$  to  $t = 1.01\text{s}$ .
- What do your results suggest about the direction of the instantaneous velocity vector at  $t = 1.0\text{s}$ ? Explain your answer.



### 101 Acceleration vectors from motion

Two particles move along the illustrated trajectories. Their locations and speeds are illustrated at instants 0.25 s apart. For each case determine the acceleration vector for the particle. (131F2024)



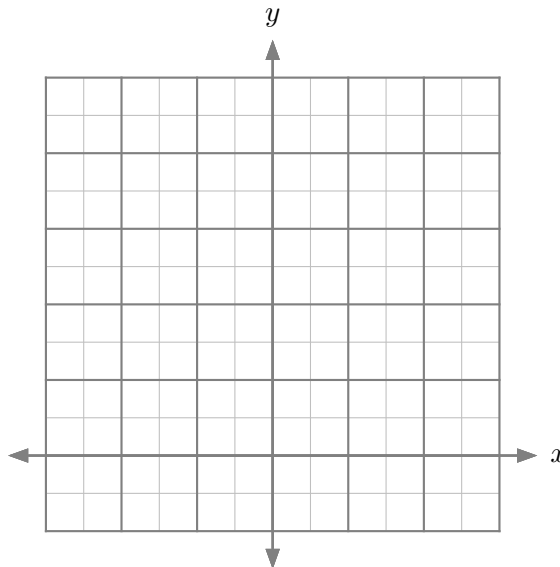
### 102 Ball moving in two dimensions

A ball travels along a trajectory described by the position vector

$$\vec{r}(t) = (t - 2)\hat{i} + \left(-\frac{(t - 2)^2}{2} + 4\right)\hat{j}.$$

from  $t = 0$  to  $t = 4$ . (131F2024)

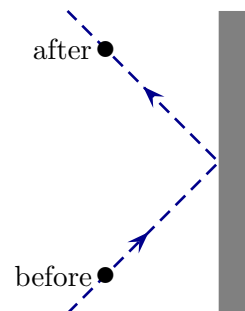
- Determine expressions for the horizontal and vertical coordinates  $x(t)$  and  $y(t)$  for the ball.
- Sketch the trajectory of the ball as accurately as possible on the illustrated grid.
- Determine the displacement vector from  $t = 1.5$  to  $t = 2$  and use this to get the average velocity vector from  $t = 1.5$  to  $t = 2$ .
- Determine the displacement vector from  $t = 2$  to  $t = 2.5$  and use this to get the average velocity vector from  $t = 2$  to  $t = 2.5$ .
- Determine an expression for the instantaneous velocity vector and use this to obtain the average acceleration from  $t = 1.8$  to  $t = 2.2$ . What does this imply for the direction of the acceleration at  $t = 2$ ?



### 103 Bouncing hockey puck

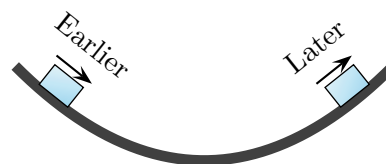
A hockey puck slides along a horizontal surface toward a board, hitting it at an angle and bouncing off with unchanged speed. The view from above is as illustrated. (131F2024)

- Draw the velocity vectors of the puck just before and just after hitting the board, use these to draw the vector  $\Delta\vec{v}$ , and use the result to draw the direction of the acceleration vector.
- If the puck traveled backwards along the same path (i.e. diagonally from top left to the board and then toward the bottom left), what would the direction of the acceleration vector be? Explain your answer.



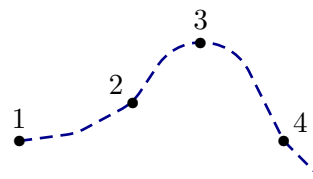
### 104 Sliding ice cube

An ice cube slides back and forth along a bowl with a parabolic cross section. It moves with the same speeds at the indicated earlier and later moments, which are symmetrical about the lowest point in the parabola. Determine the direction of the average acceleration, using velocity. (131S2025)



### 105 Ants moving along a curved path

Various ants follow the same path on a horizontal surface, starting at point 1. The path is as illustrated. Ant A moves with a constant speed, ant B gradually speeds up and ant C gradually slows. (131F2024)



- Draw the velocity vector at points 1, 2, 3 and 4.
- Does any of the ants have zero acceleration at all times? Explain your answer.

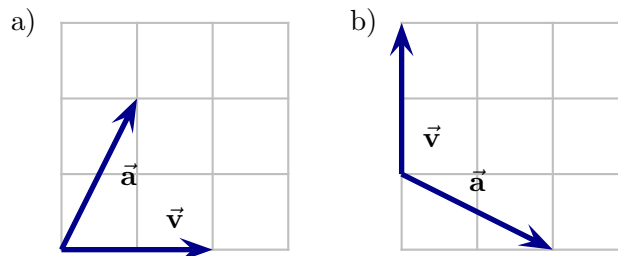
### 106 Clock second hand

The second hand in a grandfather clock moves at a constant rate. Consider the interval of motion from when the second hand passes the 3 mark until it passes the 9 mark. Estimate the average acceleration of the tip of the second hand for this interval of motion for a clock with dimensions like that a grandfather clock or the clock on the classroom wall. Explain all the steps in your process of estimation. (131F2025)

*Note: Modern electric clocks with an analog display (hands) do not operate exactly like this. The second hand actually jumps every second. However, in older purely mechanical clocks the second hand does move continuously.*

### 107 Velocity and acceleration vectors and motion

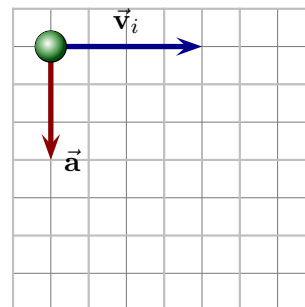
Velocity and acceleration vectors for two particles at one instant are illustrated. For each case, describe whether the particle is speeding up, slowing down or moving with constant speed at the illustrated instant. Also describe how the direction of the particle's motion changes. Explain your answers. *Note that the question refers to what is happening at the illustrated instant. The answers could be different if the particles were observed significantly later.* (131F2024)





### 108 Constant vertical acceleration

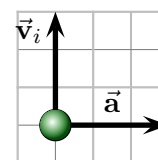
A ball launches off a horizontal surface. At the moment of launch its velocity is  $\vec{v}_i$ . At all later times it accelerates with a constant acceleration,  $\vec{a}$ . The situation with the vectors drawn to scale is illustrated (for the velocity vector, the grid unit is the standard unit of velocity and for acceleration the grid unit is the standard unit of acceleration). (131F2024)



- Draw, as accurately as possible, the velocity vector,  $\vec{v}_f$ , at an instant 1.0 s after the initial instant.
- Using  $\vec{v}_f$  describe whether the object is moving faster at the 1.0 s instant than at the initial instant.
- Using  $\vec{v}_f$  describe the direction in which the object is moving at the 1.0 s instant.

### 109 Constant acceleration in two dimensions

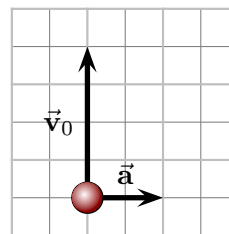
A ball can slide along a horizontal surface. At an initial instant its velocity is  $\vec{v}_i$ . At all later times it accelerates with a constant acceleration,  $\vec{a}$ . The situation with the vectors drawn to scale as viewed from above is illustrated (for the velocity vector, the grid unit is the standard unit of velocity and for acceleration the grid unit is the standard unit of acceleration). (131F2024)



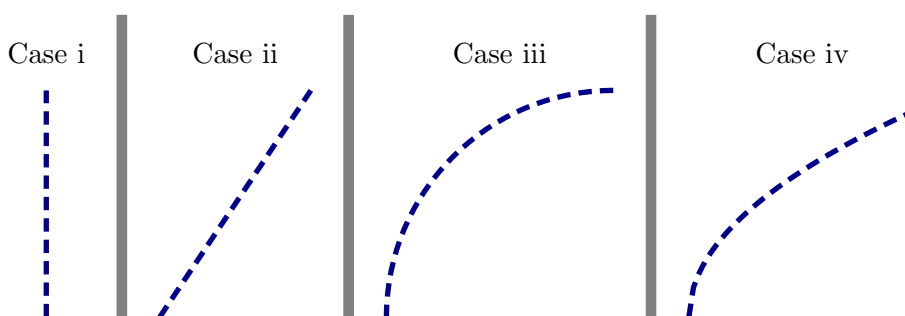
- Draw, as accurately as possible, the velocity vector at an instant 1.0 s after the initial instant.
- Let  $v_i$  denote the speed at the initial instant. Determine an expression for the speed, in terms of  $v_i$ , at an instant 1.0 s after the initial instant.
- Determine the time for the speed to be twice the speed at the initial moment.

### 110 Motion with constant acceleration

A ball can slide along a horizontal surface. At an initial instant its velocity,  $\vec{v}_0$ , is as illustrated. At all later times it accelerates with a constant acceleration,  $\vec{a}$ , as illustrated. (131F2024)



- Draw, as accurately as possible, the velocity vector at an instant 0.5 s after the initial instant.
- Draw, as accurately as possible, the velocity vector at an instant 1.0 s after the initial instant.
- Which of the following best represents the trajectory of the particle? Explain your answer.



### 111 Dropping a coin on a ship

A person is on the deck of a ship. While the ship is at rest in a harbor, the person drops a coin from rest and the coin lands on the deck of the ship. The person marks the spot where the coin lands. Later the ship sails with constant speed in a straight line and the person returns to the same spot and drops the coin again. Does the coin land at the original mark, in front of it or behind it? Explain your answer. (131F2024)

### 112 Throwing a ball on a train

A person is inside a train that moves horizontally in a straight line with speed  $v_{\text{train}}$ . The person launches a ball vertically with speed,  $v_{\text{launch}}$  (from the perspective of the person in the train). Let  $v$  be the speed of the ball at the moment of launch as viewed from outside the train. Which of the following is true? Explain your answer. (131F2024)

- $v = v_{\text{train}}$
- $v = v_{\text{launch}}$
- $v = v_{\text{train}} + v_{\text{launch}}$
- $v = \sqrt{v_{\text{train}}^2 + v_{\text{launch}}^2}$

### 113 Jumping monkey

A monkey jumps leaving the Earth at an angle of  $45^\circ$  from the surface. Which of the following is true when the monkey reaches the highest point along the trajectory? Explain your answer. (131F2024)

- i) The acceleration of the monkey is zero.
- ii) The acceleration of the monkey is horizontal in the forwards direction.
- iii) The acceleration of the monkey is horizontal in the backwards direction.
- iv) The acceleration of the monkey is vertical downward.
- v) The acceleration of the monkey is vertical upward.

### 114 Red vs. blue ball

Two balls, one red and the other blue, are on a horizontal table. They are made to roll off the table. At the instants that they leave the speed of the blue ball is four times that of the red ball. Which of the following is true regarding the times taken to hit the horizontal floor on which the table stands? Explain your answer. (131F2024)

- i) The time taken for blue ball is a quarter of that for the red ball.
- ii) The time taken for blue ball is half that for the red ball.
- iii) The time taken for blue ball is the same as that for the red ball.
- iv) The time taken for blue ball is twice that for the red ball.
- v) The time taken for blue ball is four times that for the red ball.

### 115 Heavier vs. lighter ball

Two balls, one heavier and the other lighter, roll off the edge of a horizontal table. (131S2025)

- a) Suppose that the balls are launched with the same speed. Which of the following is true? Explain your answer.
  - i) The time taken for the heavier ball to hit the floor is the same as the time taken for the lighter ball.
  - ii) The time taken for the heavier ball to hit the floor is larger than the time taken for the lighter ball.
  - iii) The time taken for the heavier ball to hit the floor is smaller than the time taken for the lighter ball.
- b) Suppose that the balls are launched with the same speed. Which of the following is true? Explain your answer.
  - i) The distance traveled by the heavier ball is the same as that traveled by the lighter ball.
  - ii) The heavier ball travels further than the lighter ball.
  - iii) The lighter ball travels further than the heavier ball.

### 116 Balls launched off horizontal surfaces

Two balls, one red and the other blue, are on horizontal surfaces above a floor. They are each launched horizontally off the surface with the same speed. The height of the blue ball surface above the floor is nine times that of the red ball surface. Let  $t_{\text{red}}$  be the time between the moment that the red ball leaves the table until it hits the floor; similarly  $t_{\text{blue}}$  represents the same for the blue ball. Let  $\Delta x_{\text{red}}$  be the horizontal distance between the point where the red ball leaves the table and hits the ground. (131F2024)

a) Suppose that the balls are launched with the same speed. Which of the following is true? Explain your answer.

i)  $t_{\text{blue}} = \frac{1}{9} t_{\text{red}}$

ii)  $t_{\text{blue}} = \frac{1}{3} t_{\text{red}}$

iii)  $t_{\text{blue}} = t_{\text{red}}$

iv)  $t_{\text{blue}} = 3 t_{\text{red}}$

v)  $t_{\text{blue}} = 9 t_{\text{red}}$

b) Suppose that the balls are launched with the same speed. Which of the following is true? Explain your answer.

i)  $\Delta x_{\text{blue}} = \frac{1}{9} \Delta x_{\text{red}}$

ii)  $\Delta x_{\text{blue}} = \frac{1}{3} \Delta x_{\text{red}}$

iii)  $\Delta x_{\text{blue}} = \Delta x_{\text{red}}$

iv)  $\Delta x_{\text{blue}} = 3 \Delta x_{\text{red}}$

v)  $\Delta x_{\text{blue}} = 9 \Delta x_{\text{red}}$

c) Suppose that the blue ball is launched with a slower speed than the red ball. Describe how this would affect the answers of the previous questions.

### 117 Running off a roof

A person runs with speed 8.0 m/s off a flat roof that is 3.0 m above the ground. First suppose that the person travels horizontally at the moment that he leaves the roof. Determine how far horizontally from the edge of the roof the person will land. (131F2024)

- a) Sketch the situation with the “earlier” instant being that at which the person leaves the roof and the “later” instant being the moment just before the person hits the ground. List as many of the variables as possible. Use the format:

$t_i =$	$t_f =$
$x_i =$	$x_f =$
$y_i =$	$y_f =$
$v_{ix} =$	$v_{fx} =$
$v_{iy} =$	$v_{fy} =$
$a_x =$	$a_y =$

- b) Sketch the velocity vector at the earlier moment and use this to determine the components of  $\vec{v}_i$ . Enter these in the list above.
- c) Identify the variable needed to answer the question of the problem. Select and write down a kinematic equation that contains this variable and attempt to solve it. You should see to solve the variable describing the horizontal position, you first need the value for another, currently unknown variable. Which variable is this?
- d) Use the vertical aspects of the object’s motion to solve for this other unknown variable and use this result to answer the question of this problem.

Suppose that the person ran and jumped from the building at an angle of  $30^\circ$  above the horizontal. This will change how far the person travels. Before answering that question, we ask, what is the maximum height above the ground reached by the person for this running jump?

- e) Sketch the velocity vector at the earlier moment and use this to determine the components of  $\vec{v}_i$ . Reconstruct the list of variables for the problem.
- f) Sketch the velocity vector at the instant when the person reaches his highest point. Use this to add additional information to the list of variables for the problem.
- g) Use the kinematic equations to determine the maximum height that the person reaches.
- h) Determine the speed of the person at the maximum height.
- i) Determine how far horizontally from the edge of the roof the person will land.

## 118 Projectile motion range

A person runs with speed 8 m/s off a flat roof that is 3.0 m above the ground. The person can launch himself at various angles and the purpose of this exercise is to determine the speed with which the person hits the ground and where on the ground the person lands (for various angles of launch). First suppose that the person travels horizontally at the moment that he leaves the roof. It was found that the person lands 6.3 m from the base of the building. (131F2024)

- a) Sketch the situation with the “earlier” instant being that at which the person leaves the roof and the “later” instant being the moment just before the person hits the ground. List as many of the variables as possible. Use the format

	$t_f =$	
$x_i =$	$x_f =$	
$y_i =$	$y_f =$	
$v_{ix} =$	$v_{fx} =$	
$v_{iy} =$	$v_{fy} =$	
$a_x =$	$a_y =$	.

- b) Draw the velocity vector at the earlier instant and use this to determine the components of  $\vec{v}_0$ . Enter these in the list above.
- c) Draw the velocity vector at the later instant. Describe whether the components are positive, negative or zero.
- d) The speed of the object is the magnitude of the velocity vector. So to determine the speed, you will first need to determine the components of the velocity vector. Use the kinematic equations to determine the  $x$  and  $y$  components of the velocity at the later instant. You must start by writing the equation that you use, before substituting.
- e) Sketch the velocity vector at the instant just before the person hits the ground and indicate the values of the  $x$  and  $y$  components on the sketch. Use this to determine the speed of the person just before hitting the ground.

Now suppose that the person travels with the same speed but launches himself at an angle of  $45^\circ$  from the roof.

- f) Using the same “earlier” and later instants as before, list as many of the variables as possible.
- g) Draw the velocity vector at the earlier moment and use this to determine the components of  $\vec{v}_0$ .
- h) Use the kinematic equations to determine the  $x$  and  $y$  components of the velocity at the later instant.
- i) Sketch the velocity vector at the instant just before the person hits the ground and indicate the values of the  $x$  and  $y$  components on the sketch. Use this to determine the speed of the person just before hitting the ground.

- j) Determine the time taken to hit the ground.
- k) Determine how far the person lands from the building.

### 119 Aircraft dropping object

An aircraft flies horizontally with a constant speed of 600 km/h at a height of 1200 m above a flat surface. It drops an object from its underside; this object is supposed to hit a particular spot on the ground. How far (horizontally) from the spot must the aircraft be for the object to hit the spot? (131F2024)

**Answer:** 2610 m

### 120 Rifle shot

A rifle fires a bullet. At the moment that the bullet leaves the rifle it travels horizontally with speed 300 m/s. Determine how far the bullet will drop by the time that it reaches a vertical wall that is 100 m from the end of the rifle that is closest to the wall. (131F2024)

**Answer:**  $-0.54\text{ m}$

### 121 Ball thrown horizontally

A ball is thrown, leaving the hand horizontally at a height of 2.0 m above the ground. It lands a horizontal distance of 5.0 m from where it left the hand. (111F2023)

- a) Determine the time from when the ball leaves the hand until it hits the ground.
- b) Determine the speed with which the ball leaves the hand.

### 122 Rock thrown horizontally

You throw a rock, which leaves your hand 1.5 m above the ground. You would like it to land a horizontal distance of 18.0 m from where it left the hand. How fast must you throw the ball? (131F2024)

**Answer:** 33 m/s

### 123 Jumping over a ditch

A dog attempts to jump over a ditch, which is 2.0 m wide. The dog launches itself from one edge of the ditch with speed 5.0 m/s at an angle of  $30^\circ$  from the horizontal. The aim of this exercise is to determine whether the dog will reach the other side of the ditch. First we will find out whether the dog reaches its maximum height before or after it is above the middle of the ditch. (131F2024)

- a) Sketch the situation with the “earlier” instant being that at which the dog launches and the “later” instant being the moment when it reaches its highest point. List as many of the variables as possible. Use the format

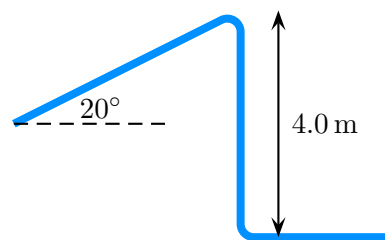
$t_i =$	$t_f =$	
$x_i =$	$x_f =$	
$y_i =$	$y_f =$	
$v_{ix} =$	$v_{fx} =$	
$v_{iy} =$	$v_{fy} =$	
$a_x =$	$a_y =$	.

- b) Draw the velocity vector at the earlier instant and use this to determine the components of  $\vec{v}_i$ . Enter these in the list above.
- c) Draw the velocity vector at the later instant. Describe whether the components are positive, negative or zero and enter as much information about these in the list above.
- d) Determine the horizontal distance traveled by the dog by the time that it reaches its maximum height.
- e) As the dog descends from its maximum height back to the ground, how much further does it travel? Does it reach the other side of the ditch?



### 124 Launching off a ski ramp, 1

A ski ramp is arranged as illustrated. A skier launches off the ramp with a speed of 15 m/s. Initially the aim of this exercise is to determine the maximum height reached by the skier and the velocity at this point. A later goal is to determine the distance at which the skier lands from the bottom of the ramp. (131F2024)



- a) Sketch the situation with the “earlier” instant being that at which the skier launches and the “later” instant being the moment when she reaches its highest point. List as many of the variables as possible. Use the format

$t_0 =$	$t_1 =$
$x_0 =$	$x_1 =$
$y_0 =$	$y_1 =$
$v_{0x} =$	$v_{1x} =$
$v_{0y} =$	$v_{1y} =$
$a_x =$	$a_y =$

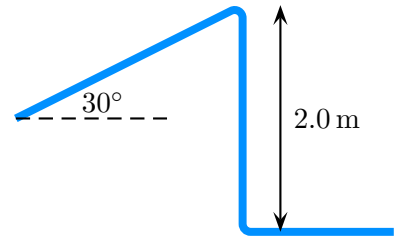
- b) Draw the velocity vector at the earlier instant and use this to determine the components of  $\vec{v}_0$ . Enter these in the list above.
- c) Draw the velocity vector at the later instant. Describe whether the components are positive, negative or zero and enter as much information about these in the list above.
- d) Determine the time taken to reach the maximum height and then the horizontal distance traveled by the skier to reach her maximum height. Determine the velocity at this point.

You will now consider the motion from the highest point back to the ground.

- e) Repeat the problem set-up with the “earlier” instant being that at which the skier is at maximum height and the “later” instant being the moment *just before* she reaches hits the ground. Determine the time taken for this portion of the motion and use it to determine the horizontal distance from the base of the ramp to the skier’s landing point.

### 125 Launching off a ski ramp, 2

A ski ramp is arranged as illustrated. A skier launches off the top of the ramp with a speed of 28 m/s. Determine the horizontal distance at which the skier lands from the bottom of the ramp. (131F2024)



### 126 Jumping grasshopper

A grasshopper jumps, leaving the ground at an angle of  $70^\circ$  from the horizontal. The grasshopper reaches a height of 0.60 m. (131F2024)

- Determine the speed with which the grasshopper leaves the ground.
- Determine how far the grasshopper lands from where it jumped.

### 127 Jumping lemur

A lemur (a type of primate) jumps, leaving the ground at an angle of  $40^\circ$  from the horizontal with speed 6.0 m/s. (131F2024)

- Determine the maximum height reached by the lemur.
- Determine the amount of time for which the lemur is airborne.
- Determine the horizontal distance traveled by the lemur from the moment it leaves the ground until it returns to the ground.

### 128 Diver splashdown

A diver launches off a platform at an angle of  $55^\circ$  above the horizontal and with speed 8.0 m/s. The platform is 3.0 m above the surface of a pool. (131F2024)

- Determine the diver's speed just before hitting the water.
- Determine the time taken by the diver to hit the water.

### 129 Stone thrown from a bridge

A person stands on a bridge over a small lake. The person throws a stone with speed 18.0 m/s at an angle of  $40^\circ$  above the horizontal. The stone leaves the hand at a height 3.0 m above the surface of the water. (131F2024)

- Determine the time taken for the stone to reach the highest point in its trajectory.
- Determine the time taken for the stone to hit the water.
- Determine the horizontal distance traveled by the stone between the throw and when it hits the water.

### 130 Ball launched from a cliff

A person hits a ball from the top of a cliff. The ball leaves at height of 12 m above the surface of the water at angle  $30^\circ$  above the horizontal. It hits the water 2.5 s later. (131F2024)

- a) Determine speed with which the ball is launched.
- b) Determine the horizontal distance traveled by the ball.

### 131 Cannonball range

A cannonball is fired from the ground (assume that the ball leaves from the ground level) and must hit a target on the ground a distance of 500 m away. Assume that the ground is horizontal and ignore air resistance. (131F2024)

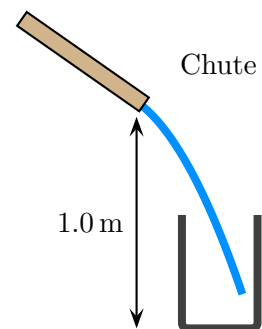
- a) Suppose that the cannonball leaves at an angle of  $50^\circ$  from the horizontal. Determine the speed with which it must be fired to hit the target.
- b) Suppose that the cannonball is fired with speed 150 m/s. Determine the angle above the horizontal at which it must be fired so that it hits the target.
- c) (*Challenging*) Determine the minimum speed with which the cannonball must be fired so that it can hit the target if the angle is adjusted correctly.

### 132 Angry tennis player

An angry tennis player hits a tennis ball into the air toward the fence that surrounds the court. The tennis player is a horizontal distance of 20 m from the fence, which is 10 m high. The tennis player hits the ball with speed 28 m/s at an angle of  $70^\circ$  above the horizontal from a height of 1.2 m above the ground. Does the ball reach the fence? Does it pass over the top of the fence? Explain your answer. (131F2024)

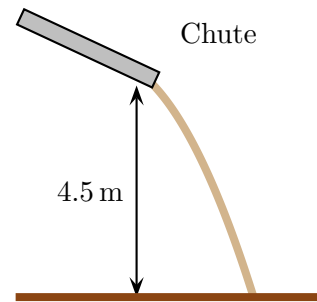
### 133 Water chute and bucket

Water slides down a chute that is at an angle of  $35^\circ$  above the horizontal. It leaves the end of the chute with speed 3.0 m/s. A bucket with sides 30 cm high and diameter 20 cm is placed beyond the chute so that it catches the water. Determine where the left edge of the bucket should be placed so that the water will land in the bucket. Provide the entire range of possible locations. (131F2024)



### 134 Grain chute

Grain pours off a chute that is at an angle of  $25^\circ$  above the horizontal. It leaves the end of the chute with speed  $5.0 \text{ m/s}$ . Determine how far it travels horizontally. (131F2024)



### 135 Object launch speed

Take a ball or small object that you can throw safely. Throw the object from the ground at an angle somewhere between  $45^\circ$  and  $60^\circ$  above the ground (you could also use any other horizontal surface as a reference). Estimate the angle at which you launch the ball and either or both of the maximum height and the horizontal distance that the object travels while in the air. Use these estimates to determine the speed with which you launched the object.

*You should be able to analyze this just using kinematic equations and your estimated data. Provide the complete analysis starting with the kinematic equations. (131F2025)*