Fundamental Mechanics: Class Exam 3

17 April 2025

Name: SOLUTION **Total:** /70

Instructions

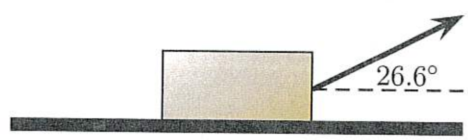
- There are 8 questions on 6 pages.
- Show your reasoning and calculations and always explain your answers.

Physical constants and useful formulae

$$g = 9.80\,\mathrm{m/s^2}$$

Question 1

A crate is dragged along a rough horizontal surface by a rope and moves 4.00 m during a certain period of observation. The rope exerts a 100 N force along the illustrated direction.



a) Determine the work done by the rope during the period of observation.

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$W_{rope} = F \Delta r \cos\theta = 100\,\mathrm{N} \times 4.0\,\mathrm{m} \cos 26.6^\circ$

$= 358\,\mathrm{J}$

b) During the period of observation, the cart moves with constant speed. Determine the work done the kinetic friction force.

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$\Delta K = W_{net}$

perpendicular to motion

$= W_{rope} + W_{grav} + W_n + W_{friction}$

$\Delta K = 0$ since speed const

$0 = W_{rope} + W_{friction} \Rightarrow W_{friction} = -W_{rope}$

$= -358\,\mathrm{J}$

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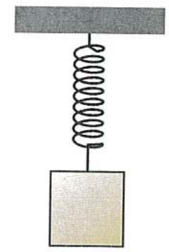
Question 2

A block is suspended from the ceiling by a spring. The block is pulled down, stretching the spring and is held at rest. The block is released, passes the equilibrium point of the spring and later it reaches a high point. Let W_{spring} denote the work done by the spring from the *equilibrium point of the spring to the high point.* Which of the following (choose one) is true?



i) $W_{\text{spring}} = 0.$

ii) $W_{\text{spring}} > 0.$

iii) $W_{\text{spring}} < 0.$

iv) Not enough information.

$\Delta\vec{r}$ $\vec{F}_{spring}$

$W = F\Delta r \cos 180°$

-1

$\Rightarrow$ negative

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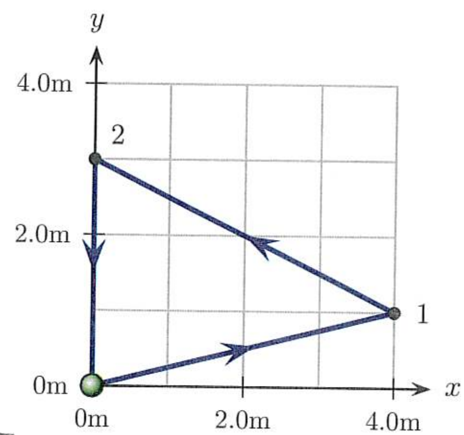
Question 3

A ball moves on a horizontal surface along the indicated path, starting and ending at the origin. A variable force acts on the particle. While it moves from the origin to point 1 and then to point 2, the force is $\mathbf{F} = 20\,\text{N}\,\hat{\imath} + 15\,\text{N}\,\hat{\jmath}$. When it returns from point 2 to the origin, the force changes to $\mathbf{F} = 10\,\text{N}\,\hat{\jmath}$.



a) Determine the work done by $\mathbf{F}$ for the entire path.

From 0→1 $W_{01} = \vec{F}\cdot\Delta\vec{r}$

$\Delta\vec{r} = 4m\hat{\imath} + 1m\hat{\jmath}$

$W_{01} = 4m \times 20N + 1m \times 15N = 95J$

From 1→2 $W_{12} = \vec{F}\cdot\Delta\vec{r}$ $\Delta\vec{r} = -4m\hat{\imath} + 2m\hat{\jmath}$

$\Rightarrow W_{12} = -80Nm + 30Nm = -50J$

From 2→0 $W_{20} = \vec{F}\cdot\Delta\vec{r}$ $\Delta\vec{r} = -3m\hat{\jmath}$

$W_{20} = -30J$ $W = W_{01} + W_{12} + W_{20} = 15J$

b) Is the force conservative? Explain your answer.

No. If it were the work in a closed loop like this would be zero

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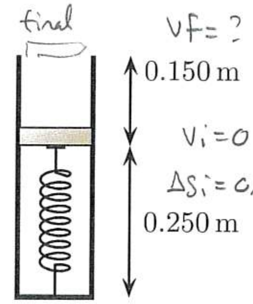
Question 4

A 3.50 kg block can move along a vertical cylinder. The block is held at rest against a spring with spring constant 2000 N/m, compressing it by 0.150 m. At this instant, the base of the block is 0.150 m beneath the top of the cylinder. The block is released and when the base reaches the top of the cylinder, it leaves the spring. While it moves up the cylinder, a constant kinetic friction force of 50.0 N acts on the block. Ignore air resistance.



$v_f = ?$ $y_f = 0.40m$

$v_i = 0$ $y_i = 0.25m$

$\Delta s_i = 0.25m$ $\Delta s_f = 0m$

$\uparrow \Delta\vec{r}$ 0.15m

$\downarrow \vec{f}_k$

a) Determine the speed of the block as it leaves the spring.

$E_f = E_i + W_{nc}$ (+1)

$K_f + U_{gf} + U_{spf} = K_i + U_{gi} + U_{spi} + W_{nc}$

$\frac{1}{2}mv_f^2 + mgy_f + \frac{1}{2}k\Delta s_f^2 = \frac{1}{2}mv_i^2 + mgy_i + \frac{1}{2}k\Delta s_i^2 + W_f$ (+1) (+1) (+1)

$W_f = F\Delta r \cos 180°$ (+1)

$= 50.0N \times 0.15m \times \cos 180°$

$= -7.5J$ (+2)

$\frac{1}{2}mv_f^2 = mgy_i - mgy_f + \frac{1}{2}k\Delta s_i^2 - 7.5J$

$\frac{1}{2}3.50kg\, v_f^2 = mg(y_i - y_f) + \frac{1}{2} 2000N/m \times (0.15m)^2 - 7.5J$

$1.75kg\, v_f^2 = 3.50kg \times 9.8m/s^2 (-0.150m) + 22.5J - 7.5J$ (+2)

$= -5.15J + 22.5J - 7.5J = 9.86J$

$v_f = \sqrt{\frac{9.86J}{1.75kg}} \Rightarrow v_f = 2.37m/s$ +3

b) Determine the maximum height that the block reaches *above the top of the cylinder.*

final

initial

$E_f = E_i$

$K_f + U_{gf} = K_i + U_{gi}$ (+1)

$\frac{1}{2}mv_f^2 + mgy_f = \frac{1}{2}mv_i^2 + mgy_i$

$\Rightarrow mg(y_f - y_i) = \frac{1}{2}mv_i^2$, where $\Delta y = y_f - y_i$

$v_i = 2.37m/s$ $v_f = 0m/s$

$y_i = 0.40m$ $y_f =$

$g\Delta y = v_i^2/2 \Rightarrow \Delta y = v_i^2/2g$ +3

$\Delta y = 0.28m$

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Total energy is conserved $E_f = E_i$

Question 5

$\frac{1}{2}mv_f^2 + mgy_f = \frac{1}{2}mv_i^2 + mgy_i$

Two balls are fired with the same speeds from a bridge above a lake. Ball A is fired horizontally and ball B is is fired at an angle of 30° below the horizontal. Both eventually hit the lake. Which of the following (choose one) is true at the instant just before the balls hit the lake? Ignore air resistance.

i) The speed of ball A is the same as that of B.
ii) The speed of ball A is smaller than that of B.
iii) The speed of ball A is larger than that of B.

$v_f^2 = v_i^2 + 2gy_i$

same for both. ⇒ v_f same for both.

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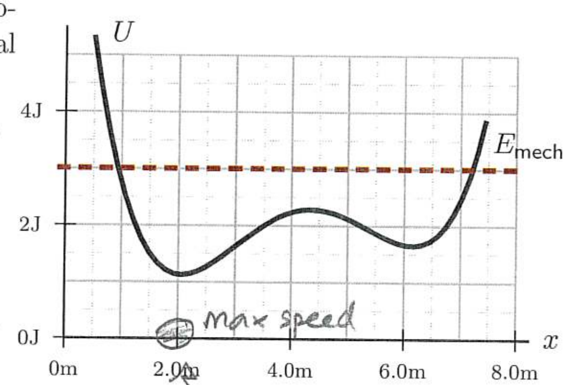
Question 6

A particle with the subject to the illustrated potential, U, (solid line) and has total mechanical energy 3 J.



a) Indicate all locations where the **speed is a maximum**. Explain your answer.

$K + U = E$

$K = E - U$ speed is max when U is min.

The minimum U occurs at about 2.0m ⇒ Max speed at about 2.0m

b) Indicate all locations where the **the force on the particle is zero**. Explain your answer.

$F = -\frac{dU}{dx} = -\text{slope}$

$F = 0$ when slope $= 0$ ≈ 2.0m, 4.25m, 6.25m

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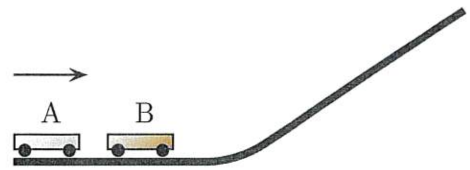
Question 7

Two carts can move along the illustrated frictionless track. Cart A, with mass 0.500 kg moves along the horizontal surface with speed 6.00 m/s toward cart B, with mass 0.750 kg and at rest on the horizontal surface. The carts collide, stick together move along the horizontal surface and up the ramp. Determine the maximum height that they reach along the ramp. Ignore friction and air resistance.



Two stages → ① initial → collision (momentum conserved)
→ ② both move after collision (energy conserved)

In stage ① m_A m_B

$V_{Ai} = 6\,m/s$ $\quad V_{Bi} = 0\,m/s$ $\quad \longrightarrow V_f$

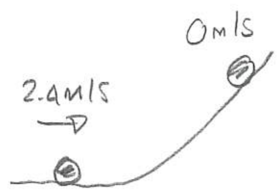
net external force = 0 ⇒ $\vec{p}$ conserved

$p_{tot\,f} = p_{tot\,i} \Rightarrow (m_A + m_B) V_f = m_A V_{Ai} + m_B V_{Bi}$ (+1) (+1)

$\Rightarrow V_f = \frac{m_A}{m_A + m_B} V_f = \frac{0.500\,kg}{1.25\,kg} 6.00\,m/s$ (+5)

$= 2.40\,m/s.$

In stage ②



Non-conservative forces do zero work

$E_f = E_i$ (+1)

$K_f + U_{gf} = K_i + U_{gi}$

$\frac{1}{2} M v_f^2 + \frac{1}{2} M g y_f = \frac{1}{2} M v_i^2 + M g y_i$ (+1) (+1) (+1)

$\Rightarrow g y_f = \frac{1}{2} V_i^2$ ← speed after collision

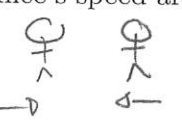
$\Rightarrow y_f = \frac{V_i^2}{2g} = \frac{(2.40\,m/s)^2}{3 \times 9.8\,m/s^2} = 0.294\,m$ /16

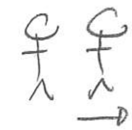
(+5)

Question 8

Two ice skaters, Alice with mass 60 kg and Bob with mass 90 kg, slide toward each other on a sheet of ice. Alice moves right with speed 10 m/s and Bob moves left with speed 5.0 m/s. They collide and Bob subsequently moves right with speed 1.5 m/s. Ignore friction and air resistance.

a) Determine Alice's speed after the collision.





$V_{Ai} = 10 m/s$ $\quad V_{Bi} = -5.0 m/s$ $\quad V_{Bf} = 1.5 m/s$

$p_{tot\,f} = p_{tot\,i}$] or +1

$m_B V_{Bf} + M_A V_{Af} = M_A V_{Ai} + M_B V_{Bi}$ +1

+2

$\Rightarrow 90kg \times 1.5 m/s + 60kg\, V_{Af} = 60kg \times 10 m/s + 90kg \times (-5.0 m/s)$

$\Rightarrow 60kg\, V_{Af} = 600 kg\,m/s - 450 kg\,m/s - 135 kg\,m/s$

$60kg\, V_{Af} = 15 kg\,m/s$

$\Rightarrow V_{Af} = 0.25 m/s$ rest +3

b) Explain whether the total mechanical energy defined as $E = K + U_{\text{grav}}$ is conserved in this collision.

$U_{grav} = 0$ so we just need $E = K$ actually calculate +2

$E_i = E_{Ai} + E_{Bi} = \frac{1}{2} M_A V_{Ai}^2 + \frac{1}{2} M_B V_{Bi}^2 = \frac{1}{2} 60kg \times (10 m/s)^2 + \frac{1}{2} 90 \times (-5.0 m/s)^2$

$= 4125 J$

$E_f = E_{Af} + E_{Bf} = \frac{1}{2} M_A V_{Af}^2 + \frac{1}{2} M_B V_{Bf}^2 = \frac{1}{2} 60kg \times (0.25 m/s)^2 + \frac{1}{2} 90kg \times (1.5 m/s)^2$

$= 103 J$

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Not conserved +1