Phys 131 2025

Lecture 3

Fri: HW 3 by 5pm 12pts each

Ex: 10, 13, 18, 20, 24, 26, 30, 34, 35

* questions on course website
* turn in answers on separate sheets of paper to my office/envelope.
* Use of resources
  - Center for Academic Support
  - Office Hours
  - use of AI
* grading

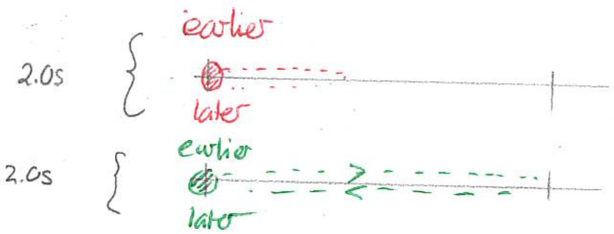
Mon: Warm Up 2 D2L

Group exercise (not graded)

Velocity

We had defined the average velocity as the rate of change of position over an entire time interval. This will not capture all the details of motion during that interval.

We need to refine this idea of average velocity to reach an idea of velocity at one instant.



Same average velocity = 0 m/s.

DEMO: PhET Moving Man → Charts

set $x_0 = +10m$

$v_0 = -6$

$a = 2$

We ask what will the velocity be at 3.0s We can arrive at an idea by computing the average over smaller intervals

4.00s → 5.00s ⇝ calculate v_{avg}

4.00s → 4.50s ⇝ " v_{avg}

DEMO: Show slides

The data illustrates that:

* as Δt decreases, Δx decreases $\to 0$
* as Δt decreases $\to 0$, $v_{avg} = \frac{\Delta x}{\Delta t} \to 2.00\,m/s$.

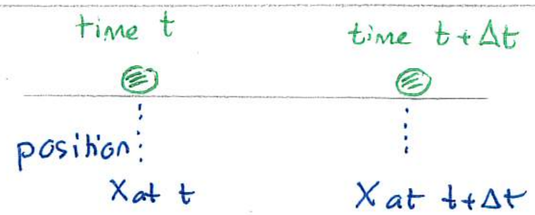
That would give a representation of the instantaneous velocity at 4.0s.

This motives the scheme:

Concept (Instantaneous) Velocity ~ rate at which position changes at one instant

Definition

The (instantaneous) velocity of an object at time t is the limiting value of the average velocity over the interval $t \to t+\Delta t$, as $\Delta t \to 0$



$$V = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{X(\text{at } t+\Delta t) - X(\text{at } t)}{\Delta t}$$

units: m/s

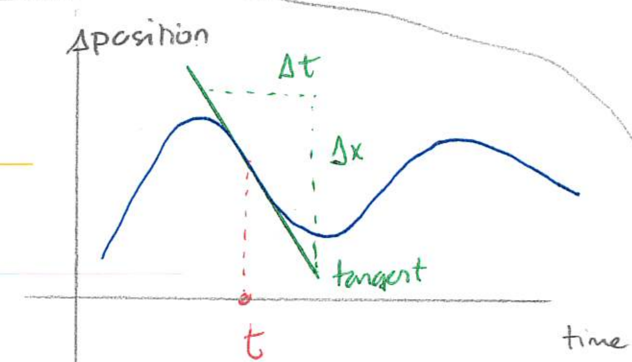
How can one calculate velocity? Some possibilities are:

1) use data for position versus time and hope for limit as $\Delta t \to 0$
2) use calculus and this require x as a function of t.

Using calculus one can show

Given a graph of position versus time one can get velocity at time t by:

* draw tangent at t
* velocity = slope of tangent.



One more definition is:

$$\boxed{\text{(Instantaneous) speed} = s = \text{magnitude of velocity}}$$

Warm Up!

Notice that there are two pieces of information to velocity

1) magnitude (size) $\Rightarrow$ speed
2) sign $\Rightarrow$ direction of travel

$v > 0 \Rightarrow$ travels right $\qquad v>0$

$v < 0 \Rightarrow$ " left $\qquad v<0$

Quiz 2 95% $\lesssim$ 95%

Position from velocity

Suppose that we are given velocity information and want to convert this to position information. We cannot determine absolute position but we can determine changes in position.

FOR UNIFORM MOTION ONLY

Given velocity v $\rightsquigarrow$ change in position over time interval Δt is $\Delta x = v \Delta t$

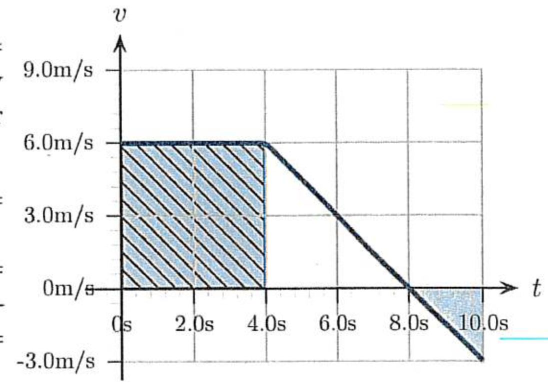
We can get the displacement Δx, rather than x. Note that

$$\boxed{\text{IF VELOCITY IS NOT CONSTANT} \quad \Delta x \neq v \Delta t}$$

For non-constant velocities we can use areas.

31 Crawling slug

A slug crawls along a straight wire, starting at $x = 0.0\,\text{m}$ at $t = 0.0\,\text{s}$. A graph of the slug's velocity versus time is illustrated. Use the graph to answer the following. *(131Sp2025)*



a) Determine the displacement of the slug from $t = 0.0\,\text{s}$ to $t = 4.0\,\text{s}$.

b) How is the displacement of the slug from $t = 0.0\,\text{s}$ to $t = 4.0\,\text{s}$ related to the shaded area *between the graph and the horizontal axis ($v = 0.0\,m/s$)*?

c) Assuming that the answer to the previous question is true in general, determine the displacement of the slug from $t = 4.0\,\text{s}$ to $t = 8.0\,\text{s}$.

d) Is the displacement of the slug from $t = 8.0\,\text{s}$ to $t = 10.0\,\text{s}$ positive or negative? How might this relate to the shaded area from $t = 8.0\,\text{s}$ to $t = 10.0\,\text{s}$?

Answer:

a) v is constant so

$\Delta x = v \Delta t$ = 6.0m/s × 4.0s = 24m

b) area = 6.0m/s × 4.0s = 24m. It's the same.

c) area = $\frac{1}{2}$ × base × height = $\frac{1}{2}$ × 4.0s × 6.0s = 12m.

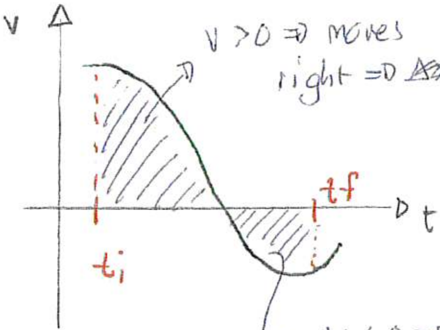
d) It moves left since $v < 0$. So $\Delta x < 0$

area = $\frac{1}{2}$ × base × height = $\frac{1}{2}$ × 2.0s × +3.0m/s = 3 m

thus $\Delta x = -3$ m

In general

Given a graph of velocity versus time



$v > 0 \Rightarrow$ moves right $\Rightarrow$ ~~Δx~~ positive area

$v < 0 \Rightarrow$ moves left $\Rightarrow$ negative area

$\Rightarrow$ Displacement from t_i to t_f is

$$\boxed{\Delta x = \text{area between graph and } t \text{ axis from } t_i \text{ to } t_f}$$

Calculating velocity from position

We can calculate velocity precisely given an equation (or function) for position versus time: Calculus gives

$$\boxed{v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} \equiv \text{"derivative of } x \text{ with respect to } t\text{"}}$$

Calculus also provides rules to determine derivatives For polynomial functions:

$$\boxed{\text{If } \quad x = a t^n \quad \text{where} \quad a, n \text{ are constants then } \frac{dx}{dt} = n a t^{n-1}}$$

38 Velocity as a derivative, 2

Suppose that the position of an object is

$$x = \left(0.25\,\mathrm{m/s^3}\right)\, t^3 + \left(6\,\mathrm{m/s}\right)\, t$$

Determine the velocity of the object at $t = 4\,\mathrm{s}$. *(131Sp2025)*

$V =$ derivative $(0.25\,\mathrm{m/s^3}\ t^3)$ + derivative $(6\,\mathrm{m/s}\ t)$

$= 0.25\,\mathrm{m/s^3} \times 3t^2 + 6\,\mathrm{m/s}$

$= 0.75\,\mathrm{m/s^2}\ t^2 + 6\,\mathrm{m/s}$

At $t = 4\,s$

$V = 0.75\,\mathrm{m/s^2}\,(4\,s)^2 + 6\,\mathrm{m/s}$

$= 18\,\mathrm{m/s}$