Lecture 7

Mon: Warm Up 3 D2L   SPS Meeting

Tues: Discussion quiz

Ex

Vector algebra

So far vectors have been defined as arrows. We now want to convert them into mathematical objects and provide algebraic operations for them. First

Two vectors $\vec{A}$ and $\vec{B}$ are equal, ie $\vec{A}=\vec{B}$	$\Leftrightarrow$	1) $\vec{A}$ and $\vec{B}$ have the same magnitude AND AND 2) $\vec{A}$ and B have " " direction

Quiz1 80% – 95% } 50% – 90%

Vector addition

One can add two vectors using the idea:

Given two vectors $\vec{A}, \vec{B}$ → Sum of $\vec{A}$ and $\vec{B}$: one vector attained by successive displacements

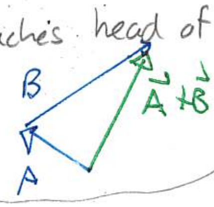
DEMO: PhET Vector Addition

→ Explore 2D → Add two vectors

We do this using:

Given: Two vectors

→ Process: Move $\vec{B}$ parallel to itself so tail of $\vec{B}$ touches head of $\vec{A}$



→ Sum: Single displacement from tail of $\vec{A}$ to head of $\vec{B}$

Quiz2 70% – 95% } 50 – 90%

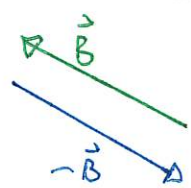
The example shows that

> The magnitude of the sum of two vectors is not necessarily the sum of the magnitudes. So given $\vec{A}, \vec{B}$ and
>
> $\underbrace{\vec{C} = \vec{A} + \vec{B}}_{\text{vectors}}$ it is <u>possible</u> that $\underbrace{C \neq A + B}_{\text{magnitudes.}}$

Vector subtraction

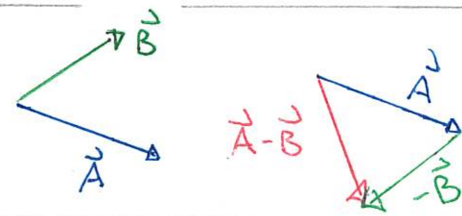
Subtraction is the inverse operation to addition. Suppose that one has a vector $\vec{B}$. Then the vector $-\vec{B}$ is the vector so that $\vec{B} + (-\vec{B}) = 0$. Thus

> If $\vec{B}$ is a vector then $-\vec{B}$ is a vector with * magnitude the same as $\vec{B}$
> * direction opposite to $\vec{B}$



Then subtraction is done via:

> For any vectors $\vec{A}, \vec{B}$
>
> $\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$



Quiz3 50%–80% § 60%–90%

Scalar multiplication

Suppose we repeatedly added the same vector, e.g. $\vec{A} + \vec{A} + \vec{A} + \vec{A}$. It would seem sensible to write this as $4\vec{A}$. So we define scalar multiplication.

> Let $\vec{A}$ be any vector and c any number. Then $c\vec{A}$ is a vector with
>
> 1) magnitude $|c|\vec{A}$
>
> 2) direction $= \begin{cases} \text{same as } \vec{A} & \text{if } c > 0 \\ \text{opposite to } \vec{A} & \text{if } c < 0 \end{cases}$

Vector components

The pictorial method for representing and manipulating vectors is not suitable for mathematics, particularly proofs. We will represent vectors using a scheme:

Given a vector $\vec{A}$ in two dimensions $\longrightarrow$ Associate two real numbers with $\vec{A}$. These are A_x, A_y

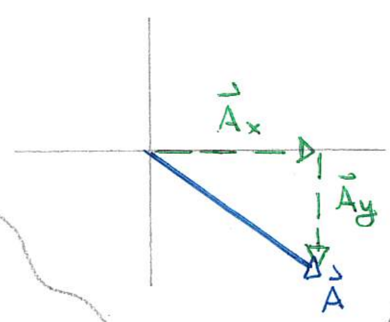
DEMO: PhET Vector addition $\rightarrow$ Explore 2D $\rightarrow$ Single vector

* component vectors
* component numbers

We associate the numbers using

Given a vector $\vec{A}$ decompose this into two vectors

* one parallel to x
* one " to y



$\rightarrow$ The components of $\vec{A}$ are two numbers:

Horizontal component A_x

$$A_x = \begin{cases} + \text{magnitude of } \vec{A}_y & \text{if } \vec{A}_x \rightarrow \\ - \text{magnitude of } \vec{A}_y & \text{if } \vec{A}_x \leftarrow \end{cases}$$

Vertical component

$$A_y = \begin{cases} + \text{magnitude of } \vec{A}_y & \text{if } \hat{A}_y \uparrow \\ - \quad " \quad " \ \vec{A}_y & \text{if } \hat{A}_y \downarrow \end{cases}$$

For every vector $\vec{A}$ there is one pair of numbers A_x, A_y that completely describes the vector

Quiz 4

Components are designed to facilitate vector algebra. So if

$\vec{S} = \vec{A} + \vec{B} + \vec{C} + \ldots$ then

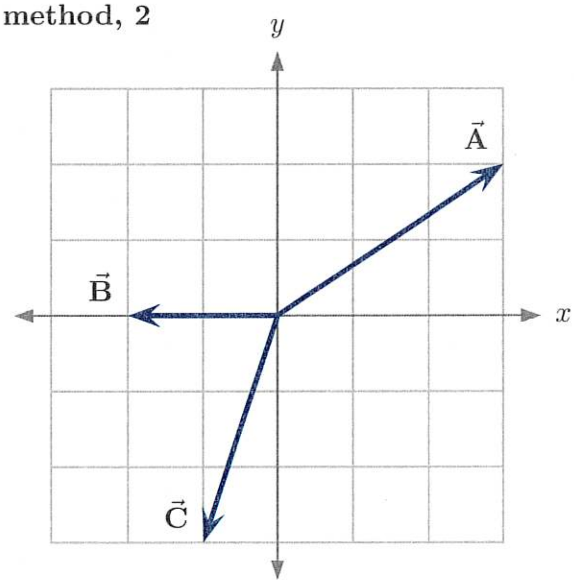
$$S_x = A_x + B_x + C_x + \ldots$$
$$S_y = A_y + B_y + C_y + \ldots$$

$\vec{S} = c\vec{A}$ then

$$S_x = c A_x$$
$$S_y = c A_y$$

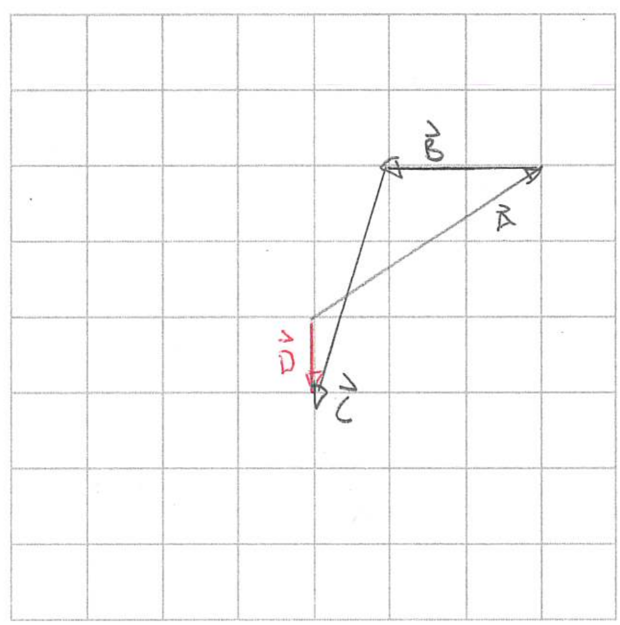
86 Vector addition: graphical and algebraic method, 2

Displacement vectors, $\vec{\mathbf{A}}, \vec{\mathbf{B}}$, and $\vec{\mathbf{C}}$ are illustrated. Let $\vec{\mathbf{D}} = \vec{\mathbf{A}} + \vec{\mathbf{B}} + \vec{\mathbf{C}}$. *(131F2025)*



a) Using the graph sheet below, determine $\vec{\mathbf{D}}$ graphically via the head–to–tail method. Use the result to determine the magnitude of $\vec{\mathbf{D}}$.

b) List the horizontal and vertical components of each of $\vec{\mathbf{A}}$, $\vec{\mathbf{B}}$, and $\vec{\mathbf{C}}$ and use these components to determine the components of $\vec{\mathbf{D}}$. Use the result to determine the magnitude of $\vec{\mathbf{D}}$.



b) $A_x = 3$ $\quad A_y = 2$ $\quad D_x = A_x + B_x + C_x = 3 - 2 - 1 = 0$

$B_x = -2$ $\quad B_y = 0$ $\quad D_y = A_y + B_y + C_y = 2 + 0 - 3 = -1$

$C_x = -1$ $\quad C_y = -3$

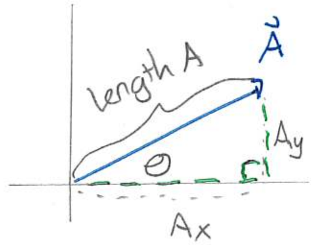
$\vec{c}$ magnitude $= 1$

Calculating vector components

We can calculate vector components from magnitude and direction information

Given vector $\vec{A}$

* magnitude A
* some angle from an axis



* let A = magnitude of vector (hypotenuse)

Then

$$A_x = A\cos\theta$$

$$A_y = A\sin\theta$$

if θ is angle c.c.w from $+x$ axis

Vector algebra

The mathematical scheme for vector algebra is.

Task: Given $\vec{A}, \vec{B}$ vectors

α, β numbers

want $\vec{D} = \alpha\vec{A} + \beta\vec{B}$

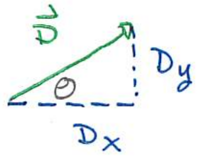
$\rightarrow$ Get components of $\vec{A} \rightarrow A_x, A_y$

Get " " $\vec{B} \rightarrow B_x, B_y$

$$D_x = \alpha A_x + \beta B_x$$

$$D_y = \alpha A_y + \beta B_y$$

Reconstruct $\vec{D}$ from D_x, D_y



Magnitude of $\vec{D}$ is

$$D = \sqrt{D_x^2 + D_y^2}$$

Direction described by θ e.g.

$$\theta = \arctan\left(\frac{D_y}{D_x}\right)$$