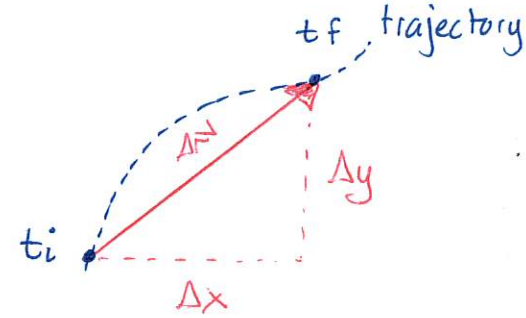
Fri: HW by 5pm

Ex 94, 95, 97, 98, 101, 104, 106, 109

Mon: Warm Up 4 D2L Group exercise

Velocity vectors in two dimensions

For two dimensional motion, position, displacement and velocity are all vectors. From time t_i to t_f the displacement vector is constructed as illustrated and



$$\Delta\vec{r} = \Delta x\,\hat{\imath} + \Delta y\,\hat{\jmath}.$$

Then velocity is (using $\Delta t = t_f - t_i$)

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta\vec{r}}{\Delta t} = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}\,\hat{\imath} + \lim_{\Delta t \to 0} \frac{\Delta y}{\Delta t}\,\hat{\jmath} = \frac{dx}{dt}\,\hat{\imath} + \frac{dy}{dt}\,\hat{\jmath}$$

We can connect this to features of the motion as:

The velocity vector is

$$\vec{v} = v_x\,\hat{\imath} + v_y\,\hat{\jmath}$$

horizontal component

vertical component

* magnitude of velocity vector ≡ speed

$$v = \sqrt{v_x^2 + v_y^2}$$

* direction of velocity = tangent to trajectory along direction of motion

DEMO: PhET Ladybug Motion - show trace and $\vec{v}$

Quiz 1 90% } 80% − 100%

Quiz 2 60% − 90% } 60% − 80%

Quiz 3 60% − 90% } 60% − 80%

Acceleration in two dimensions

Acceleration is the rate of change of the velocity vector. A preliminary definition is:

> Observe the object at two instants. Then the <u>average acceleration</u> over the interval is
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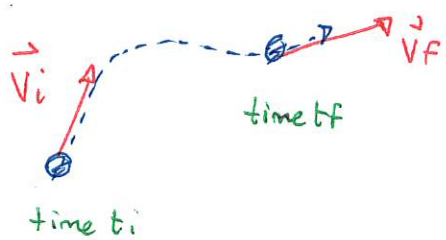
> 
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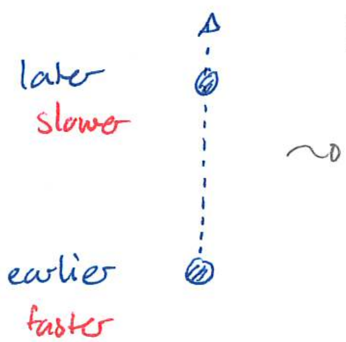
>
> $$\vec{a}_{avg} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{V}_f - \vec{V}_i}{\Delta t} = \frac{1}{\Delta t}\left(\vec{V}_f - \vec{V}_i\right)$$

The definition implies that average acceleration is a <u>vector</u>. Its construction requires <u>vector subtraction</u>. We can illustrate this with a ball moving upwards under Earth's gravity

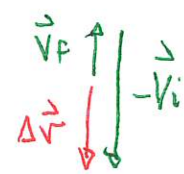


① Construct velocity vectors

$\uparrow \vec{V}_f$ → smaller ⟺ slower

$\uparrow \vec{V}_i$ → larger ⟺ faster.

② subtract to get $\Delta \vec{v} = \vec{V}_f - \vec{V}_i$



③ Direction of average acceleration is same as direction of $\Delta \vec{v}$

$\Delta \vec{v} \downarrow \quad \downarrow \vec{a}_{avg}$

Quiz 4 70% − 95% } 70% − 90%

Acceleration may vary with time and the true definition of acceleration is.

The acceleration of an object is

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt} = \frac{dv_x}{dt}\hat{\imath} + \frac{dv_y}{dt}\hat{\jmath}$$

Important points

1) acceleration is a vector. So

$$\vec{a} = \underbrace{a_x}_{\frac{dv_x}{dt}}\hat{\imath} + \underbrace{a_y}_{\frac{dv_y}{dt}}\hat{\jmath}$$

2) the direction of acceleration is not always aligned with the direction of motion It is determined by $\Delta\vec{v}$.

Motion with constant acceleration

We consider situations where the acceleration vector is constant. Then it is exactly true that, regardless of the time interval

$$\vec{a} = \frac{\Delta\vec{v}}{\Delta t} \Rightarrow \Delta\vec{v} = \vec{a}\Delta t$$

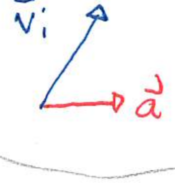
$$\Rightarrow \vec{v}_f - \vec{v}_i = \vec{a}\,\Delta t \Rightarrow$$

If acceleration is constant

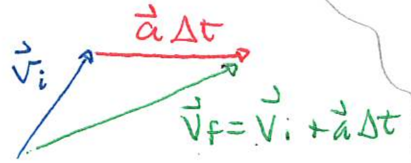
$$\vec{v}_f = \vec{v}_i + \vec{a}\,\Delta t$$

For example:

suppose that at one initial moment



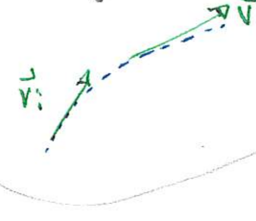
add vectors



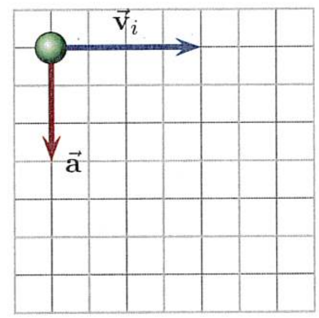
use $\vec{v}_f$ to describe motion

* v_f larger $\Rightarrow$ faster

* direction implies object curves left



108 Constant vertical acceleration



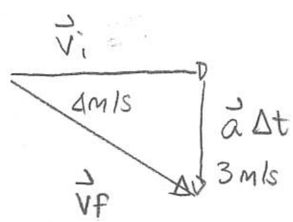
A ball launches off a horizontal surface. At the moment of launch its velocity is $\vec{\mathbf{v}}_i$. At all later times it accelerates with a constant acceleration, $\vec{\mathbf{a}}$. The situation with the vectors drawn to scale is illustrated (for the velocity vector, the grid unit is the standard unit of velocity and for acceleration the grid unit is the standard unit of acceleration). *(131F2024)*

a) Draw, as accurately as possible, the velocity vector, $\vec{\mathbf{v}}_f$, at an instant 1.0 s after the initial instant.

b) Using $\vec{\mathbf{v}}_f$ describe whether the object is moving faster at the 1.0 s instant than at the initial instant.

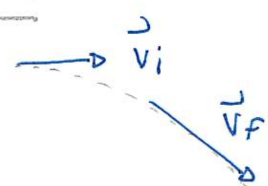
c) Using $\vec{\mathbf{v}}_f$ describe the direction in which the object is moving at the 1.0 s instant.

Answer: a) $\vec{V}_f = \vec{V}_i + \vec{a}\,\Delta t$



b) $V_f = \sqrt{(4\,m/s)^2 + (3\,m/s)^2} = 5\,m/s$ $\Rightarrow$ moves faster

c) angled downward



If the acceleration is constant then we can learn the trajectory from this process. For example

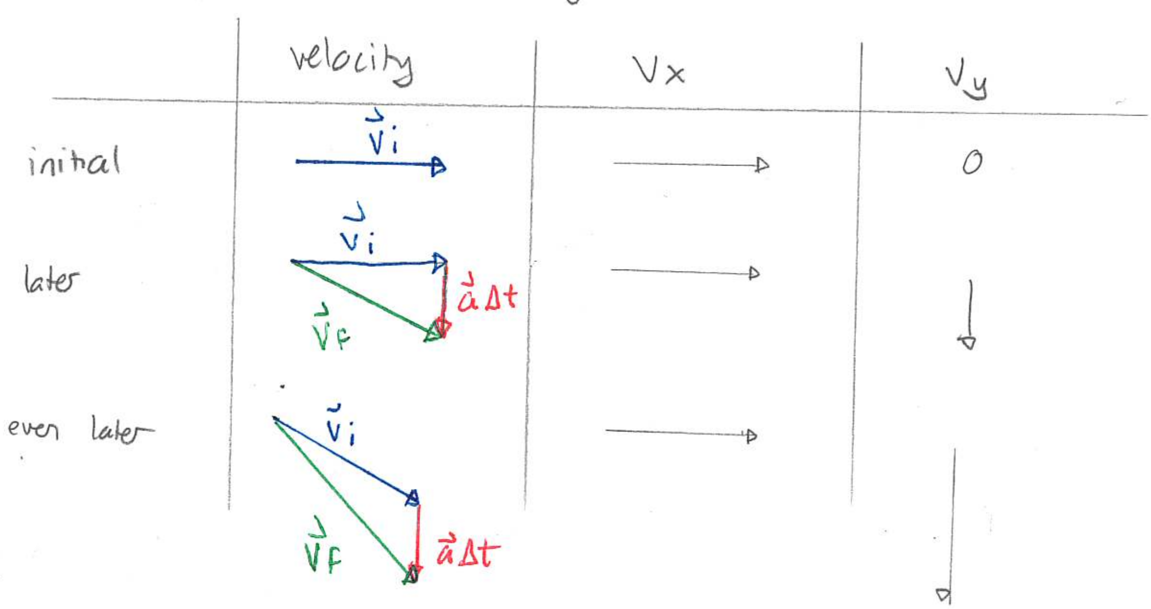
If $\vec{a} = a_y \hat{j}$

then $\vec{V}_f = \vec{V}_i + \vec{a}\,\Delta t \Rightarrow$

$$V_{fx} = V_{ix} + \cancel{a_x \Delta t} \Rightarrow V_{fx} = V_{ix}$$

$$V_{fy} = V_{iy} + a_y \Delta t$$

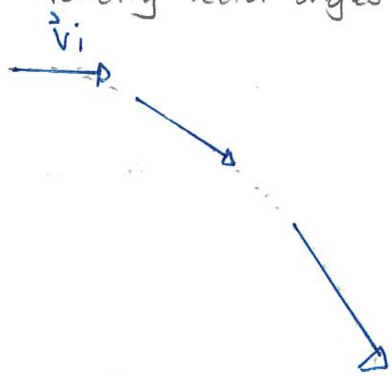
and V_x is constant



velocity vector is larger

$\Rightarrow$ speeds up

velocity vector angles down more $\Rightarrow$ curves down



One can show exactly that the trajectory is a parabola