Fundamental Mechanics: Final Exam (Version 2)

11 December 2024

Name: SOLUTION **Total:** **/139**

Instructions

- There are 17 questions on 10 pages.
- Show your reasoning and calculations and always explain your answers.

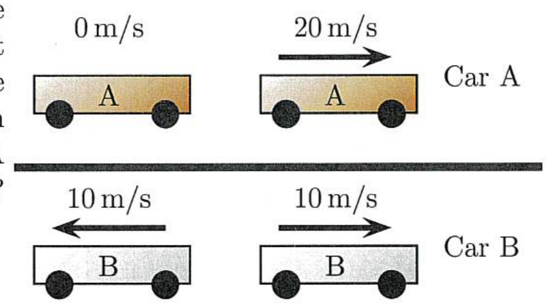
Physical constants and useful formulae

$g = 9.80\,\text{m/s}^2$ $G = 6.67 \times 10^{-11}\,\text{Nm}^2/\text{kg}^2$ Disk/solid cylinder: $I = \frac{1}{2}MR^2$

Hoop/hollow cylinder: $I = MR^2$ Hollow sphere: $I = \frac{2}{3}MR^2$ Solid sphere: $I = \frac{2}{5}MR^2$

✓✓ **Question 1**

Two cars each travel along a horizontal surface. The speed and direction of travel of each car is indicated at the same initial instant (left of diagram) and at the same final instant (right of diagram). The diagram is NOT to scale. Is the average acceleration of car A smaller than, larger than or the same as that of car B? **Explain your answer**



$a = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{\Delta t}$

For Car A $v_f = 20\text{m/s}$ $v_i = 0\text{m/s}$ $\Rightarrow$ $\Delta v = 20\text{m/s}$

Car B $v_f = 10\text{m/s}$ $v_i = -10\text{m/s}$ $\Rightarrow$ $\Delta v = 20\text{m/s}$

same $\Rightarrow$ a same

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✓✓ **Question 2**

A bug walks at a constant speed around the inside of a loop which is oriented vertically. Point A is at the bottom of the loop and point B is at the top. Which of the following (choose one) is true regarding the directions of the acceleration, $\vec{\mathbf{a}}$ of the bug at the two illustrated points?

i) $\vec{\mathbf{a}}$ is $\uparrow$ at A; $\vec{\mathbf{a}}$ is $\uparrow$ at B.

ii) $\vec{\mathbf{a}}$ is $\uparrow$ at A; $\vec{\mathbf{a}}$ is $\downarrow$ at B.

iii) $\vec{\mathbf{a}}$ is $\downarrow$ at A; $\vec{\mathbf{a}}$ is $\uparrow$ at B.

iv) $\vec{\mathbf{a}}$ is $\downarrow$ at A; $\vec{\mathbf{a}}$ is $\downarrow$ at B.

For uniform circular motion acceleration is radially inward

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✓✓ **Question 3**

A person launches a 0.250 kg ball from the ground. The ball leaves the person's hand with speed 20 m/s at an angle of 30° above the horizontal.

a) Determine the time for the ball to reach its maximum height.

$t_i = 0s$ $t_f = ?$

$x_i = 0m$ $x_f =$

$y_i = 0m$ $y_f =$

$V_{ix} = 17m/s$ $V_{fx} =$

$V_{iy} = 10m/s$ $V_{fy} = 0m/s$

$a_x = 0m/s^2$

$a_y = -9.8m/s^2$

$V_{ix} = V_i\cos\theta$

$= 20m/s\cos 30°$

$= 17m/s$ +2

$V_{iy} = V_i\sin\theta$

$= 20m/s\sin 30°$

$= 10m/s$ +2

$V_{fy} = V_{iy} + a_y\Delta t$ +1

$0m/s = 10m/s - 9.8m/s^2\Delta t$

$\Rightarrow \Delta t = \dfrac{-10m/s}{-9.8m/s^2}$

$\boxed{\Delta t = 1.0s}$

+3

b) Determine the velocity (including direction) at the highest point in the ball's trajectory.

At max $\longrightarrow \vec{V}$

$V_{fy} = 0m/s$

$V_{fx} = V_{ix} + \cancel{a_x\Delta t} = 17m/s$

+2

$\xrightarrow{17m/s}$

horizontal

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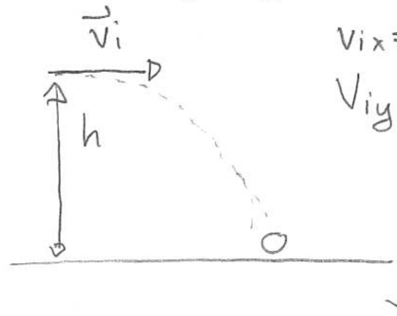
✓✓ **Question 4**

A 0.10 kg red ball and a 0.20 kg blue ball are launched horizontally from the same height above a horizontal floor. The red ball is launched with speed 8.0 m/s and the blue ball with twice the speed, 16.0 m/s. Let Δt_{red} be the time taken for the red ball to hit the floor and Δt_{blue} that for the blue ball to hit the floor. Ignoring air resistance, which of the following (choose one) is true?

i) $\Delta t_{\text{blue}} = \Delta t_{\text{red}}$.
ii) $\Delta t_{\text{blue}} < \Delta t_{\text{red}}$.
iii) $\Delta t_{\text{blue}} > \Delta t_{\text{red}}$.

Explain your answer.



$V_{ix} = V_i$

$V_{iy} = 0\,m/s \quad y_i = h \quad y_f = 0$

$y_f = y_i + \cancel{V_{iy}\Delta t} + \frac{1}{2} a_y \Delta t^2$

$0 = h + \frac{1}{2}(-g)\Delta t^2$

$\Delta t^2 = \frac{2h}{g} \Rightarrow \Delta t = \sqrt{\frac{2h}{g}}$ independent of mass and launch speed

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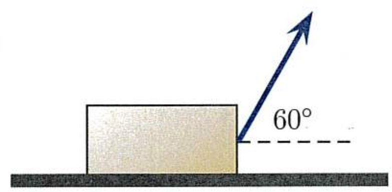
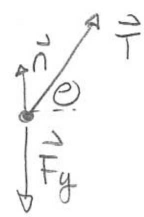
✓✓ **Question 5**

A rope pulls a block, with mass m, along a horizontal frictionless surface. The block moves horizontally while the rope pulls at the illustrated angle. Which of the following (choose one) is true regarding the magnitude of the normal force, n, exerted by the floor on the block?



i) $n = mg$
ii) $n > mg$
iii) $n < mg$



$\sum F_{iy} = ma_y = 0$

$n + T\sin 60° - mg = 0$

$n = mg - T\sin 60° < mg$

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✓✓ **Question 6**

A 0.040 kg ball is initially at rest at the bottom of a frictionless vertical tube of length 1.2 m. The ball is pushed by air which exerts a constant upward force of 1.5 N on the ball. Determine the **acceleration** of the ball while it is in the tube **and the velocity** of the ball when it reaches the top of the tube.

$\sum F_{iy} = ma_y$

$F_{air} - mg = ma_y$

$1.5\,N - 9.8\,m/s^2 \times 0.040\,kg = 0.040\,kg\, a_y$

$1.1\,N = 0.040\,kg\, a_y$

$a_y = 28\,m/s^2$

$V_{fy}^2 = V_{iy}^2 + 2a_y(y_f - y_i)$

$= (0\,m/s)^2 + 2 \times 28\,m/s^2 \times (1.2\,m)$

$= 66\,m^2/s^2$

$\Rightarrow V_{fy} = \sqrt{66\,m^2/s^2}$

$= 8.2\,m/s$

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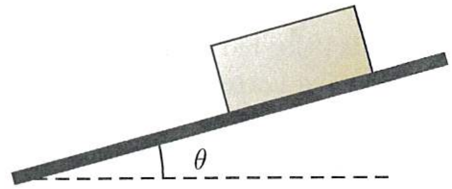
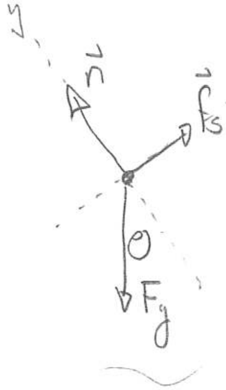
Question 7



A 40 kg box can move along a ramp which can be inclined at variable angles. The coefficient of kinetic friction between the box and the ramp is 0.20 and the coefficient of static friction is 0.30.

a) Determine the largest angle above the horizontal, θ, so that the box remains at rest on the ramp.



$$\sum F_{ix} = ma_x = 0$$
$$\sum F_{iy} = ma_y = 0$$
$$F_g = mg \quad f_s \le \mu_s n$$

	x	y
$\vec{n}$	0	n
$\vec{f_s}$	f_s	0
$\vec{F_g}$	$-mg\sin\theta$	$-mg\cos\theta$

$\sum F_{iy} = 0 \Rightarrow n - mg\cos\theta = 0 \Rightarrow n = mg\cos\theta$

$\sum F_{ix} = 0 \Rightarrow f_s - mg\sin\theta = 0 \Rightarrow f_s = mg\sin\theta$

Then $f_s \le \mu_s n$

$\Rightarrow \cancel{m}\cancel{g}\sin\theta \le \mu_s \cancel{m}\cancel{g}\cos\theta$

$\Rightarrow \dfrac{\sin\theta}{\cos\theta} \le \mu_s$

$\Rightarrow \tan\theta \le \mu_s$

$\Rightarrow \theta \le \arctan(\mu_s)$

$\Rightarrow \theta \le \arctan(0.30)$

$\Rightarrow \theta \le 16.6^\circ \Rightarrow 16.6^\circ$ largest

+3

b) Suppose that the angle of the ramp is increased beyond the maximum of the previous part. Which of the following (choose one) is true?

i) The box remains at rest.

ii) The box slides down the ramp with constant speed.

iii) The box slides down the ramp with constantly increasing speed.

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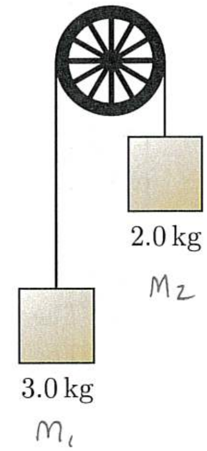
The kinetic friction force is less than static friction $\Rightarrow$ net force down ramp $\Rightarrow$ accelerates $\Rightarrow$ speed increases constantly

Question 8

Two blocks are connected by a massless string that runs over a massless pulley. Ignore air resistance. *In the following, show the entire argument and derivation starting with Newton's Second Law.*



a) Determine the acceleration of the block on the right.

For m_1

$\vec{T}$

$\vec{F}_y$

$\sum F_{iy} = m_1 a_{1y}$

$\Rightarrow T - m_1 g = m_1 a_{1y}$

For m_2

$\vec{T}$

$\vec{F}_y$

$\sum F_{iy} = m_2 a_{2y}$

$\Rightarrow T - m_2 g = m_2 a_{2y}$ (A)

Then $a_{1y} = -a_{2y}$

$\Rightarrow T - m_1 g = -m_1 a_{2y} \Rightarrow T = m_1 g - m_1 a_{2y}$ (B)

Combine (A) and (B) $\Rightarrow m_1 g - m_1 a_{2y} - m_2 g = m_2 a_{2y}$

$\Rightarrow m_1 g - m_2 g = m_1 a_{2y} + m_2 a_{2y} = (m_1 + m_2) a_{2y}$

$$\Rightarrow a_{2y} = \frac{(m_1 - m_2) g}{m_1 + m_2} = \frac{(3.0\,kg - 2.0\,kg)}{3.0\,kg + 2.0\,kg} 9.8\,m/s^2 = 1.96\,m/s^2$$

b) Determine the tension in the string.

+3

Use $T = m_1 g - m_1 a_{2y} = m_1 (g - a_{2y})$

$= 3.0\,kg (9.8\,m/s^2 - 1.96\,m/s^2)$

$T = 24\,N$

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Question 9

Two boxes on a frictionless surface are separated by a compressed spring. The mass of box B is twice the mass of box A. They are held at rest and then released. Which of the following (choose one) is true *at any instant* while the spring extends as they move apart?

Newton's 3rd Law $\Rightarrow$ forces equal.

i) The net force on A equals that on B. They have the same speed.
ii) The net force on A equals that on B. A has a greater speed than B.
iii) The net force on A is larger than on B. A has a greater speed than B.
iv) The net force on A is larger than on B. A has a smaller speed than B.
v) The net force on A is larger than on B. A has the same speed as B.

$P_{tot} = m_A v_A + m_B v_B = 0$

$\Rightarrow v_A = -v_B \frac{m_B}{m_A}$.

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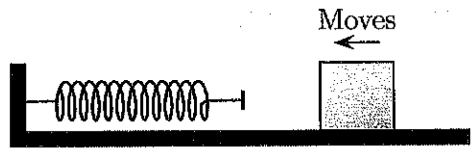
Question 10

A box slides along a rough horizontal surface toward a spring. The box hits the spring and compresses it. *While the spring is being compressed by the block*, let W_{spring}, be the work done by the spring and W_f the work done by friction.



a) Which of the following (choose one) is true *while the spring is being compressed?*

i) $W_{\text{spring}} = 0$
ii) $W_{\text{spring}} > 0$
iii) $W_{\text{spring}} < 0$

Explain your answer.

$\Delta\vec{r}$ ϕ F_{spring}

$W_{sp} = F\Delta r\cos\phi = F\Delta r\cos 180^\circ = -F\Delta r < 0$

b) Which of the following (choose one) is true *while the spring is being compressed?*

i) $W_f = 0$
ii) $W_f > 0$
iii) $W_f < 0$

Explain your answer.

$\Delta\vec{r}$ 180° $\vec{f}_k$ friction

$W_f = f_k\Delta r\cos 180^\circ = -f_k\Delta r < 0$

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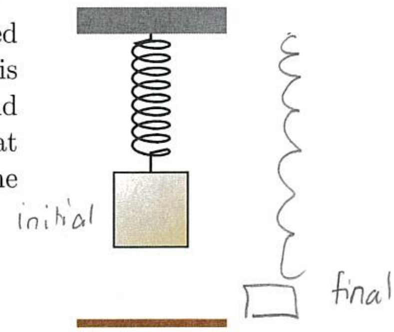
✓✓

Question 11

A block is attached to a spring (spring constant 2000 N/m) that is suspended from the ceiling. The block is initially held at rest so that the spring is stretched by 0.40 m and the bottom of the block is 0.60 m above the ground The block is released and drops toward the Earth. It reverses direction at the moment that it would be about to touch the ground. Determine the mass of the block so that this occurs.



$E_f = E_i$

$y_i = 0.60\,m \qquad y_f = 0\,m$

$\Delta s_i = 0.40\,m \qquad \Delta s_f = 1.0\,m$

$V_i = 0\,m/s \qquad V_f = 0\,m/s$

$K_f + U_{gF} + U_{spf} = K_i + U_{gi} + U_{spi}$

$\cancel{\frac{1}{2} m v_f^2} + \cancel{m g y_f} + \frac{1}{2} k (\Delta s_f)^2$

$= \cancel{\frac{1}{2} m v_i^2} + m g y_i + \frac{1}{2} k (\Delta s_i)^2$

$\frac{1}{2} k (\Delta s_f)^2 = m g y_i + \frac{1}{2} k (\Delta s_i)^2$

$\frac{1}{2} k (\Delta s_f)^2 - \frac{1}{2} k (\Delta s_i)^2 = m g y_i$

$\frac{1}{2}\, 2000\,N/m\,(1.0\,m)^2 - \frac{1}{2}\, 2000\,N/m\,(0.40\,m)^2 = m \times 9.8\,m/s^2 \times 0.60\,m$

$1000\,J - 160\,J = m \times 5.88\,m^2/s^2$

$\Rightarrow 840\,J = 5.88\,m^2/s^2 \times m$

$\Rightarrow m = \frac{840\,J}{5.88\,m^2/s^2} = 140\,kg$

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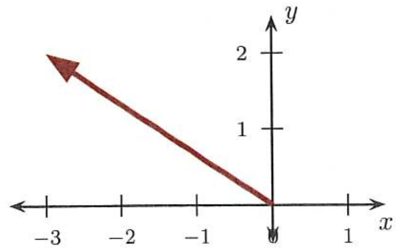
✓✓ **Question 12**

A particle moves along the indicated straight path in the xy plane. While this happens the following two forces act on the particle: $\vec{\mathbf{F}}_{\mathrm{A}} = 2\,\mathrm{N}\hat{\mathbf{i}} + 3\,\mathrm{N}\hat{\mathbf{j}}$ and $\vec{\mathbf{F}}_{\mathrm{B}} = -2\,\mathrm{N}\hat{\mathbf{i}} + 3\,\mathrm{N}\hat{\mathbf{j}}$. Let W_{A} be the work done by $\vec{\mathbf{F}}_{\mathrm{A}}$ and W_{B} be the work done by $\vec{\mathbf{F}}_{\mathrm{B}}$. Which of the following (choose one) is true?



i) $W_{\mathrm{A}} = W_{\mathrm{B}}$.

ii) $W_{\mathrm{A}} > W_{\mathrm{B}}$.

iii) $W_{\mathrm{A}} < W_{\mathrm{B}}$.

$W = \vec{F} \cdot \Delta\vec{r}$ $\quad \Delta\vec{r} = -3\mathrm{m}\hat{i} + 2\mathrm{m}\hat{j}$

$W_A = 2N \times (-3m) + 3N \times 2m = 0J$

$W_B = -2N \times (-3m) + 3N \times 2m = 12J$

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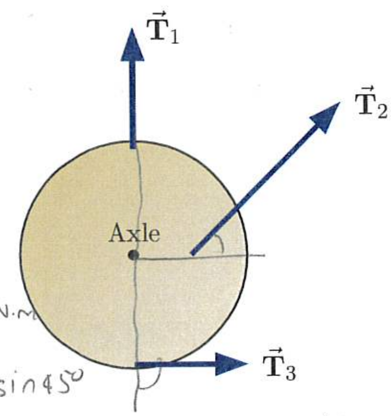
✓✓ **Question 13**

Ropes pull on a disk as indicated. The magnitudes of the tensions are: $T_1 = 10\,\mathrm{N}, T_2 = 15\,\mathrm{N}$, and $T_3 = 8\,\mathrm{N}$. The angle between $\vec{\mathbf{T}}_2$ and the horizontal is 45° and this rope is attached halfway from the axle to the disk edge. Let τ_i be the magnitude of the torque (about the center of the disk) produced by $\vec{\mathbf{T}}_i$. Which of the following (choose one) is true?



i) $\tau_3 < \tau_1 < \tau_2$

ii) $\tau_1 < \tau_3 < \tau_2$

iii) $\tau_2 < \tau_3 < \tau_1$

iv) $\tau_1 < \tau_2 < \tau_3$

$\tau_1 = r_1 F_1 \sin\phi_1 = R T_1 \sin 0^\circ$
$= 0N \cdot m$

$\tau_2 = r_2 F_2 \sin\phi_2 = R/2 \times 15N \times \sin 45^\circ$
$= 5.3N \times R$

$\tau_3 = r_3 F_3 \sin\phi_3 = R \times 8N \times \sin 90^\circ = 8N \times R$

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✓ **Question 14**

A disk in a hard drive has radius 0.0625 m and rotates about an axle at its center, speeding up from rest to 7000 rpm in 0.150 s.

a) Determine the angular acceleration of the disk.

$\omega_f = 7000\,\mathrm{rpm} = 7000\,\frac{\mathrm{rev}}{\mathrm{min}} \frac{1\,\mathrm{min}}{60\,\mathrm{s}} \frac{2\pi\,\mathrm{rad}}{1\,\mathrm{rev}}$
$= 733\,\mathrm{rad/s}$

$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{\omega_f - \cancel{\omega_i}}{0.150\,\mathrm{s}} = \frac{733\,\mathrm{rad/s}}{0.150\,\mathrm{s}} = 4900\,\mathrm{rad/s^2}$

b) Determine the tangential acceleration of a point on the disk midway between the axle and the edge of the disk.

$a_t = \alpha r = 4900\,\mathrm{rad/s^2} \times \left(\frac{0.0625\,\mathrm{m}}{2}\right)$

$= 150\,\mathrm{m/s^2}$

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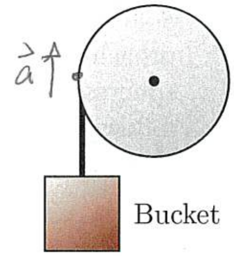
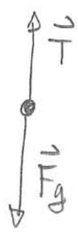
✓✓ **Question 15**

A bucket with mass m_1 is suspended from a massless string that is wrapped around a cylinrical drum. The drum has with radius, R, and uniformly distributed mass, m_2 and rotates about a frictionless axle through its center. The bucket is released from rest and the string moves across the drum without slipping. Determine an expression for the magnitude of the acceleration of the bucket (the only symbols allowed are m_1, m_2, R and g).



Bucket



$\sum F_{iy} = m_1 a_{iy}$

$\Rightarrow T - m_1 g = m_1 a_{1y}$

$T = m_1 g + m_1 a_{1y}$

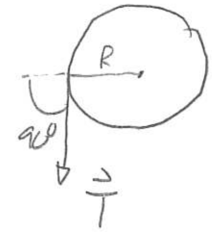
Drum

$\tau_{net} = I\alpha$

$\tau_{string} = \frac{1}{2} M_2 R^2 \alpha$

$\cancel{R} T = \frac{1}{2} m_2 R^{\cancel{2}} \alpha$

$T = \frac{1}{2} M_2 R \alpha$



$\tau_{string} = r F \sin\phi$

$= RT \sin 90°$

$= RT$

Let $\vec{a}_{edge}$ be the acceleration of the left edge of the drum.

Then

$a_{edge\,y} = a_{1y}$

and

$a_{edge\,y} = -\alpha R \Rightarrow a_{1y} = -\alpha R$

$\Rightarrow \alpha = -a_{1y}/R$

Thus $T = m_1 g + m_1 a_{1y}$ $\quad T = -\frac{1}{2} m_2 \cancel{R} \frac{a_{1y}}{\cancel{R}}$

$\Rightarrow -\frac{1}{2} M_2 a_{1y} = m_1 g + m_1 a_{1y}$

$\Rightarrow -m_1 a_{1y} - \frac{1}{2} M_2 a_{1y} = m_1 g$

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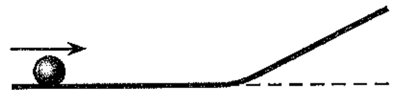
$\Rightarrow a_{1y}(m_1 + \frac{1}{2} M_2) = -M_1 g \Rightarrow a_{1y} = \frac{-M_1}{M_1 + \frac{1}{2} M_2} g$

$\Rightarrow a = \frac{m_1}{m_1 + \frac{1}{2} m_2} g$

√✓ **Question 16**

Solid spheres with various uniformly distributed masses approach on an inclined track with the same speeds. The spheres roll without slipping along the track. Which of the following is true regarding the maximum heights reached by the spheres along the ramp?



i) A heavier sphere reaches the same maximum height as a lighter sphere.
ii) A heavier sphere reaches a higher maximum height than a lighter sphere.
iii) A heavier sphere reaches a lower maximum height than a lighter sphere.

Explain your answer.

$$E_f = E_i$$

$$\cancel{K_{trans\,f}} + \cancel{K_{rot\,f}} + U_{gf} = K_{trans\,i} + K_{rot\,i} + \cancel{U_{gi}}$$

$$\cancel{m} g y_f = \frac{1}{2} m v_i^2 + \frac{1}{2} I \omega_i^2$$

$$= \frac{1}{2}\cancel{m} v_i^2 + \frac{1}{2}\frac{2}{5}\cancel{m} R^2 \omega_i^2$$

$$\rightarrow g y_f = \frac{1}{2} v_i^2 + \frac{1}{5} v_i^2$$

mass and radius do not affect the max height

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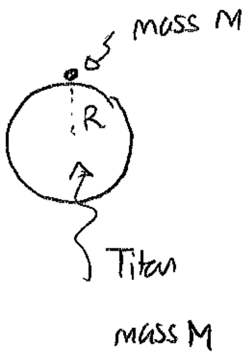
Question 17

Titan is a moon of the planet Saturn. Titan has mass 1.35×10^{23} kg and radius 2.57×10^{6} m. Determine the acceleration due to Titan's gravity at the surface of Titan. **Note: to receive full credit for this problem, your solution must with Newton's second law and use this to derive the answer.**



$$\sum F_{iy} = m a_y$$

$$G\frac{\cancel{m} M}{R^2} = \cancel{m} a_y$$

$$a_y = G\frac{M}{R^2}$$

$$= 6.67 \times 10^{-11}\,\mathrm{Nm^2/kg^2} \times \frac{1.35 \times 10^{23}\,\mathrm{kg}}{(2.57 \times 10^{6}\,\mathrm{m})^2}$$

$$= 1.36\,\mathrm{m/s^2}$$

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