Intermediate Laboratory: Homework 4

Due: 5 February 2024

Read chapter 4 and complete the following problems.

1 Taylor, *Error Analysis, 2nd ed.*, 4.2, page 111. *(4.2 in 3rd ed.)*

2 Taylor, *Error Analysis, 2nd ed.*, 4.6, page 111. *(4.8 in 3rd ed.)*

3 Taylor, *Error Analysis, 2nd ed.*, 4.16, page 114. *(4.18 in 3rd ed.)*

4 Taylor, *Error Analysis, 2nd ed.*, 4.18, page 115. *(4.20 in 3rd ed.)*

5 Taylor, *Error Analysis, 2nd ed.*, 4.20, page 115. *(4.22 in 3rd ed.)*

6 Taylor, *Error Analysis, 2nd ed.*, 4.24, page 116. *(4.26 in 3rd ed.)*

7 Statistical Analysis of Random Numbers

Excel has a random number generator and we can use this to explore the statistics of sampling distributions. The Excel function RAND() returns a number between $0 \leqslant x \leqslant 1$ and it is claimed that the distribution which governs this is uniform (i.e. each number is equally likely). One can show that the (true) mean of this distribution is $\frac{1}{2}$ and the (true) standard deviation is $\frac{1}{\sqrt{12}}$.

Set up an Excel spreadsheet. Create a column of 20 randomly generated numbers; this forms one "data set." Note that Excel will update this, changing the numbers, when you hit any return on nearly any cell. In order to preserve the original numbers copy them into a new column using the "values" option when you paste.

a) Determine the sample average of the numbers in the single data set and their standard deviation. Are these similar to the true mean and true standard deviation?

Repeat the above procedure 15 times (copy the original column). You will have 15 data sets.

b) For each data set obtain the sample average and standard deviation (call this σ). Do these fluctuate from one data set to another? If so, what are their ranges?

c) Now consider the 15 sample averages as a single set of data. Determine the average and standard deviation (call this σ_{averages}) of all of these sample averages. Does the true mean lie within this range? How does σ_{averages} compare to the various σ that each data set yielded?

d) For each data set, determine the standard deviation of the mean (call this σ_{mean}). How does it compare to σ_{averages}? Which of σ or σ_{mean} more accurately reflects the fluctuations in the sample averages, σ_{averages}?

e) Treat all of the data points as one set. Determine the average and standard deviation of these. Is the standard deviation over the 300 data points much different to that over any single set of 20 data points?

f) Which better reflects the uncertainty in a sample average: the standard deviation or the standard deviation of the mean?