Tues: Discussion / quiz

Ex 39, 41, 42, 45, 48, 49, 53, 57

- complete before + bring to class
- do not turn in
- quiz 10min ~~like~~ ~~one~~ question like one of these → 5pts

Weds - ~~Class~~ Group Exercise for credit → 3pts
Fri: HW 5pm

Acceleration framework

Conceptual Idea: Acceleration is rate of change of velocity

Preliminary definition: Observe an object at two instants. The average acceleration over the interval is

$$a_{avg} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i}$$

t_f	time	t_i
v_f	velocity	v_i

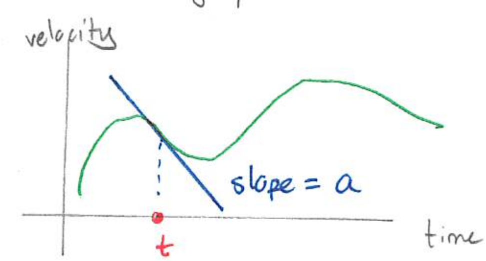
Definition: The (instantaneous) acceleration at time t is the limit as $\Delta t \to 0$ of the average acceleration over the interval $t \to t+\Delta t$. Then

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$$

units: m/s^2

Calculation: Given a formula for $v = v(t)$: acceleration = derivative of v w.r.t. t

Given a graph of v vs t



acceleration = slope of tangent to v vs t

Quiz 1 75% - 95% ↝ 95% - 100%

Quiz 2 40% - 75% ↝ 40% - 70%

Quiz 3

Quiz 4

Motion with constant acceleration

We will encounter many situations where acceleration is constant. For example:

* an object sliding down a ramp
* an object in free fall

In such case we want to relate kinematical quantities at two instants

initial (earlier)		final (later)
t_i	time	t_f
x_i	position	x_f
v_i	velocity	v_f

Physics can provide acceleration $\vec{a}$

E.g. know initial quantities t_i, x_i, v_i

Want quantities at t_f: x_f, v_f

Variants of this process include situations:

* where we know all later information and want earlier informations
* mixtures of earlier and later information

In general, only the elapsed time matters. So let

$$\Delta t = t_f - t_i$$

Warm Up 1

Warm up 2

It is not always true that $\Delta x = v_i \Delta t$. This requires acceleration $=0$. In general.

> If an object moves in one dimension with constant acceleration, then
>
> $$v_f = v_i + a\Delta t$$
>
> $$x_f = x_i + v_i \Delta t + \tfrac{1}{2} a \Delta t^2$$
>
> $$v_f^2 = v_i^2 + 2a\,\Delta x = v_i^2 + 2a(x_f - x_i)$$
>
> KINEMATIC EQUATIONS

Proofs: First, consider $v_f = v_i + a\Delta t$.

If the acceleration is constant then $a = \frac{\Delta v}{\Delta t}$

$$\Rightarrow a\Delta t = \Delta v \Rightarrow \Delta v = a\Delta t$$

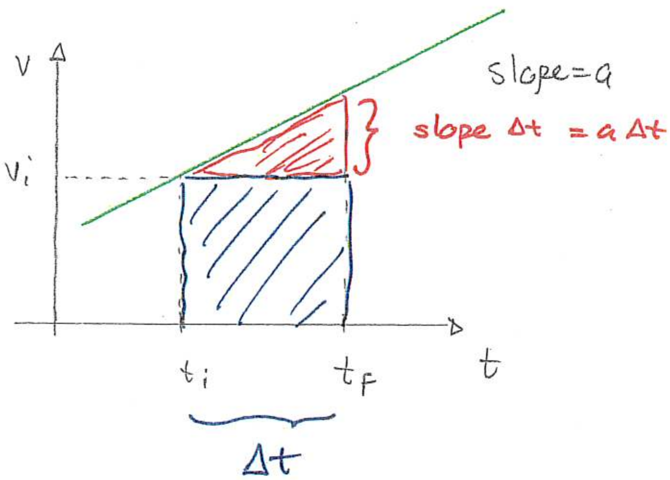
$$\Rightarrow v_f - v_i = a\Delta t \Rightarrow v_f = v_i + a\Delta t \checkmark\checkmark$$

Second, consider $x_f = x_i + \ldots$ We can use a graphical proof

Then Δx = area under v vs t

= area blue rectangle

+ area triangle



Area blue rectangle $= v_i(t_f - t_i)$

$= v_i \Delta t$

Area red triangle $= \frac{1}{2} b h$

$$= \frac{1}{2}\Delta t(a\Delta t) = \frac{1}{2}a(\Delta t)^2$$

Thus $\Delta x = v_i\Delta t + \frac{1}{2}a(\Delta t)^2$

$$\Rightarrow x_f - x_i = v_i\Delta t + \frac{1}{2}a(\Delta t)^2 \Rightarrow x_f = x_i + v_i\Delta t + \frac{1}{2}a(\Delta t)^2 \checkmark\checkmark$$

Third $v_f^2 = v_i^2 \ldots$

From $v_f - v_i = a\Delta t \Rightarrow \Delta t = \frac{v_f - v_i}{a}$

Then $\Delta x = v_i\Delta t + \frac{1}{2}a(\Delta t)^2$

$$= v_i\left(\frac{v_f - v_i}{a}\right) + \frac{1}{2}a\frac{(v_f - v_i)^2}{a^2}$$

$$\Rightarrow a\Delta x = v_i(v_f - v_i) + \frac{1}{2}(v_f - v_i)^2$$

$$\Rightarrow 2a\Delta x = 2v_iv_f - 2v_i^2 + v_f^2 + v_i^2 - 2v_fv_i = v_f^2 - v_i^2 \checkmark\checkmark$$

50 Person moving with constant acceleration

A person is initially at rest and subsequently moves right with a constant acceleration. The person's reaches speed 6.0 m/s at a point 9.0 m to the right of the starting location. The aim of this exercise will be to determine the time taken to reach this point. A first step will be to determine the acceleration of the person.

a) Sketch the situation, illustrating the person at the two instants described above. List all relevant variables for the two instants:

$$t_\mathrm{i} = \qquad\qquad t_\mathrm{f} =$$
$$x_\mathrm{i} = \qquad\qquad x_\mathrm{f} =$$
$$v_\mathrm{i} = \qquad\qquad v_\mathrm{f} =$$

b) Determine the acceleration by selecting one of the kinematic equations, substituting and solving for a.

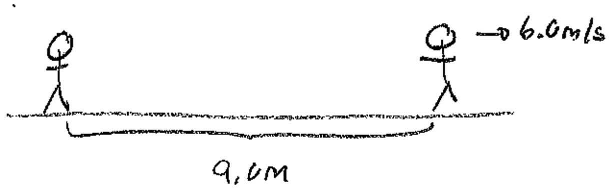
c) Using a different kinematic equation, find the time that it takes the person to reach speed 6.0 m/s.

d) Suppose that you had tried to find the time taken to reach speed 6.0 m/s by using

$$v = \frac{\Delta x}{\Delta t} \Rightarrow 6.0\,\mathrm{m/s} = \frac{9.0\,\mathrm{m}}{\Delta t}.$$

What time does this give? Does it agree with the answer that you obtained to the previous part? Is it correct?

Answer: a)



$t_i = 0s \qquad t_f =$

$x_i = 0m \qquad x_f = 9.0m$

$v_i = 0m/s \qquad v_f = 6.0m/s$

$a = ?$

b) $v_f^2 = v_i^2 + 2a(x_f - x_i) \Rightarrow (6.0m/s)^2 = (0m/s)^2 + 2a\,9.0m$

$\Rightarrow 36\,m^2/s^2 = 18m\,a \Rightarrow \boxed{a = 2.0m/s^2}$

c) $v_f = v_i + a\Delta t \Rightarrow 6.0m/s = 0m/s + 2.0m/s^2\Delta t$

$\Rightarrow \Delta t = 3.0s$

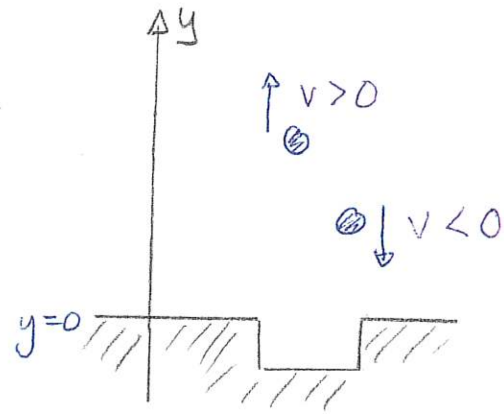
d) that would give $\Delta t = \frac{9.0m}{6.0m/s} = 1.5s$ NO

Vertical motion

Although we described kinematics for horizontal motion, the same system applies for vertical motion with these modifications:

1) position variable : y
2) velocity $v > 0 \Rightarrow$ moves up
   $v < 0 \Rightarrow$ moves down

The kinematic equations then apply with x replaced by y



Free fall

An example of vertical motion is

Free fall motion $\Rightarrow$ vertical motion (up or down) only under the influence of Earth's gravity.

Longstanding questions about this are:

1) Does the motion depend on the mass of the object?
2) Does the object accelerate constantly or not?
3) Does the acceleration depend on the state of motion (up/down / faster/slower)?

DEMO: Free fall demo