Phys 131 2025

Lecture 8

Tues: Discussion / Quiz

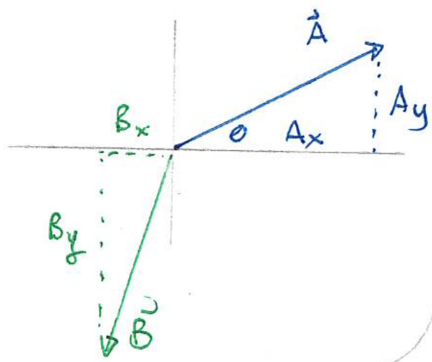
Ex: 78, 80, 81, 82, 89, 92, 93

Weds: ....

Vector algebra with components

Vector algebra can be done using the scheme.

Task: Given $\vec{A}$, $\vec{B}$ and numbers α, β, determine $\vec{D} = \alpha\vec{A} + \beta\vec{B}$



① Determine components of each vector, e.g.

$$A_x = A\cos\theta$$

$$A_y = A\sin\theta$$

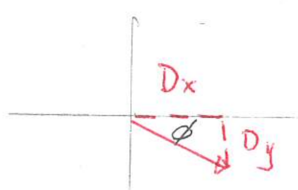
$$\vdots$$

② Calculate components of $\vec{D}$ via

$$D_x = \alpha A_x + \beta B_x$$

$$D_y = \alpha A_y + \beta B_y$$

③ Construct $\vec{D}$ using D_x, D_y



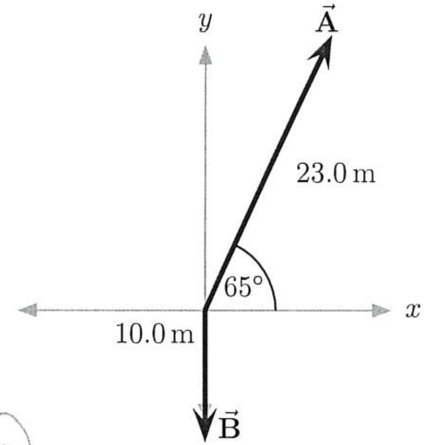
magnitude $D = \sqrt{D_x^2 + D_y^2}$

$$\phi = \arctan(D_y/D_x)$$

Warm Up!

90 Vector addition: algebraic method, 4

Two displacement vectors, $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ are illustrated. [redacted]



a) Determine the components of $\vec{\mathbf{C}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$.

b) Determine the magnitude of $\vec{\mathbf{C}}$.

Answer: Scheme:

Determine components of $\vec{A}, \vec{B}$ → Use these to determine components of $\vec{C}$ → Reconstruct $\vec{C}$ and find its magnitude

a) $C_x = A_x + B_x$

$C_y = A_y + B_y$

Components of $\vec{A}$

$A_x = A\cos 65^\circ = 23.0\,\text{m} \cos 65^\circ \Rightarrow A_x = 9.7\,\text{m}$

$A_y = A\sin 65^\circ = 23.0\,\text{m} \sin 65^\circ \Rightarrow A_y = 20.8\,\text{m}$

Components of B

$B_x = 0\,\text{m}$

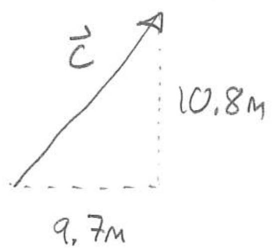
$B_y = -10.0\,\text{m}$

$C_x = 9.7\,\text{m} + 0\,\text{m} = 9.7\,\text{m}$

$C_y = 20.8\,\text{m} - 10.0\,\text{m} = 10.8\,\text{m}$

	x-comp	y-comp
$\vec{A}$	9.7 m	20.8 m
$\vec{B}$	0 m	-10.0 m
$\vec{C}$	9.7 m	10.8 m

b)



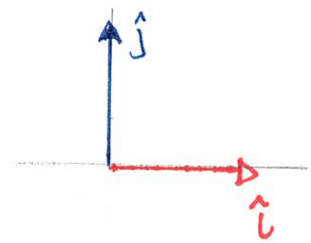
$C = \sqrt{C_x^2 + C_y^2} = \sqrt{(9.7\,\text{m})^2 + (10.8\,\text{m})^2}$

$\Rightarrow \boxed{C = 14.5\,\text{m}}$

Unit vector notation

Vector components allow for accurate vector algebra. But the pair of numbers is not very convenient for algebraic manipulation. We can incorporate this into a simple algebraic system using unit vectors. These are:

$\hat{\imath}$ = unit vector along +x axis
$\hat{\jmath}$ = " " " +y axis
$\hat{k}$ = " " " +z axis



Then

> For any vector in two dimensions
>
> $$\vec{A} = \underbrace{A_x\hat{\imath} + A_y\hat{\jmath}}_{\text{vector}}$$
>
> where A_x = x component of $\vec{A}$, A_y = y component of $\vec{A}$ $\Big]$ NUMBERS (each ~~is~~ positive or negative)

Slide 1
Slide 2
Slide 3

Quiz 1 80% - 100%

↳ Here

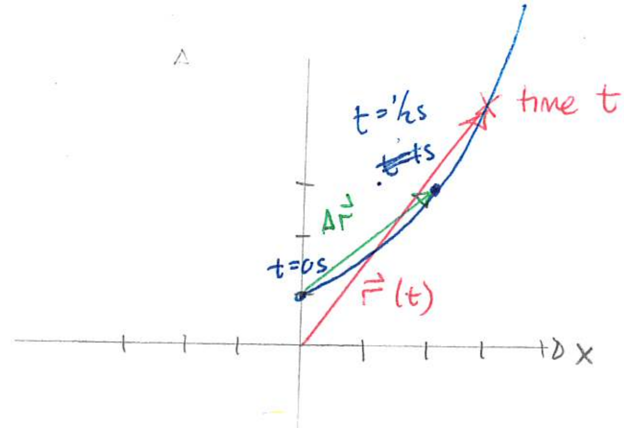
$$\left.\begin{aligned}\vec{A} &= \hat{\imath} + 3\hat{\jmath} \\ \vec{B} &= -\hat{\imath} + 2\hat{\jmath}\end{aligned}\right\}$$

$$\vec{A} - \vec{B} = \hat{\imath} + 3\hat{\jmath} - (-\hat{\imath} + 2\hat{\jmath})$$
$$= \hat{\imath} + 3\hat{\jmath} + \hat{\imath} - 2\hat{\jmath} = 2\hat{\imath} + \hat{\jmath}$$

Warm Up 2

Kinematics in Two dimensions

Consider an object moving in the xy plane. We can chart its trajectory or path, parameterizing (marking) instants along the path at various times. We illustrate this with an example:



$x(t)$ = horizontal position

$y(t)$ = vertical position

Example Position represent via:

horizontal: $x(t) = 2\,m/s\; t$

vertical: $y(t) = 8\,m/s^2\, t^2 + 1m$

position vector

$$\vec{r}(t) = x(t)\,\hat{\imath} + y(t)\,\hat{\jmath}$$

$$\vec{r}(t) = 2m/s\, t\,\hat{\imath} + (8m/s^2 t^2 + 1m)\,\hat{\jmath}$$

displacement from time t_i to t_f

Let $x_i = x(t_i)$ $\quad x_f = x(t_f)$

$y_i = y(t_i)$ $\quad y_f = x(t_f)$

$\Delta x = x_f - x_i$

$\Delta y = y_f - y_i$

displacement vector is

$$\Delta\vec{r} = \Delta x\,\hat{\imath} + \Delta y\,\hat{\jmath}$$

e.g. from $t_i = 0s$ to $t_i = \tfrac{1}{2}s$

$$\Delta x = x_f - x_i$$

$$= 2m - 0m = 2m$$

$$\Delta y = y_f - y_i$$

$$3m - 0m = 2m$$

$$\Delta\vec{r} = \Delta x\,\hat{\imath} + \Delta y\,\hat{\jmath} = 2m\,\hat{\imath} + 2m\,\hat{\jmath}$$

Average velocity vector is

$$\vec{V}_{avg} = \frac{\Delta x}{\Delta t}\hat{\imath} + \frac{\Delta y}{\Delta t}\hat{\jmath} = \frac{\Delta\vec{r}}{\Delta t}$$

from $t_i = 0s$ to $t_i = \tfrac{1}{2}s$

$$\vec{V}_{avg} = \frac{2m}{0.5s}\hat{\imath} + \frac{2m}{0.5s}\hat{\jmath}$$

$$= 4m/s\,\hat{\imath} + 4m/s\,\hat{\jmath}$$

Quiz 2 40% – 70%

The instantaneous velocity is a vector

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}\hat{\imath} + \lim_{\Delta t \to 0} \frac{\Delta y}{\Delta t}\hat{\jmath}$$

$$\vec{v} = \frac{dx}{dt}\hat{\imath} + \frac{dy}{dt}\hat{\jmath}$$

In example

$$\frac{dx}{dt} = 2\,m/s$$

$$\frac{dy}{dt} = 16\,m/s^2\,t$$

$$\Rightarrow \vec{v} = 2\,m/s\,\hat{\imath} + 16\,m/s^2\,t\,\hat{\jmath}$$

In general

For motion in two dimensions, velocity is a vector with two components

$$\vec{v} = v_x\hat{\imath} + v_y\hat{\jmath}$$

where

$$v_x = \frac{dx}{dt}$$

$$v_y = \frac{dy}{dt}.$$