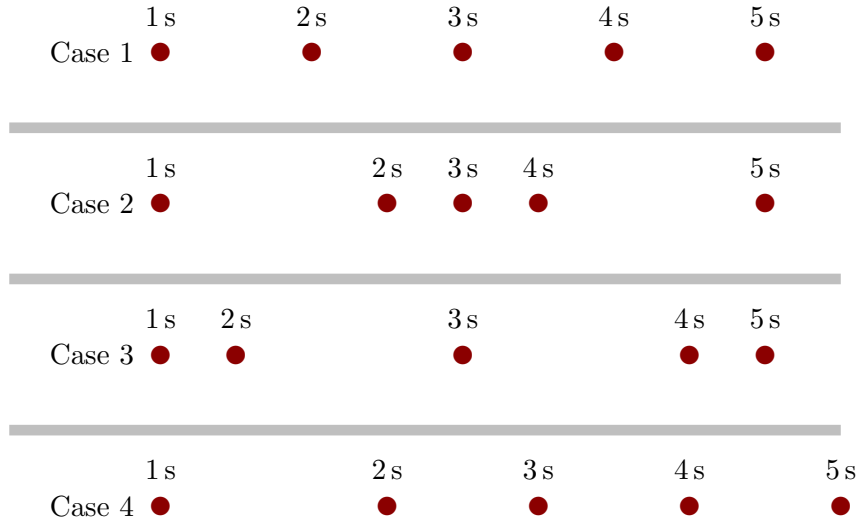


Phys 131: Exercises

One-Dimensional Kinematics

1 Motion diagrams: car slowing and accelerating

A car moves to the right. Earlier in its motion it slows down and later it speeds up. Which of the following (choose one) best represents its location as time passes? (131F2024)



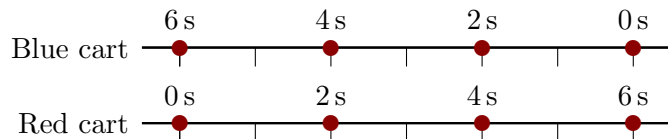
Briefly explain your choice.

2 Drone motion diagram

A drone (unmanned aircraft) hovers at one height above the ground. The drone is subsequently allowed to descend in a straight line toward the ground. It initially descends at an increasing rate, then at a steady rate and, a few feet before the ground, the descent rate decreases. Draw a motion diagram for the drone while it descends. (131F2024)

3 Motion diagrams: racing carts

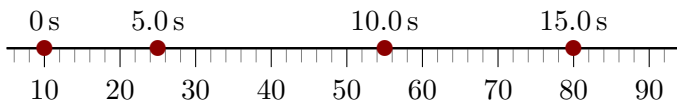
Motion diagrams for two carts are as illustrated. (131F2024)



- Is the speed of the red cart the same as or different to that of the blue cart? Explain your answer.
- Is the velocity of the red cart the same as or different to that of the blue cart? Explain your answer.

4 Motion diagrams and average velocity

A car moves from left to right and its position, measured in meters, is recorded every 5.0 s. The resulting motion diagram is illustrated.

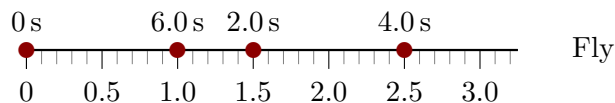


(131F2024)

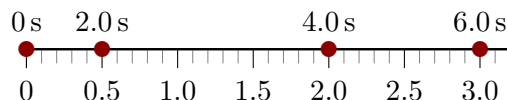
- Determine the average velocity from 0 s to 5.0 s.
- Determine the average velocity from 5.0 s to 10.0 s.
- Determine the average velocity from 5.0 s to 15.0 s.
- Determine the average velocity from 0 s to 15.0 s. Is this the average of the two velocities in the interval $0\text{ s} \rightarrow 5.0\text{ s}$ and $5.0\text{ s} \rightarrow 15.0\text{ s}$?

5 Motion diagrams, speed and average velocity

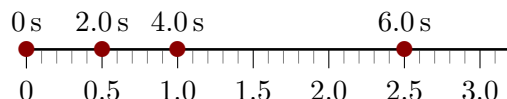
Several insects move along a straight line and their positions, measured in meters, are recorded every 2.0 s. The resulting motion diagrams are illustrated. (131F2024)



Fly



Beetle



Cricket

- Rank the insects in order of average speed from 0.0 s to 6.0 s.
- Rank the insects in order of average velocity from 0.0 s to 6.0 s.

6 Katie Ledecky splits

Katie Ledecky, one of the greatest female swimmers, won the women's 800 m freestyle race at the 2023 World Aquatics Championships. In the final, she covered the first 200 m in 1:58.29 (means 1 min and 58.29 s), the second 200 m in 2:03.63, the third 200 m in 2:04.00 and the final 200 m in 2:01.50. Determine her average speed for each 200 m section and also the entire race. Which section most resembled her speed for the entire race? (131F2024)

7 Bouncing hockey puck

A hockey puck can slide along a horizontal surface. Starting at an initial location, it slides right and takes 3.0 s to hit a board 18 m away. It bounces back to the left and reaches a final point 4.0 m from the board 2.0 s from the time it first hit the board. Determine the average velocity of the puck from the initial to final moments. (131F2024)

8 Red car, blue car, green car

Determine the average speed and the average velocity for cars moving in a straight line as follows:

- Red car moves right for 50 m in 10.0 s.
- Blue car moves right for 16 m in 8.0 s and then continues right for 34 m in 2.0 s.
- Red car moves right for 30 m in 5.0 s reverses and moves left for 20 m in 5.0 s. (131F2024)

9 Man versus dog

The following objects lie along a straight line: a bicycle, a coffee cup and a soccer ball. The distance from the coffee cup to the bicycle is 400 m and from the cup to the ball is 500 m. A man starts at the cup and travels in a straight line to the ball. This takes 200 s. A dog is initially at the cup and runs at constant speed to the bicycle, taking 50 s to do so. The dog immediately turns around and runs to ball; this takes the dog an additional 150 s. Consider the entire trip from the cup to the ball for each. Who has the larger average velocity for this entire trip? Explain your answer. (131F2024)



10 Displacement and average velocity

Various people move as described below over a total interval of 40 s. In each case determine the total displacement and the average velocity over the 40 s period. (131F2024)

- Anna takes a trip in two stages. In the first stage, lasting 20 s, she moves 30 m to the right. In the second stage, lasting 20 s, she moves 10 m to the left.
- Bill takes a trip in two stages. In the first stage, lasting 10 s, he moves 10 m to the left. In the second stage, lasting 30 s, he moves 30 m to the right.
- Explain whether displacement and average velocity capture all the information about the motion of each. If not, how might one modify the description of the motion in terms of displacement and velocity to describe the motion more accurately?

11 Average velocity, speed for motion with changing directions

- A person takes a trip, first walking 30 m to the right in 5.0 s and then returning to his starting point in another 10 s. Determine the average velocity of the person for the entire trip.
- Anna takes a trip in two stages. First she moves 100 m right in 40 s. She briefly stops and then she moves 120 m to the left in another 60 s. Determine her average velocity for each stage of the trip and also for the entire trip.

- c) Bill takes a trip in two stages. First he moves 200 m left in 100 s. He briefly stops and then he moves 150 m to the right in another 100 s. Determine his average velocity for each stage of the trip and also for the entire trip. (131F2024)

12 Average velocity and direction of motion

- a) An object moves in such a way that during a certain period the average velocity of the object is negative. Is it possible that at the end of the period the object is to the right of the origin of the coordinate system/axis? Explain your answer.
- b) An object moves in such a way that over a certain period the average velocity of the object is positive. Is it possible that at some point during the period the object moves to the left? Explain your answer. (131F2024)

13 Average velocity and displacement down a field

Three people, Alice, Bob and Charlie leave one end of a 100 m long field at the same instant and travel in the same direction toward the other end. (131F2024)

- a) Alice travels at a constant speed of 8.0 m/s for half the distance and then at 4.0 m/s for the remaining half. Determine Alice's average velocity for the entire trip.
- b) Bob travels at a constant speed of 10.0 m/s for half the distance and then at 2.0 m/s for the remaining half. Determine Bob's average velocity for the entire trip.
- c) Charlie travels for half of the time at a constant speed of 8.0 m/s for the remaining half of the time at a constant speed of 4.0 m/s. Determine Charlie's average velocity for the entire trip.
- d) Determine the order in which they arrive at the other end of the field.

14 El Capitan climb

The rock climbing route called "The Nose" ascends the El Capitan formation in Yosemite National Park. The elevation gain on the climb is 2900 ft and the quickest ascent took 1 hr, 58 min, 7 s. Determine the average speed for this ascent in m/s. (131F2024)

15 Average speed of the Moon

The Moon orbits the Earth in an ellipse with average radius 239,000 miles. It completes one orbit in 27 days, 7 hrs, 43 min, 11.5 s. Suppose that it orbits in a perfect circle with radius 239,000 miles. (131F2024)

- a) Determine the average speed of the Moon in mph.
- b) Determine the average speed of the Moon in m/s.

16 Running speed

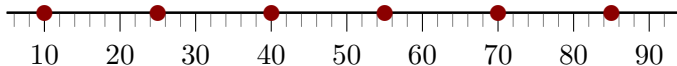
Estimate the speed with which you could run a distance of about 100 yards. Convert your answer to standard metric units. (131F2024)

17 Grass growing

Estimate the speed with typical lawn grass that one might find on campus grows. Express your answer in standard metric units. Describe all of the steps that give your estimate. (131F2024)

18 Motion diagrams and position vs. time graphs, 1

A car moves from left to right and its position, measured in meters, is recorded every 5.0 s. The resulting motion diagram is illustrated.

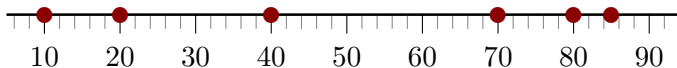


(131F2024)

- Produce a table of numerical data for position versus time for the car for the duration of the motion.
- Produce a position versus time graph for the car for the duration of the motion. This graph must be drawn by hand using axes that are clearly labeled.

19 Motion diagrams and position vs. time graphs, 2

A car moves from left to right and its position, measured in meters, is recorded every 5.0 s. The resulting motion diagram is illustrated.



(131F2024)

- Produce a table of numerical data for position versus time for the car for the duration of the motion.
- Produce a position versus time graph for the car for the duration of the motion. This graph must be drawn by hand using axes that are clearly labeled.

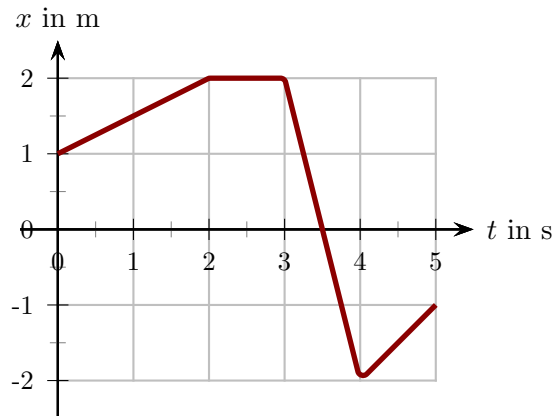
20 Hockey puck trip time

A hockey puck travels along a horizontal surface to the right for a distance of 12 m at speed 5.0 m/s. It hits a board and bounces to the left traveling a distance of 20 m at speed 4.0 m/s. Determine the total time for this trip. (131F2024)

21 Angry ant on a stick

An angry ant walks along a straight stick. The graph illustrates the ant's position vs. time. (131F2024)

- a) Describe in words qualitatively how the ant moves during the period from 0 s to 5 s. Include as much detail without using any numbers.
- b) During which period is the ant's speed largest? During which period is it smallest? Explain your answers.



22 Slug on a stick

A slippery slug crawls along a straight stick. The graph illustrates the slug's position vs. time. Answer the following, giving explanations for each answer. (131F2024)

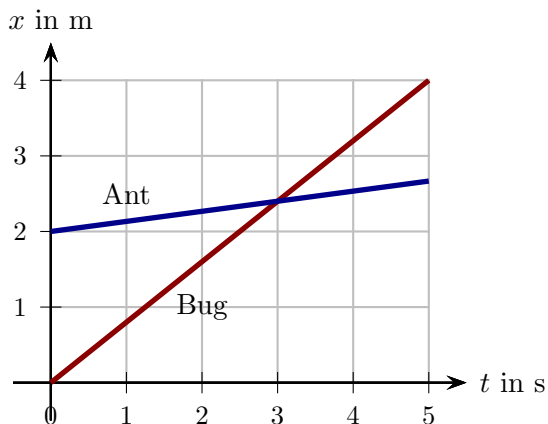
- a) Determine the velocity of the slug at 1.0 s.
- b) Determine the velocity of the slug at 4.0 s.



23 Ant and bug on a stick

An ant and a bug walk along a straight stick. The graph illustrates their position vs. time. Answer the following, explaining each answer. (131F2024)

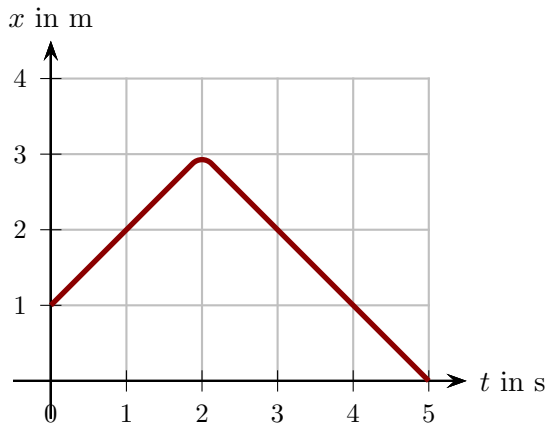
- a) When and where, if ever, are the ant and the bug at the same location?
- b) When, if ever, do the ant and the bug have the same speed?
- c) When, if ever, does the ant move faster than the bug?



24 Red ant on a stick

A red ant walks along a straight stick. The graph illustrates the ant's position vs. time. Answer the following, giving explanations for each answer. (131F2024)

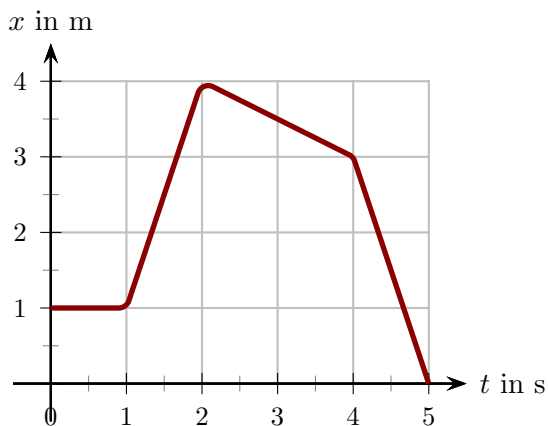
- a) During which times is the ant moving right? During which times is it moving left?
- b) When, if ever, is the velocity of the ant 0 m/s?
- c) How does the speed of the ant at 1.0 s compare to its speed at 4.0 s?
- d) How does the velocity of the ant at 1.0 s compare to its velocity at 4.0 s?



25 Timid tick on a stick

A timid tick walks along a straight stick. The graph illustrates the tick's position vs. time. (131F2024)

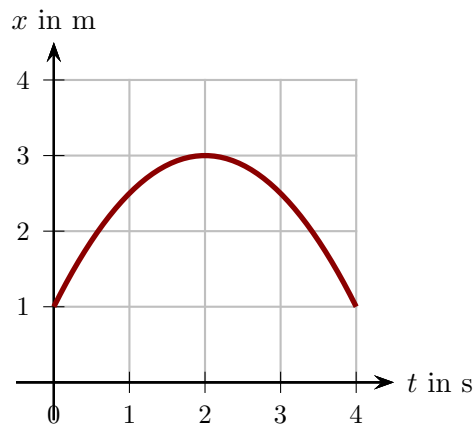
- a) Determine the velocity of the tick at 0.5 s.
- b) Determine the velocity of the tick at 1.5 s.
- c) Determine the velocity of the tick at 3.0 s.
- d) Determine the velocity of the tick at 4.5 s.



26 Bug on a stick

A bug walks along a straight stick. The graph illustrates the bug's position vs. time. In each of the following, explain your answers. (131F2024)

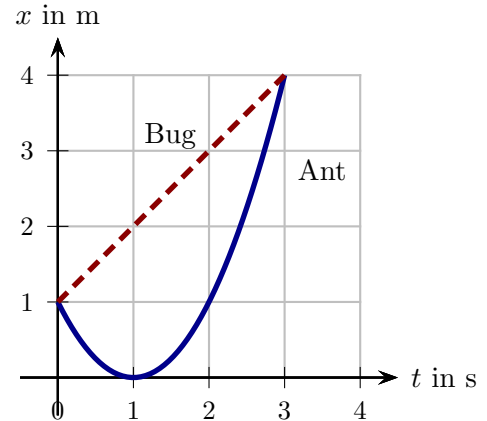
- a) Does the bug ever return to the position that it occupied initially ($t = 0$ s)? If so, when?
- b) Does the bug ever reverse direction? If so when?
- c) Does the bug ever have zero velocity? If so, when?



27 Ant and bug on a stick: speeds

An ant and a bug walk along straight sticks. The solid graph illustrates the ant's position vs. time. The dashed graph indicates the bug's position vs. time. Answer the following, giving explanations for each answer. (131F2024)

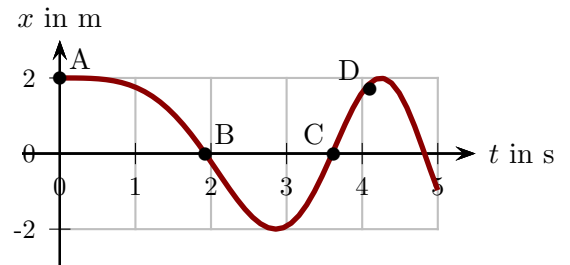
- At what time(s) are the ant and bug at the same location?
- Which is moving faster at 2 s?
- Do the ant and bug ever have the same velocity? If so when?



28 Graceful ladybug on a stick

A ladybug insect walks gracefully back and forth along a straight stick. A graph of position vs. time for its motion is illustrated with several instants labeled A, B, C and D. Explain your answers for the following. (131F2024)

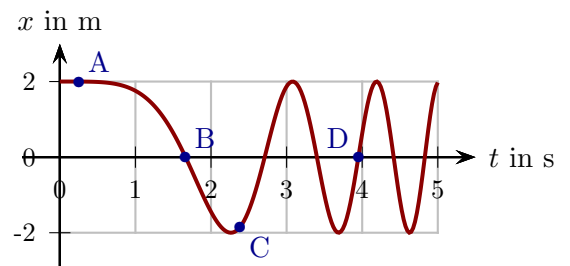
- At which of these instants is the ladybug moving right?
- At which of these instants is the ladybug slowing down?
- Rank the moments in order of increasing instantaneous speed.
- Sketch a graph of velocity versus time for the ladybug.



29 Agitated cat on a wall

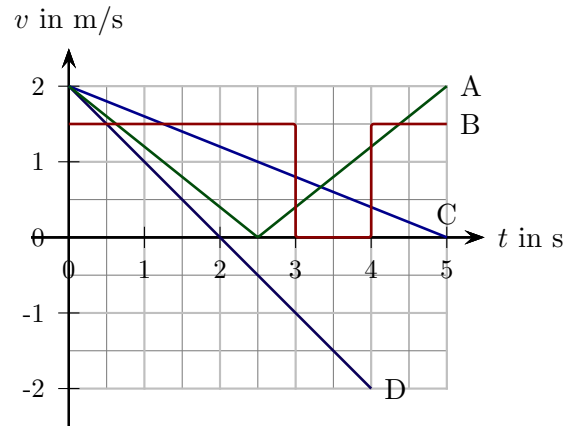
An increasingly agitated cat walks back and forth along a straight wall. A graph of position vs. time for its motion is illustrated with several instants labeled A, B, C and D. Explain your answers for the following. (131F2024)

- At which of these instants is the cat moving right?
- Rank the instants in order of increasing instantaneous speed.
- Describe an interval during which the cat slows down. Provide approximate initial and final times for this interval.
- Sketch a graph of velocity versus time for the cat from 0 s to 3 s.



30 Bead sliding along a wire

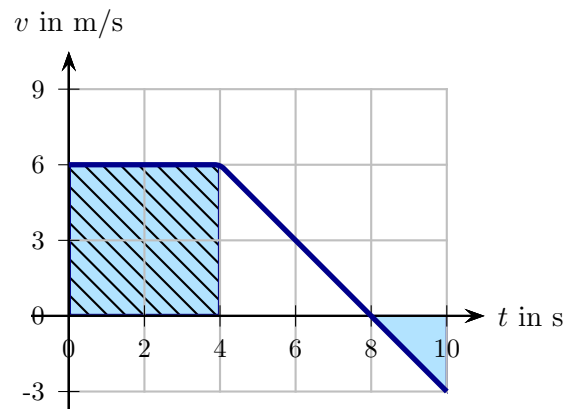
A bead slides along a straight piece of wire. After an initial moment it slides right with decreasing speed, coming to a brief stop. Immediately after this it slides right with increasing speed. Taking right as positive, which of the following represents a graph of *velocity versus time* for the bead? Explain your choice. (131F2024)



31 Crawling slug

A slug crawls along a straight wire, starting at $x = 0.0\text{ m}$ at $t = 0.0\text{ s}$. A graph of the slug's velocity versus time is illustrated. Use the graph to answer the following. (131F2024)

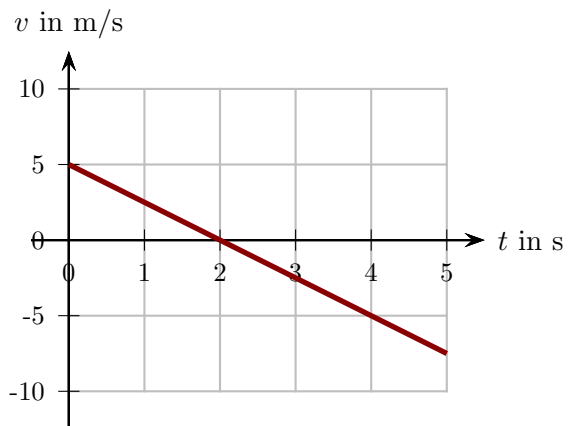
- Determine the displacement of the slug from $t = 0.0\text{ s}$ to $t = 4.0\text{ s}$.
- How is the displacement of the slug from $t = 0.0\text{ s}$ to $t = 4.0\text{ s}$ related to the shaded area between the graph and the horizontal axis ($v = 0.0\text{ m/s}$)?
- Assuming that the answer to the previous question is true in general, determine the displacement of the slug from $t = 4.0\text{ s}$ to $t = 8.0\text{ s}$.
- Is the displacement of the slug from $t = 8.0\text{ s}$ to $t = 10.0\text{ s}$ positive or negative? How might this relate to the shaded area from $t = 8.0\text{ s}$ to $t = 10.0\text{ s}$?



32 Stick insect on a stick

A stick insect walks back and forth along a straight stick. A graph of velocity vs. time for its motion is illustrated. (131F2024)

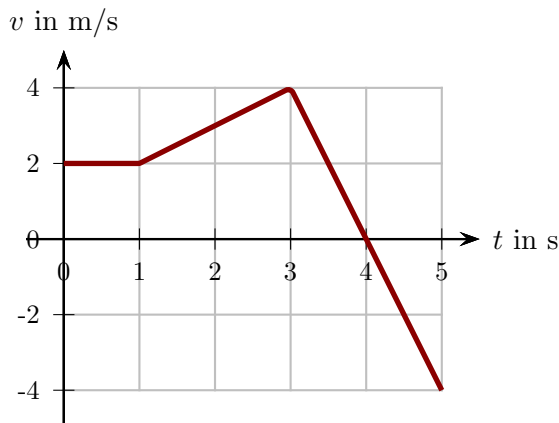
- Determine the displacement of the stick insect from $t = 0\text{ s}$ to $t = 2\text{ s}$.
- Determine the displacement of the stick insect from $t = 2\text{ s}$ to $t = 4\text{ s}$.
- Determine the displacement of the stick insect from $t = 0\text{ s}$ to $t = 4\text{ s}$.



33 Ladybug on a stick

A ladybug insect walks back and forth along a straight stick. A graph of velocity vs. time for its motion is illustrated. The ladybug is initially at $x = -1.0$ m. (131F2024)

- Determine the position of the ladybug at $t = 1$ s.
- Determine the position of the ladybug at $t = 3$ s.
- Determine the position of the ladybug at $t = 4$ s.
- Determine the position of the ladybug at $t = 5$ s.
- Describe in words how the position of the ladybug evolves with time during the illustrated interval.



34 Slithering worm

A worm is initially at rest and subsequently slithers along a straight wire. The worm's velocity increases steadily from 0.000 m/s to 0.015 m/s over a period of 120 s. Determine the distance traveled by the worm during this period. *Hint: A graph will help!* (131F2024)

35 Instantaneous velocity as a limit

An atom is trapped in such a way that it can move back and forth along one straight line. Its position is tracked as time passes and is represented by the function $x = t^2 - 2t - 1$ (the coefficients all have units such that x is in units of meters and t in seconds). (131F2024)

- Produce a list of positions at every 0.5 s from $t = 0.0$ s to $t = 4.0$ s. Use this data to plot an accurate graph of position versus time for $t = 0.0$ s $\leq t \leq 4.0$ s. *The graph must be drawn accurately enough to draw and calculate slopes of tangent lines.*
- Determine the velocity of the atom at 3.0 s by using a tangent line construction for the graph of position versus time.

The aim of the next parts of this problem is to determine the instantaneous velocity at 3.0 s.

- Use the function of position versus time to determine the average velocity over the time interval from $t = 3.0$ s to $t = 3.1$ s.
- Use the function of position versus time to determine the average velocity over the time interval from $t = 3.00$ s to $t = 3.01$ s.
- Does the value of average velocity at 3.0 s appear to approach a limit as the time interval decreases? If so what does the limit appear to be?
- Use the derivative of position to determine the exact instantaneous velocity at 3.0 s. Does the result agree with your answer to the previous part?
- At what time is the instantaneous velocity exactly zero? Explain your answer.

36 Velocity as a derivative, 1

Suppose that the position of an object is

$$x = (5 \text{ m/s}^2) t^2 + (3 \text{ m/s}) t$$

Determine the velocity of the object at $t = 3$ s. (131F2024)

37 Velocity as a derivative, 2

Suppose that the position of an object is

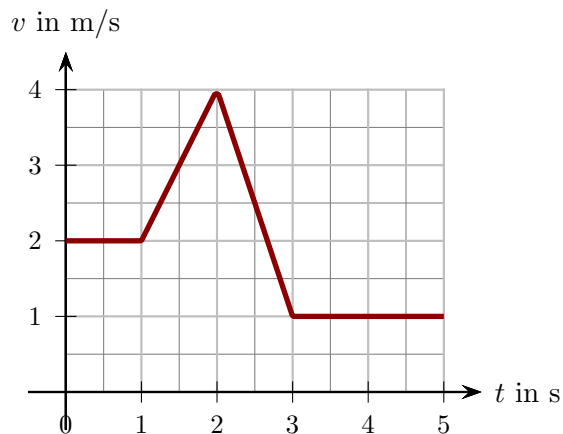
$$x = (2 \text{ m/s}^3) t^3 + (6 \text{ m/s}^2) t^2$$

Determine the velocity of the object at $t = 2$ s. Is this the same as x/t ? (131F2024)

38 Wandering ant

An ant walks along a straight stick. The graph illustrates the ant's velocity vs. time. Answer the following, giving explanations for each answer. (131F2024)

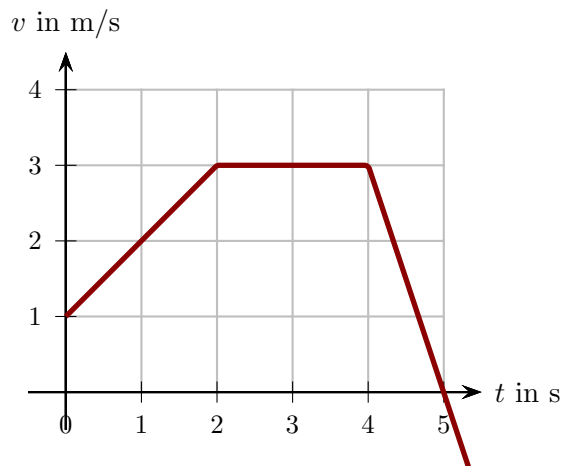
- How does the magnitude of the ant's acceleration at 1.5 s compare (larger, smaller, same) to its acceleration at 2.5 s?
- How does the magnitude of the ant's acceleration at 0.5 s compare (larger, smaller, same) to its acceleration at 2.5 s?
- How does the magnitude of the ant's acceleration at 0.5 s compare (larger, smaller, same) to its acceleration at 4.0 s?



39 Accelerating bug on a stick

A bug walks along a straight stick. The graph illustrates the bug's velocity vs. time. At $t = 0$ s the bug is at the $x = 2.0$ m mark. (131F2024)

- Determine the position, velocity and acceleration of the bug at 1.0 s.
- Determine the position, velocity and acceleration of the bug at 3.0 s.
- Determine the position, velocity and acceleration of the bug at 5.0 s.

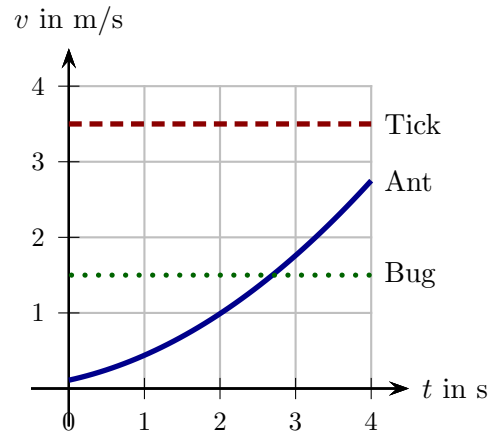


40 Cyclist versus skateboarder

A skateboarder moves down a gentle slope. She passes a cyclist, who is at rest and at this moment the skater's speed is 4.0 m/s. At an instant 10 s later the skater's speed is 10.0 m/s and the cyclist's is 8.0 m/s. During this period, who has the greater acceleration? Explain your answer. (131F2023)

41 Insects on sticks

An ant, a tick and a bug walk along straight sticks. The solid graph illustrates the ant's velocity vs. time. The dashed graph indicates the tick's velocity vs. time. The dotted line indicates the bug's velocity versus time. (131F2023)



- a) Which of the following is true during the period from 0s to 4s? Explain your answer.
- The acceleration of the ant is the same as that of the tick.
 - The acceleration of the tick is always larger than that of the ant.
 - The acceleration of the tick is always smaller than that of the ant.
 - The acceleration of the tick is sometimes smaller than that of the ant, sometimes larger.
- b) Which of the following is true during the period from 0s to 4s? Explain your answer.
- The acceleration of the ant is the same as that of the bug.
 - The acceleration of the bug is always larger than that of the ant.
 - The acceleration of the bug is always smaller than that of the ant.
 - The acceleration of the bug is sometimes smaller than that of the ant, sometimes larger.

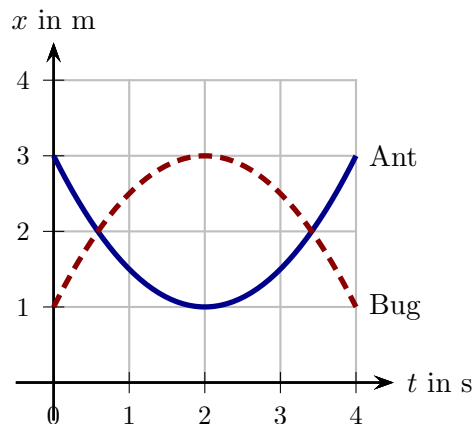
42 Acceleration sign

A bicycle can move east (positive) or west (negative). (131F2024)

- If the bicycle moves east can the acceleration be negative? Explain your answer.
- If the bicycle moves west can the acceleration be positive? Explain your answer.

43 Ant and bug on sticks

An ant and a bug walk along straight sticks. The solid graph illustrates the ant's position vs. time. The dashed graph indicates the bug's position vs. time. (131F2024)



- a) For the bug and, separately, the ant, which of the following is true during the period from 0s to 4s? Explain your answers.
- Velocity is zero at all times.
 - Velocity is positive at all times.
 - Velocity is negative at all times.
 - Velocity is first positive and later negative.
 - Velocity is first negative and later positive.
- b) For the bug and, separately, the ant, which of the following is true during the period from 0s to 4s? Explain your answers.
- Acceleration is zero at all times.
 - Acceleration is positive at all times.
 - Acceleration is negative at all times.
 - Acceleration is first positive and later negative.
 - Acceleration is first negative and later positive.

44 Moving man, 1

A man in an animation walks along a horizontal line. Positions are marked in meters along the line with positive numbers to the right of the origin and negative to the left. His positions and velocities are recorded at equally spaced intervals in time. The data is:

Time in s	Position in m	Velocity in m/s
0.0	-5.00	-4.00
2.0	-7.00	2.00
4.0	3.00	8.00

- Determine the man's average acceleration from 0.0s to 2.0s.
- Determine the man's average acceleration from 2.0s to 4.0s. (131F2024)

45 Person walking left and right

A person walks along a horizontal line. Positions are marked in meters along the line with positive numbers to the right of the origin and negative to the left. His velocity is recorded at equally spaced intervals in time. The data is:

Time in s	Velocity in m/s
10.00	-1.00
10.10	-0.80
10.20	-0.60
10.30	-0.40
10.40	-0.20
10.50	0.00
10.60	0.20
10.70	0.40
10.80	0.60
10.90	0.80

- During which period is the person moving left? During which period is the person moving right?
- Determine the person's acceleration while moving left.
- Determine the person's acceleration while moving right.
- Does the person's acceleration appear to change during the period from 10.10 s to 10.90 s?
- At what moment does the person reverse direction? According to the data is the acceleration zero or not at this moment? (*131F2024*)

46 Bungee jumper

A bungee jumper falls downward stretching the cord, reaching a low point, after which the cord pulls him up again. His velocity is recorded at equally spaced intervals in time. The data is:

Time in s	Velocity in m/s
10.0	-20.0
10.5	-15.0
11.0	-10.0
11.5	-5.0
12.0	0.0
12.5	5.0
13.0	10.0
13.5	15.0
14.0	20.0

- a) During which period is the man falling? When is he rising?
- b) By how much does the man's velocity change per second? Is this change constant throughout the recorded motion?
- c) Determine the man's acceleration while he is falling and also while he is rising. Are these accelerations the same or not?
- d) What is the man's acceleration (according to the data) at his low point? (131F2024)

47 Moving man activity

Go to the moving man animation at:

<http://phet.colorado.edu/en/simulation/moving-man>

Run the moving man animation. Click on the charts tab. Set the position to 0.00 m, the velocity to -5.00 m/s and the acceleration to 2.00 m/s². Run the animation, stopping it just before the man hits the wall. The animation will have recorded the motion. Check the playback button at the bottom. You can slide the light blue bar left and right to get data for the motion. Gray zoom icons at the right will let you rescale the charts. (131F2024)

- a) Consider the interval from 2.0 s to 3.0 s. Describe the motion verbally during this time.
- b) How does the speed of the man at 2.0 s compare to that at 3.0 s? Explain your answer.
- c) How does the velocity of the man at 2.0 s compare to that at 3.0 s? Explain your answer.
- d) Will the average acceleration over the interval from 2.0 s to 3.0 s be positive, negative or zero? Explain your answer.
- e) If the acceleration is not zero, does it vary during this interval? Explain your answer.
- f) Determine the average acceleration over the interval from 2.0 s to 3.0 s.

48 Person moving with constant acceleration

A person is initially at rest and subsequently moves right with a constant acceleration. The person's reaches speed 6.0 m/s at a point 9.0 m to the right of the starting location. The aim of this exercise will be to determine the time taken to reach this point. A first step will be to determine the acceleration of the person. (131F2024)

- a) Sketch the situation, illustrating the person at the two instants described above.

List all relevant variables for the two instants:

$t_i =$	$t_f =$
$x_i =$	$x_f =$
$v_i =$	$v_f =$

- b) Determine the acceleration by selecting one of the kinematic equations, substituting and solving for a .
- c) Using a different kinematic equation, find the time that it takes the person to reach speed 6.0 m/s.

d) Suppose that you had tried to find the time taken to reach speed 6.0 m/s by using

$$v = \frac{\Delta x}{\Delta t} \Rightarrow 6.0 \text{ m/s} = \frac{9.0 \text{ m}}{\Delta t}.$$

What time does this give? Does it agree with the answer that you obtained to the previous part? Is it correct?

49 Avoid the wall!

A skateboarder slides toward a wall. Initially the skateboarder is 18 m left of the wall and moving with speed 6.0 m/s to the right. The aim of this exercise will be to determine the minimum acceleration to barely avoid hitting the wall. (131F2024)

a) Sketch the situation, illustrating the skateboarder at the initial instant and the instant just before reaching the wall.

List all relevant variables for the two instants:

$$\begin{array}{ll} t_i = & t_f = \\ x_i = & x_f = \\ v_i = & v_f = \end{array}$$

b) Determine the acceleration by selecting one of the kinematic equations, substituting and solving for a .

c) Use one of the kinematic equations to determine the time that it takes for the skateboarder to reach the wall.

d) Would the equation

$$v = \frac{\Delta x}{\Delta t} \Rightarrow 6.0 \text{ m/s} = \frac{18 \text{ m}}{\Delta t}$$

allow one to find the time taken to reach the wall correctly? Why or why not?

e) Set up the moving man animation at:

<http://phet.colorado.edu/en/simulation/moving-man>

and run this to check your prediction. In order to verify that you have done this, use the animation to provide the times at which the man is 10 m to the left of the wall.

50 Braking car, 1

A car travels to the right with speed 30.0 m/s (about 67 mph). The car brakes and slows to a stop with constant acceleration. It does this in a distance of 80.0 m (about 260 ft). The aim of this exercise is to find the time taken to do this. (131F2024)

- a) Sketch the situation, illustrating the car at two key instants.

List all relevant variables for the two instants:

- b) Determine the acceleration using one of the kinematic equations. Write down the equation, substitute from your list of variables and solve for a . (is acceleration positive or negative?)

Answer: -5.63 m/s^2

- c) Using a different kinematic equation, find the time that it takes the car to stop. Write down the equation, substitute from your list of variables and solve for time.

Answer: 5.33 s

51 Braking car, 2

A car travels to the right with speed 24.6 m/s (about 55 mph). The car brakes, coming to a stop in 50 m (about 165 ft). The acceleration during the braking process is constant. (131F2024)

- a) Determine the acceleration of the car while it brakes.
b) Determine the time taken for the car to stop.

52 Accelerating aircraft

An aircraft accelerates from rest to a speed of 120.0 m/s. It does this with a constant acceleration of 4.00 m/s^2 . The aim of this exercise is to find the distance traveled by the aircraft during this process. (131F2024)

- a) Sketch the situation, illustrating the aircraft at two key instants. List all relevant variables for the two instants and list the acceleration.
b) Determine the distance traveled by the aircraft using one of the kinematic equations. Write down the equation, substitute from your list of variables and solve for the relevant variable.

Answer: 1800 m

- c) Using a different kinematic equation, find the time that it takes for the aircraft to reach this speed. Write down the equation, substitute from your list of variables and solve for time.

Answer: 30 s

- d) Would it have worked to use $v = \frac{\Delta x}{\Delta t} \Rightarrow 120.0 \text{ m/s} = \frac{1800 \text{ m}}{\Delta t}$?

53 Accelerating cart

A cart travels in one direction. At an initial instant it passes the 4.0 m mark while traveling to the right with speed 15 m/s. (131F2024)

- a) Determine the location of the cart at an instant 6.0 s later if its acceleration is 3.0 m/s^2 .
- b) Determine the location of the cart at an instant 6.0 s later if its acceleration is -3.0 m/s^2 .

54 Accelerating shrew

A shrew (a small mammal) travels in one direction. At an initial instant it travels to the right with speed 3.0 m/s. It subsequently moves with constant acceleration and after traveling an additional 2.0 m to the right it reaches a speed of 7.0 m/s. Determine the time taken to do this. (131F2024)

55 Moving sloth

A sloth (a tree-climbing mammal) travels along a straight branch. At an initial instant it travels to the right with speed 0.030 m/s. It subsequently crawls with constant acceleration and after an additional 5.0 s has traveled an additional 0.40 m to the right. Determine its speed at the end of this period. (131F2024)

56 Car starting from rest

A car is initially at rest and subsequently moves in one direction with constant acceleration. After 12 s its speed is 50 mph. Determine the total distance that it has traveled in this time. (131F2024)

57 Feasible Acceleration?

Suppose that one aims to push a cart along a horizontal surface, starting from rest, with a constant acceleration. Could one sustain this over a period of several seconds? This problem will try to help you answer this. When doing this, assume that the cart is light enough that one can easily exert the necessary force. Explain your answers, preferably with calculations, and describe any assumptions you made. (131F2024)

- a) Your friend claims that they can push the cart with a constant acceleration of 3.0 m/s^2 for a period of 10 s. Considering the state motion of the cart at the end of the 10 s period, does this seem reasonable?
- b) Suppose that you pushed the cart, starting from rest along a 100 m long straight track. What is the highest acceleration that you could feasibly attain? *Hint: Consider the highest speed you could attain at the end of the track.*

58 Reversing cart

At an initial instant a cart travels to the left with speed 12 m/s. Subsequently the cart's acceleration is 4.0 m/s^2 . (131F2024)

- a) How long does it take for the cart to reverse its direction of travel?
- b) How far does the cart travel before it reverses its direction of travel?

59 Racing cyclists

Two cyclists can ride along a straight road. Juliette moves right with a constant speed of 20 m/s. She passes Elisa, who is at rest. At the moment that Juliette passes, Elisa begins to move, accelerating at a constant rate of 2.0 m/s^2 . Elisa maintains this for 12.0 s and after that she moves with constant velocity. (131F2024)

- a) Explain how you can be sure that Elisa eventually passes Juliette.
- b) Determine the time (after Elisa starts to move) that it takes Elisa to catch up to Juliette.
- c) Determine the distance traveled by each when Elisa catches up with Juliette.

60 Runway design, 1

The takeoff speed for an aircraft under certain conditions is 260 km/h. During takeoff, one wants the acceleration to be as low as possible. (131F2024)

- a) Suppose that the runway has length 1500 m. Determine the aircraft's minimum acceleration so that it can take off on this runway.
- b) Suppose that the acceleration is only $0.20g = 1.96 \text{ m/s}^2$. Determine the minimum length of the runway so that the aircraft could take off successfully.

Note: It is useful to describe acceleration in terms of g as this is correlated to what humans can feel. An acceleration of g is what you feel when jumping off a diving board. An acceleration of around $6g$ will cause a person to blackout.

61 Aircraft takeoff acceleration

The takeoff speed for an aircraft under certain conditions is 260 km/h. A typical runway has length 3500 m. (131F2024)

- a) Suppose that the aircraft starts at rest somewhere along the runway and uses $2/3$ of the runway length to takeoff. Determine the acceleration of the aircraft, assuming that it is constant.
- b) It is useful to describe acceleration in terms of g as this is correlated to what humans can feel. An acceleration of g is what you feel when jumping off a diving board. An acceleration of around $6g$ will cause a person to blackout. What multiple of g is the aircraft's acceleration?

- c) Determine the fraction of the runway that the aircraft would have to use so that the acceleration were g .

62 Jumping cat

A cat jumps, launching itself vertically. During its subsequent motion up and down, air resistance is negligible. In the following let the upward direction be positive. (131F2024)

- a) Which of the following is true in the period after the cat's feet have left the ground and before the cat reaches its highest point above the ground?
- i) Acceleration is zero at all times.
 - ii) Acceleration is positive at all times.
 - iii) Acceleration is negative at all times.
- b) Which of the following is true in the period while the cat drops back to the ground?
- i) Acceleration is zero at all times.
 - ii) Acceleration is positive at all times.
 - iii) Acceleration is negative at all times.
- c) Which of the following is true when the cat is at its highest point above the ground?
- i) Acceleration is zero at all times.
 - ii) Acceleration is positive at all times.
 - iii) Acceleration is negative at all times.

63 Ball launched vertically

A ball is launched vertically from Earth's surface with speed 10 m/s. The aim of the first part of this exercise is to determine the maximum height reached by the ball and time taken to reach the maximum height. (131F2024)

- a) Sketch the situation, illustrating the ball at two key instants.
List all relevant variables, including the acceleration, for the two instants:
- b) Determine the maximum height reached by the ball by using one of the kinematic equations. Write down the equation, substitute from your list of variables and solve for the maximum height variable.
Answer: 5.1 m
- c) Determine the time taken to reach the maximum height by using one of the kinematic equations. Write down the equation, substitute from your list of variables and solve for the maximum height variable.
Answer: 1.0 s

The second part of this exercise aims to find the speed of the ball just before returning to the ground using the fall from its maximum height.

- d) Sketch the situation for the falling ball, illustrating the ball at two key instants.
List all relevant variables, including the acceleration, for the two instants:
- e) Determine the speed of the ball just before hitting the ground the ball by using one of the kinematic equations. Write down the equation, substitute from your list of variables and solve for the velocity variable.

Answer: *velocity: -10 m/s , speed: 10 m/s .*

64 Ball maximum height

A ball is launched vertically upward from Earth's surface with speed 25 m/s . (131F2024)

- a) Determine the maximum height reached by the ball.
b) Determine the speed of the ball when it is 16 m above Earth's surface.

65 Bullet return speed

A bullet is fired vertically upward from Earth's surface with various speeds. The bullet will return to Earth at its launch position. (131F2024)

- a) Suppose that the bullet is launched with speed 600 m/s . Ignoring air resistance, determine the speed of the bullet just before it hits the ground.
b) Is the speed with which the bullet returns the same as that with which it is launched (regardless of the launch speed)? Explain your answer.

66 Thrown rock

A rock is thrown vertically down from a bridge. It leaves the hand with speed 20 m/s and hits the water 15 m below. Determine its speed at the instant before it hits the water. (131F2024)

Answer: *26 m/s*

67 Ball thrown from above the ground, 1

A ball is thrown vertically upwards, leaving the hand at a height of 1.2 m above the ground. It hits the ground 2.5 s after leaving the hand. (131F2024)

- a) Determine the speed with which the ball left the hand.
- b) Determine the maximum height above the ground reached by the ball.

68 Ball thrown from above the ground, 2

A ball is thrown vertically upwards, leaving the hand with speed 4.0 m/s at a height of 1.2 m above the ground. Determine the speed of the ball just before it hits the ground. (131F2024)

69 Diver

A diver stands on a diving board that is 10 m above the surface of the water. The diver launches herself vertically up, leaving the board with a speed of 5.0 m/s. (131F2024)

- a) Determine the time taken for the diver to hit the water. **Answer:** 2.0 s
- b) Determine the diver's speed when she hits the water. **Answer:** 15 m/s

70 Jumping flea

A flea is at rest on a bed and launches itself vertically. A child watching this thinks that the flea is airborne for a total time of about 2 s. Determine the maximum height above the bed that the flea would reach if this were true. Ignore air resistance. Is it plausible that the flea could have been airborne for that long? What might be a more realistic range of times? (131F2024)

Answer: 4.9 m

71 Penny and a well

A penny is held at rest at the top of a well. The penny is released, falls freely and takes 1.75 s to hit the water below. Determine the depth (distance from the top to the water) of the well. (131F2024)

Answer: 15 m

72 Parachuting package

A package is released from rest at a height of 100 m above Earth's surface. It falls freely until it is 40 m above Earth's surface. At that instant it deploys a parachute and after this it falls with a constant speed. The aim of this exercise is to determine the time taken to reach Earth.

To do this we will calculate the time taken for the free fall motion, Δt_A , and separately that time taken to fall with the parachute open, Δt_B .

The aim of the first part of this exercise is to determine Δt_A . (131F2024)

- a) Sketch the situation, illustrating the ball at two key instants that will allow one to determine the time for the free fall portion of the motion.
List all relevant variables for the two instants and the acceleration.
- b) Determine Δt_A by using one of the kinematic equations. Write down the equation, substitute from your list of variables and solve for Δt_A .

Answer: 3.5 s.

The second part of this exercise aims to find Δt_B .

- c) Sketch the situation for the falling ball, illustrating the ball at two key instants that will allow one to determine the time for the parachuting portion of the motion.
List all relevant variables for the two instants and the acceleration.
- d) There is one quantity that one can obtain from the free fall part of the motion that will be needed to analyze the parachuting portion of the motion. Identify and compute this and insert it in to the list of variables from the previous part.
- e) Now determine Δt_B by using one of the kinematic equations. Write down the equation, substitute from your list of variables and solve for Δt_B .

Answer: 1.2 s.

- f) Determine the total time taken to fall.

Answer: 4.7 s.

73 Rocket flight

A rocket is at rest on the ground. Its engine then fires, producing a constant acceleration of 40 m/s^2 for a period of 15 s and propelling the rocket vertically. The engine then stops and the rocket continues to move upward. (131F2024)

- a) Determine the maximum altitude (height) reached by the rocket.
- b) Determine the time taken by the rocket to reach its maximum altitude.
- c) Determine the time taken for the rocket to fall back to the ground after reaching its maximum height.

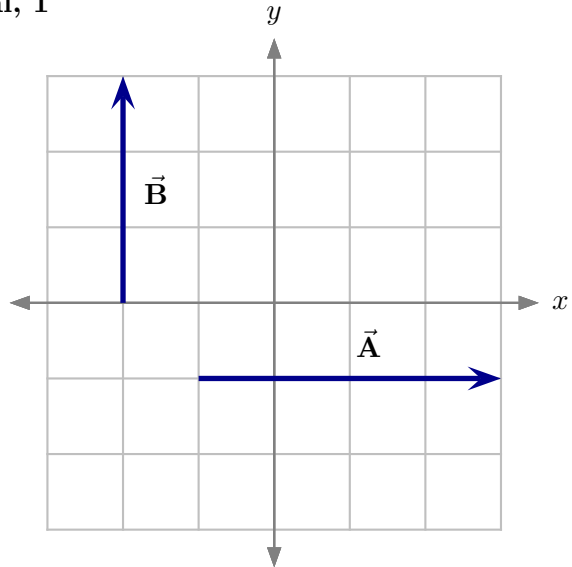
Answer: a) 23 km b) 76 s c) 69 s

Vectors

74 Vector addition and subtraction: graphical, 1

Two displacement vectors, \vec{A} and \vec{B} , are illustrated. (131F2024)

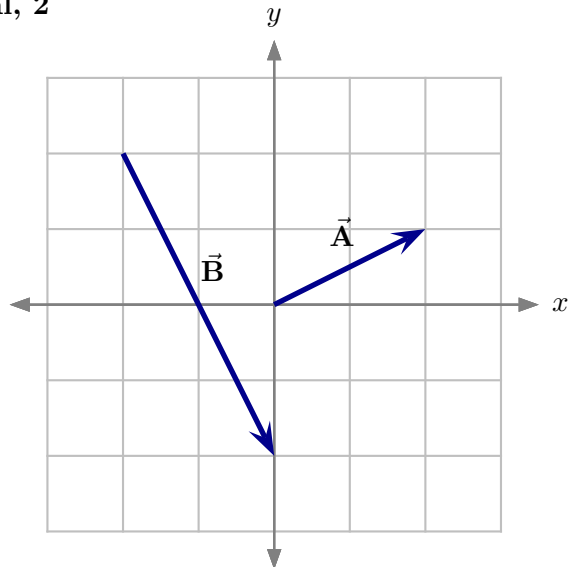
- Sketch $\vec{C} = \vec{A} + \vec{B}$. Determine the magnitude and direction of \vec{C} .
- Let A be the magnitude of \vec{A} , let B be the magnitude of \vec{B} and C be the magnitude of \vec{C} . In this case, do these magnitudes satisfy $C = A + B$? Explain your answer.



75 Vector addition and subtraction: graphical, 2

Two displacement vectors, \vec{A} and \vec{B} , are illustrated. (131F2024)

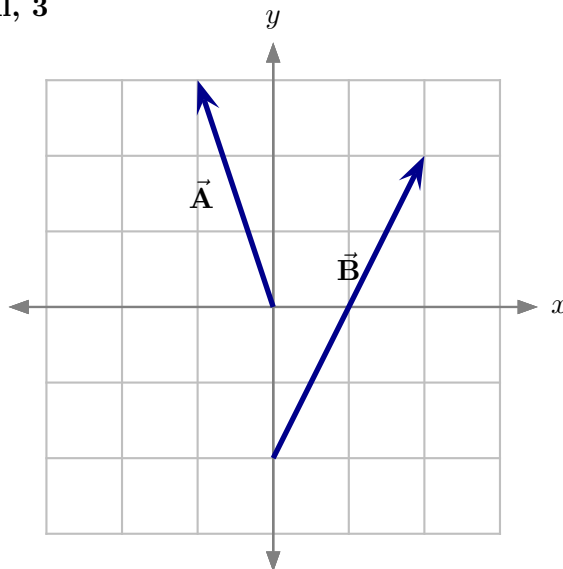
- Sketch $\vec{C} = \vec{A} + \vec{B}$. Determine the magnitude and direction of \vec{C} .
- Sketch $\vec{C} = \vec{A} - \vec{B}$. Determine the magnitude and direction of \vec{C} .
- Sketch $\vec{C} = 2\vec{A} - \frac{1}{2}\vec{B}$. Determine the magnitude and direction of \vec{C} .



76 Vector addition and subtraction: graphical, 3

Two displacement vectors, \vec{A} and \vec{B} , are illustrated. (131F2024)

- Sketch $\vec{C} = \vec{A} + \vec{B}$. Determine the magnitude and direction of \vec{C} .
- Sketch $\vec{C} = \vec{A} - \vec{B}$. Determine the magnitude and direction of \vec{C} .
- Sketch $\vec{C} = 2\vec{A} - \frac{1}{2}\vec{B}$. Determine the magnitude and direction of \vec{C} .



77 Vector algebra: conceptual

Consider two vectors labeled \vec{A} and \vec{B} . (131F2024)

- Suppose that \vec{A} and \vec{B} are perpendicular. Explain whether it is possible that $\vec{A} - \vec{B} = 0$.
- Suppose that the magnitudes satisfy $B = 2A$. Let $\vec{C} = \vec{A} + \vec{B}$. Explain whether it is possible that the magnitude of \vec{C} satisfies $C = A$. Explain whether it is possible that the magnitude of \vec{C} satisfies $C = 2A$.

78 Adding two vectors

Consider displacement vectors \vec{A} , whose magnitude is 4.0 m, and \vec{B} , whose magnitude is 3.0 m. Their directions are not specified. Let $\vec{C} = \vec{A} + \vec{B}$. (131F2024)

- What is the maximum possible magnitude for \vec{C} ? How must the vectors be arranged to give this?
- What is the minimum possible magnitude for \vec{C} ? How must the vectors be arranged to give this?

79 Vector components: conceptual

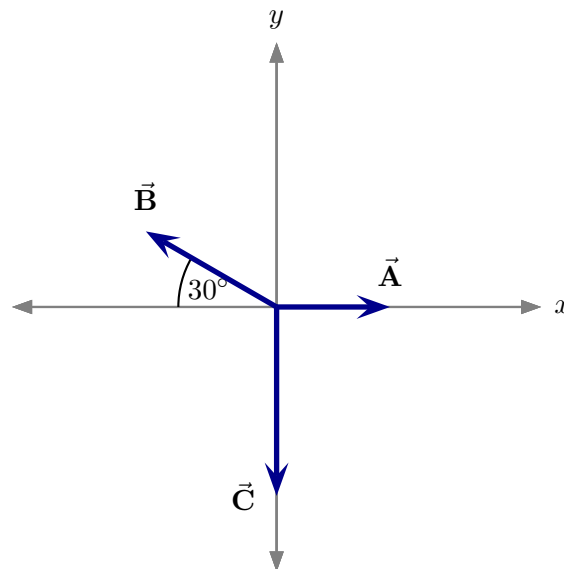
Consider a vector \vec{A} in two dimensions. (131F2024)

- Is it possible that either A_x or A_y is larger than the magnitude, A ? Explain your answer.
- Is it possible that both components of \vec{A} can be non-zero and the magnitude of \vec{A} is zero? Explain your answer.

80 Vector components: algebraic, 1

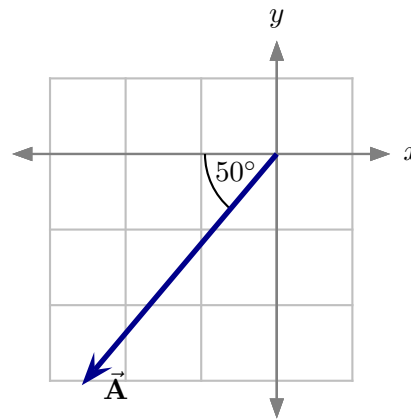
Displacement vectors, \vec{A} , \vec{B} , and \vec{C} are illustrated. Their magnitudes are $A = 15\text{ m}$, $B = 20\text{ m}$ and $C = 25\text{ m}$. (131F2024)

- Determine the x and y components of each vector.
- Determine the components of $\vec{D} = \vec{A} + \vec{B} + \vec{C}$. Determine the magnitude of \vec{D} .



81 Vector components: algebraic, 2

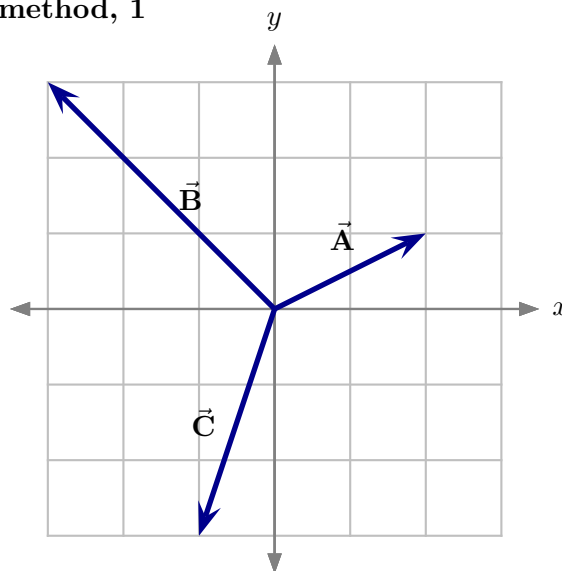
Determine the components of \vec{A} , whose magnitude is 20.0 m . (131F2024)



82 Vector addition: graphical and algebraic method, 1

Displacement vectors, \vec{A} , \vec{B} , and \vec{C} are illustrated. Let $\vec{D} = \vec{A} + \vec{B} + \vec{C}$. (131F2024)

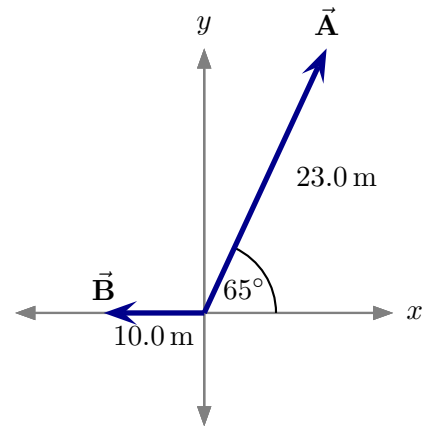
- Using the graph sheet below, determine \vec{D} graphically via the head-to-tail method. Use the result to determine the magnitude of \vec{D} .
- List the horizontal and vertical components of each of \vec{A} , \vec{B} , and \vec{C} and use these components to determine the components of \vec{D} . Use the result to determine the magnitude of \vec{D} .



83 Vector addition: algebraic method, 1

Two displacement vectors, \vec{A} and \vec{B} are illustrated. (131F2024)

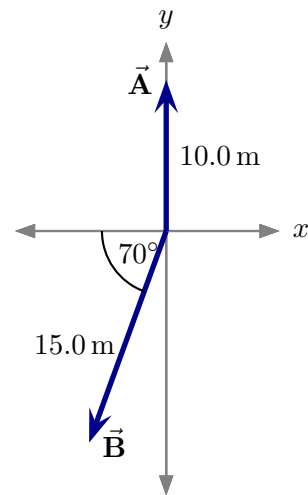
- a) Determine the components of \vec{A} .
- b) Determine the components of \vec{B} .
- c) Determine the components of $\vec{C} = \vec{A} + \vec{B}$.
- d) Determine the magnitude of \vec{C} .



84 Vector addition: algebraic method, 2

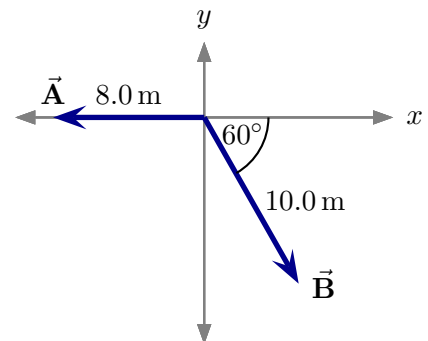
Two displacement vectors, \vec{A} and \vec{B} are illustrated. (131F2024)

- a) Determine the components of \vec{A} .
- b) Determine the components of \vec{B} .
- c) Determine the components of $\vec{C} = \vec{A} + \vec{B}$.
- d) Determine the magnitude of \vec{C} .



85 Vector addition: algebraic method, 3

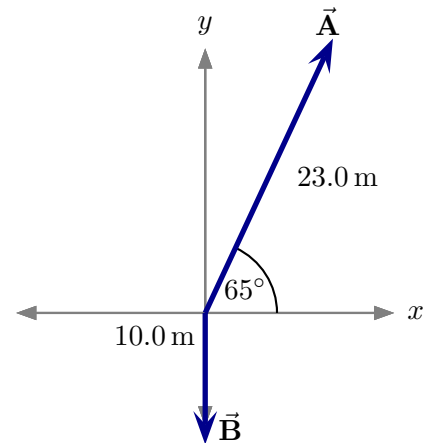
Two displacement vectors, \vec{A} and \vec{B} are illustrated. Determine the magnitude of $\vec{C} = \vec{A} + \vec{B}$. (131F2024)



86 Vector addition: algebraic method, 4

Two displacement vectors, \vec{A} and \vec{B} are illustrated. (131F2024)

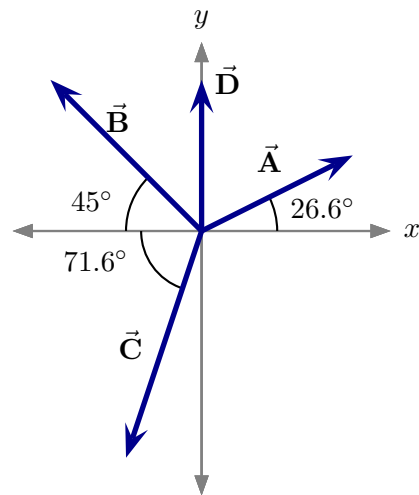
- Determine the components of $\vec{C} = \vec{A} + \vec{B}$.
- Determine the magnitude of \vec{C} .



87 Vector algebra using components

Displacement vectors, \vec{A} , \vec{B} , \vec{C} , and \vec{D} are illustrated. Their magnitudes are $A = 2.0$ m, $B = 2.5$ m, $C = 3.0$ m and $D = 1.5$ m. (131F2024)

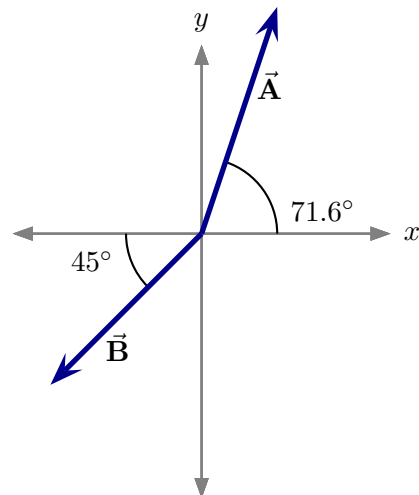
- Determine the x and y components for each vector.
- Determine the x and y components for $\vec{E} = \vec{A} + \vec{B}$. Determine the magnitude of \vec{E} .
- Determine the x and y components for $\vec{E} = \vec{B} + \vec{C}$. Determine the magnitude of \vec{E} .
- Determine the x and y components for $\vec{E} = \vec{B} + \vec{D}$. Determine the magnitude of \vec{E} .
- Determine the x and y components for $\vec{E} = \vec{B} + \vec{C} + \vec{D}$. Determine the magnitude of \vec{E} .



88 Vector subtraction using components

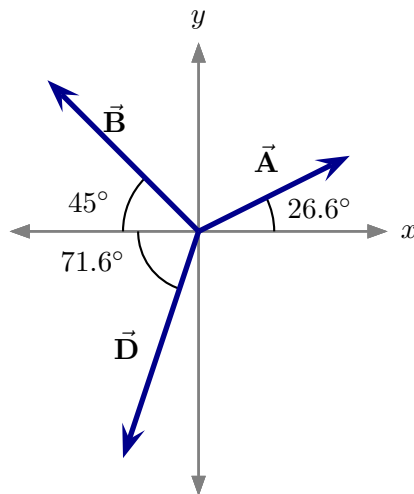
Displacement vectors, \vec{A} , \vec{B} are illustrated. Their magnitudes are $A = 3.16$ m and $B = 2.83$ m. (131F2024)

- Determine the components of $\vec{C} = \vec{A} - \vec{B}$. Determine the magnitude of \vec{C} .
- Determine the components of $\vec{C} = 4\vec{A} - 3\vec{B}$. Determine the magnitude of \vec{C} .



89 Vector algebra using unit vectors

Displacement vectors, \vec{A} , \vec{B} , and \vec{D} are illustrated. Let $\vec{D} = 3\vec{A} + 2\vec{B} + \vec{C}$. Their magnitudes are $A = 2.0\text{ m}$, $B = 3.5\text{ m}$ and $D = 6.0\text{ m}$. Express each of the vectors in terms of unit vectors and use these to determine \vec{C} . (131F2024)

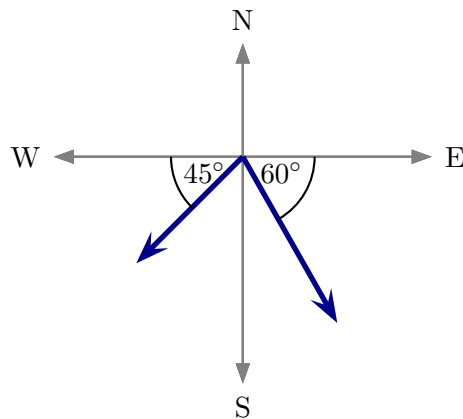


90 Marching soldier

A soldier marches around a playing field whose edges are along North-South (N-S) and East-West (E-W) lines. The soldier starts in the southwest corner, marches in a straight line in the direction 30° N of E for 40 m. He then marches straight south for 12 m. Finally he marches in a straight line in the direction 50° N of W for 30 m. After this, how far is the soldier from his starting point? (131F2024)

91 Successive displacements

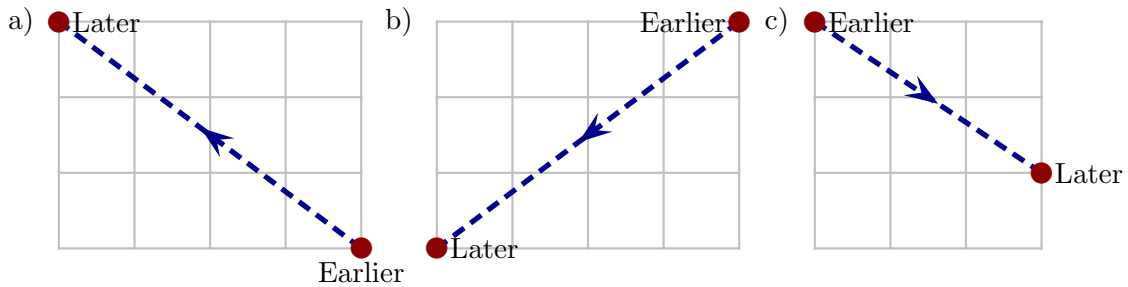
An insect crawls along a horizontal two dimensional surface in two straight stages. First the insect walks 8.0 cm at an angle of 45° south of west. Second, the insect walks 10.0 cm in a straight line at an angle 60° south of east. Determine an expression for the displacement of the insect and how far the insect ends up from its starting point. (131F2024)



Two dimensional kinematics

92 Velocity vectors from motion

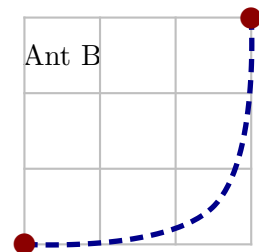
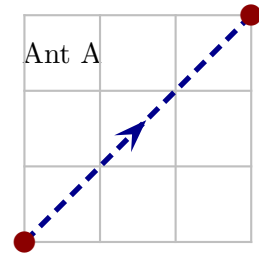
Particles move along the illustrated trajectories. Their locations are illustrated at instants 4.0 s apart. The grid units are meters. For each case determine the average velocity vector for the particle over the entire interval and express it in terms of the conventional unit vectors. (131F2024)



93 Ants on a table

Two ants move on a flat surface between two grains of sugar. Their trajectories are illustrated. The ants take the same time to move between the grains but follow different trajectories. Which of the following is true about the average velocities of the ants for the entire trip? Explain your answer. (131F2024)

- The average velocities are the same.
- The average velocity of ant A is smaller than that for ant B.
- The average velocity of ant A is larger than that for ant B.
- The average velocities are different but one cannot say which is larger.



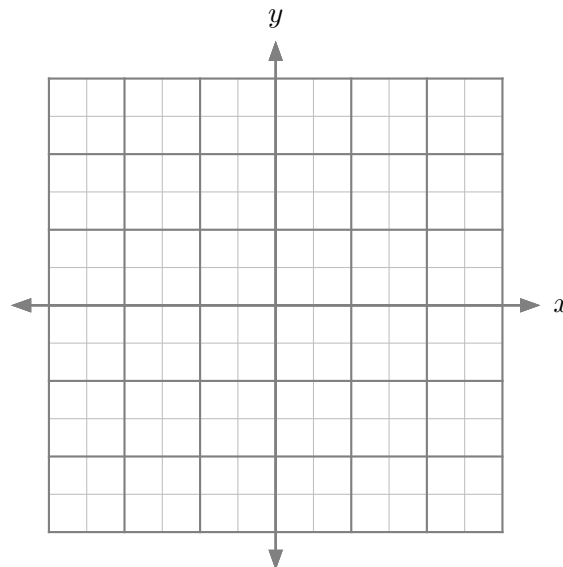
94 Model moon motion

A moon of a planet in a demonstration model of a planetary system travels a trajectory described by the position vector

$$\vec{r}(t) = 2.0 \text{ m} \cos(\omega t) \hat{i} + 2.0 \text{ m} \sin(\omega t) \hat{j}.$$

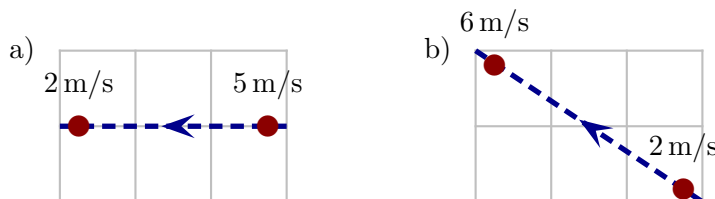
where $\omega = 0.50\pi \text{ rad s}^{-1}$ (note that this expresses the arguments of the trigonometric functions in terms of radians). (131F2024)

- Determine expressions for the horizontal and vertical coordinates $x(t)$ and $y(t)$ for the moon from $t = 0\text{s}$ to $t = 2.0\text{s}$.
- Sketch the trajectory of the moon as accurately as possible on the illustrated grid.
- Determine the displacement vector from $t = 1.0\text{s}$ to $t = 1.5\text{s}$ and use this to get the average velocity vector from $t = 1.0\text{s}$ to $t = 1.5\text{s}$.
- Determine the displacement vector from $t = 1.0\text{s}$ to $t = 1.1\text{s}$ and use this to get the average velocity vector from $t = 1.0\text{s}$ to $t = 1.1\text{s}$.
- Determine the displacement vector from $t = 1.0\text{s}$ to $t = 1.01\text{s}$ and use this to get the average velocity vector from $t = 1.0\text{s}$ to $t = 1.01\text{s}$.
- What do your results suggest about the direction of the instantaneous velocity vector at $t = 1.0\text{s}$? Explain your answer.



95 Acceleration vectors from motion

Two particles move along the illustrated trajectories. Their locations and speeds are illustrated at instants 0.25 s apart. For each case determine the acceleration vector for the particle. (131F2024)



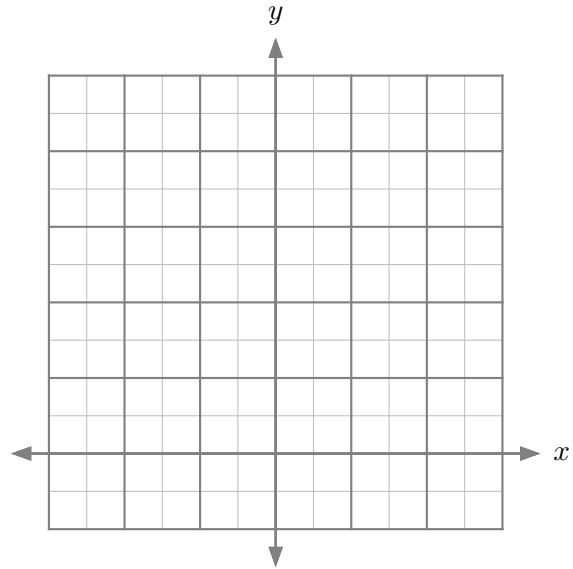
96 Ball moving in two dimensions

A ball travels along a trajectory described by the position vector

$$\vec{r}(t) = (t - 2)\hat{i} + \left(-\frac{(t - 2)^2}{2} + 4\right)\hat{j}.$$

from $t = 0$ to $t = 4$. (131F2024)

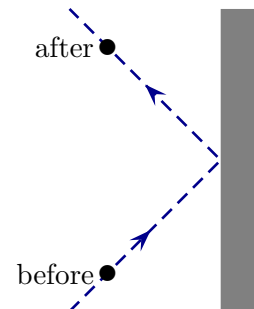
- Determine expressions for the horizontal and vertical coordinates $x(t)$ and $y(t)$ for the ball.
- Sketch the trajectory of the ball as accurately as possible on the illustrated grid.
- Determine the displacement vector from $t = 1.5$ to $t = 2$ and use this to get the average velocity vector from $t = 1.5$ to $t = 2$.
- Determine the displacement vector from $t = 2$ to $t = 2.5$ and use this to get the average velocity vector from $t = 2$ to $t = 2.5$.
- Determine an expression for the instantaneous velocity vector and use this to obtain the average acceleration from $t = 1.8$ to $t = 2.2$. What does this imply for the direction of the acceleration at $t = 2$?



97 Bouncing hockey puck

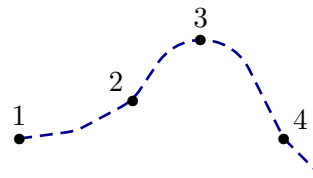
A hockey puck slides along a horizontal surface toward a board, hitting it at an angle and bouncing off with unchanged speed. The view from above is as illustrated. (131F2024)

- Draw the velocity vectors of the puck just before and just after hitting the board, use these to draw the vector $\Delta\vec{v}$, and use the result to draw the direction of the acceleration vector.
- If the puck traveled backwards along the same path (i.e. diagonally from top left to the board and then toward the bottom left), what would the direction of the acceleration vector be? Explain your answer.



98 Ants moving along a curved path

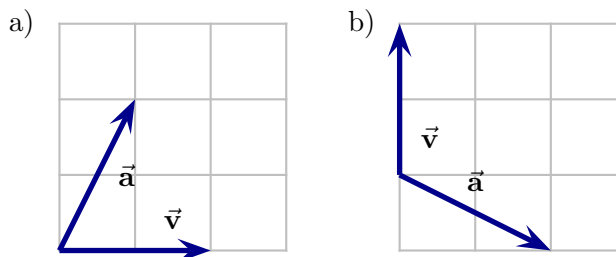
Various ants follow the same path on a horizontal surface, starting at point 1. The path is as illustrated. Ant A moves with a constant speed, ant B gradually speeds up and ant C gradually slows. (131F2024)



- a) Draw the velocity vector at points 1, 2, 3 and 4.
- b) Does any of the ants have zero acceleration at all times? Explain your answer.

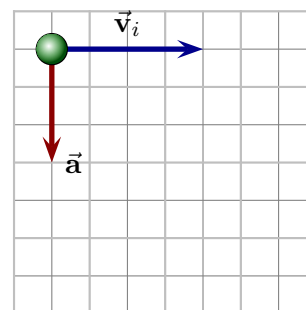
99 Velocity and acceleration vectors and motion

Position and acceleration vectors for two particles at one moment are illustrated. For each case, describe whether the particle is speeding up, slowing down or moving with constant velocity. Also describe how the direction of the particle's motion changes. Explain your answers. (131F2024)



100 Constant vertical acceleration

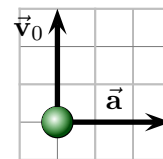
A ball launches off a horizontal surface. At the moment of launch its velocity is \vec{v}_i . At all later times it accelerates with a constant acceleration, \vec{a} . The situation with the vectors drawn to scale is illustrated (for the velocity vector, the grid unit is the standard unit of velocity and for acceleration the grid unit is the standard unit of acceleration). (131F2024)



- a) Draw, as accurately as possible, the velocity vector, \vec{v}_f , at an instant 1.0s after the initial instant.
- b) Using \vec{v}_f describe whether the object is moving faster at the 1.0s instant than at the initial instant.
- c) Using \vec{v}_f describe the direction in which the object is moving at the 1.0s instant.

101 Constant acceleration in two dimensions

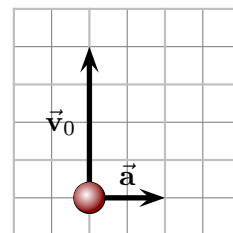
A ball can slide along a horizontal surface. At an initial instant its velocity is \vec{v}_0 . At all later times it accelerates with a constant acceleration, \vec{a} . The situation with the vectors drawn to scale as viewed from above is illustrated (for the velocity vector, the grid unit is the standard unit of velocity and for acceleration the grid unit is the standard unit of acceleration). (131F2024)



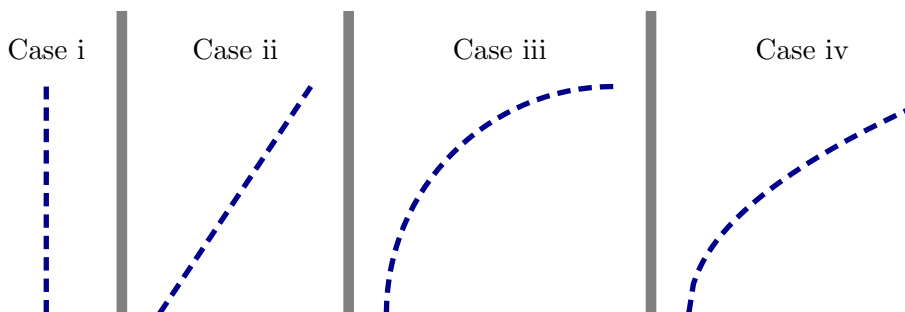
- Draw, as accurately as possible, the velocity vector at an instant 1.0s after the initial instant.
- Let v_0 denote the speed at the initial instant. Determine an expression for the speed, in terms of v_0 , at an instant 1.0s after the initial instant.
- Determine the time for the speed v to be twice the speed at the initial moment.

102 Motion with constant acceleration

A ball can slide along a horizontal surface. At an initial instant its velocity, \vec{v}_0 , is as illustrated. At all later times it accelerates with a constant acceleration, \vec{a} , as illustrated. (131F2024)



- Draw, as accurately as possible, the velocity vector at an instant 0.5s after the initial instant.
- Draw, as accurately as possible, the velocity vector at an instant 1.0s after the initial instant.
- Which of the following best represents the trajectory of the particle? Explain your answer.



103 Dropping a coin on a ship

A person is on the deck of a ship. While the ship is at rest in a harbor, the person drops a coin from rest and the coin lands on the deck of the ship. The person marks the spot where the coin lands. Later the ship sails with constant speed in a straight line and the person returns to the same spot and drops the coin again. Does the coin land at the original mark, in front of it or behind it? Explain your answer. (131F2024)

104 Throwing a ball on a train

A person is inside a train that moves horizontally in a straight line with speed v_{train} . The person launches a ball vertically with speed, v_{launch} (from the perspective of the person in the train). Let v be the speed of the ball at the moment of launch as viewed from outside the train. Which of the following is true? Explain your answer. (131F2024)

- i) $v = v_{\text{train}}$
- ii) $v = v_{\text{launch}}$
- iii) $v = v_{\text{train}} + v_{\text{launch}}$
- iv) $v = \sqrt{v_{\text{train}}^2 + v_{\text{launch}}^2}$

105 Jumping monkey

A monkey jumps leaving the Earth at an angle of 45° from the surface. Which of the following is true when the monkey reaches the highest point along the trajectory? Explain your answer. (131F2024)

- i) The acceleration of the monkey is zero.
- ii) The acceleration of the monkey is horizontal in the forwards direction.
- iii) The acceleration of the monkey is horizontal in the backwards direction.
- iv) The acceleration of the monkey is vertical downward.
- v) The acceleration of the monkey is vertical upward.

106 Red vs. blue ball

Two balls, one red and the other blue, are on a horizontal table. They are made to roll off the table. At the instants that they leave the speed of the blue ball is four times that of the red ball. Which of the following is true regarding the times taken to hit the floor on which the table stands? Explain your answer. (131F2024)

- i) The time taken for blue ball is a quarter of that for the red ball.
- ii) The time taken for blue ball is half that for the red ball.
- iii) The time taken for blue ball is the same as that for the red ball.
- iv) The time taken for blue ball is twice that for the red ball.
- v) The time taken for blue ball is four times that for the red ball.

107 Balls launched off horizontal surfaces

Two balls, one red and the other blue, are on horizontal surfaces above a floor. They are each launched horizontally off the surface with the same speed. The height of the blue ball surface above the floor is nine times that of the red ball surface. Let t_{red} be the time between the moment that the red ball leaves the table until it hits the floor; similarly t_{blue} represents the same for the blue ball. Let Δx_{red} be the horizontal distance between the point where the red ball leaves the table and hits the ground. (131F2024)

- a) Suppose that the balls are launched with the same speed. Which of the following is true? Explain your answer.
- i) $t_{\text{blue}} = \frac{1}{9} t_{\text{red}}$
 - ii) $t_{\text{blue}} = \frac{1}{3} t_{\text{red}}$
 - iii) $t_{\text{blue}} = t_{\text{red}}$
 - iv) $t_{\text{blue}} = 3 t_{\text{red}}$
 - v) $t_{\text{blue}} = 9 t_{\text{red}}$
- b) Suppose that the balls are launched with the same speed. Which of the following is true? Explain your answer.
- i) $\Delta x_{\text{blue}} = \frac{1}{9} \Delta x_{\text{red}}$
 - ii) $\Delta x_{\text{blue}} = \frac{1}{3} \Delta x_{\text{red}}$
 - iii) $\Delta x_{\text{blue}} = \Delta x_{\text{red}}$
 - iv) $\Delta x_{\text{blue}} = 3 \Delta x_{\text{red}}$
 - v) $\Delta x_{\text{blue}} = 9 \Delta x_{\text{red}}$
- c) Suppose that the blue ball is launched with a slower speed than the red ball. Describe how this would affect the answers of the previous questions.

108 Running off a roof

A person runs with speed 8.0 m/s off a flat roof that is 3.0 m above the ground. First suppose that the person travels horizontally at the moment that he leaves the roof. Determine how far horizontally from the edge of the roof the person will land. (131F2024)

- a) Sketch the situation with the “earlier” instant being that at which the person leaves the roof and the “later” instant being the moment just before the person hits the ground. List as many of the variables as possible. Use the format:

$$\begin{array}{ll} t_i = & t_f = \\ x_i = & x_f = \\ y_i = & y_f = \\ v_{ix} = & v_{fx} = \\ v_{iy} = & v_{fy} = \\ a_x = & a_y = \end{array} .$$

- b) Sketch the velocity vector at the earlier moment and use this to determine the components of \vec{v}_i . Enter these in the list above.
- c) Identify the variable needed to answer the question of the problem. Select and write down a kinematic equation that contains this variable and attempt to solve it. You should see to solve the variable describing the horizontal position, you first need the value for another, currently unknown variable. Which variable is this?
- d) Use the vertical aspects of the object’s motion to solve for this other unknown variable and use this result to answer the question of this problem.

Suppose that the person ran and jumped from the building at an angle of 30° above the horizontal. This will change how far the person travels. Before answering that question, we ask, what is the maximum height above the ground reached by the person for this running jump?

- e) Sketch the velocity vector at the earlier moment and use this to determine the components of \vec{v}_i . Reconstruct the list of variables for the problem.
- f) Sketch the velocity vector at the instant when the person reaches his highest point. Use this to add additional information to the list of variables for the problem.
- g) Use the kinematic equations to determine the maximum height that the person reaches.
- h) Determine the speed of the person at the maximum height.
- i) Determine how far horizontally from the edge of the roof the person will land.

109 Projectile motion range

A person runs with speed 8 m/s off a flat roof that is 3.0 m above the ground. The person can launch himself at various angles and the purpose of this exercise is to determine the speed with which the person hits the ground and where on the ground the person lands (for various angles of launch). First suppose that the person travels horizontally at the moment that he leaves the roof. It was found that the person lands 6.3 m from the base of the building. (131F2024)

- a) Sketch the situation with the “earlier” instant being that at which the person leaves the roof and the “later” instant being the moment just before the person hits the ground. List as many of the variables as possible. Use the format

$$\begin{array}{ll} & t_f = \\ x_i = & x_f = \\ y_i = & y_f = \\ v_{ix} = & v_{fx} = \\ v_{iy} = & v_{fy} = \\ a_x = & a_y = \end{array} .$$

- b) Draw the velocity vector at the earlier instant and use this to determine the components of \vec{v}_0 . Enter these in the list above.
- c) Draw the velocity vector at the later instant. Describe whether the components are positive, negative or zero.
- d) The speed of the object is the magnitude of the velocity vector. So to determine the speed, you will first need to determine the components of the velocity vector. Use the kinematic equations to determine the x and y components of the velocity at the later instant. You must start by writing the equation that you use, before substituting.
- e) Sketch the velocity vector at the instant just before the person hits the ground and indicate the values of the x and y components on the sketch. Use this to determine the speed of the person just before hitting the ground.

Now suppose that the person travels with the same speed but launches himself at an angle of 45° from the roof.

- f) Using the same “earlier” and later instants as before, list as many of the variables as possible.
- g) Draw the velocity vector at the earlier moment and use this to determine the components of \vec{v}_0 .
- h) Use the kinematic equations to determine the x and y components of the velocity at the later instant.
- i) Sketch the velocity vector at the instant just before the person hits the ground and indicate the values of the x and y components on the sketch. Use this to determine the speed of the person just before hitting the ground.

- j) Determine the time taken to hit the ground.
- k) Determine how far the person lands from the building.

110 Aircraft dropping object

An aircraft flies horizontally with a constant speed of 600 km/h at a height of 1200 m above a flat surface. It drops an object from its underside; this object is supposed to hit a particular spot on the ground. How far (horizontally) from the spot must the aircraft be for the object to hit the spot? (131F2024)

111 Rifle shot

A rifle fires a bullet. At the moment that the bullet leaves the rifle it travels horizontally with speed 300 m/s. Determine how far the bullet will drop by the time that it reaches a vertical wall that is 100 m from the end of the rifle that is closest to the wall. (131F2024)

112 Ball thrown horizontally

A ball is thrown, leaving the hand horizontally at a height of 2.0 m above the ground. It lands a horizontal distance of 5.0 m from where it left the hand. (131F2024)

- a) Determine the time from when the ball leaves the hand until it hits the ground.
- b) Determine the speed with which the ball leaves the hand.

113 Rock thrown horizontally

You throw a rock, which leaves your hand 1.5 m above the ground. You would like it to land a horizontal distance of 18.0 m from where it left the hand. How fast must you throw the ball? (131F2024)

114 Jumping over a ditch

A dog attempts to jump over a ditch, which is 2.0 m wide. The dog launches itself from one edge of the ditch with speed 5.0 m/s at an angle of 30° from the horizontal. The aim of this exercise is to determine whether the dog will reach the other side of the ditch. First we will find out whether the dog reaches its maximum height before or after it is above the middle of the ditch. (131F2024)

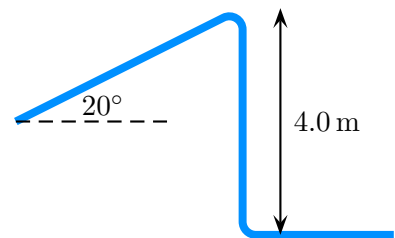
- a) Sketch the situation with the “earlier” instant being that at which the dog launches and the “later” instant being the moment when it reaches its highest point. List as many of the variables as possible. Use the format

$t_i =$	$t_f =$	
$x_i =$	$x_f =$	
$y_i =$	$y_f =$	
$v_{ix} =$	$v_{fx} =$	
$v_{iy} =$	$v_{fy} =$	
$a_x =$	$a_y =$.

- b) Draw the velocity vector at the earlier instant and use this to determine the components of \vec{v}_i . Enter these in the list above.
- c) Draw the velocity vector at the later instant. Describe whether the components are positive, negative or zero and enter as much information about these in the list above.
- d) Determine the horizontal distance traveled by the dog by the time that it reaches its maximum height.
- e) As the dog descends from its maximum height back to the ground, how much further does it travel? Does it reach the other side of the ditch?

115 Launching off a ski ramp, 1

A ski ramp is arranged as illustrated. A skier launches off the ramp with a speed of 15 m/s. Initially the aim of this exercise is to determine the maximum height reached by the skier and the velocity at this point. A later goal is to determine the distance at which the skier lands from the bottom of the ramp. (131F2024)



- a) Sketch the situation with the “earlier” instant being that at which the skier launches and the “later” instant being the moment when she reaches its highest point. List as many of the variables as possible. Use the format

$t_0 =$	$t_1 =$
$x_0 =$	$x_1 =$
$y_0 =$	$y_1 =$
$v_{0x} =$	$v_{1x} =$
$v_{0y} =$	$v_{1y} =$
$a_x =$	$a_y =$

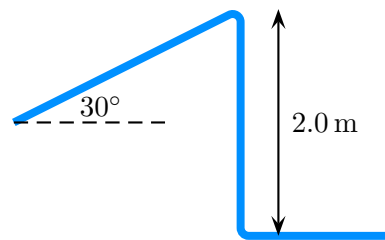
- b) Draw the velocity vector at the earlier instant and use this to determine the components of \vec{v}_0 . Enter these in the list above.
- c) Draw the velocity vector at the later instant. Describe whether the components are positive, negative or zero and enter as much information about these in the list above.
- d) Determine the time taken to reach the maximum height and then the horizontal distance traveled by the skier to reach her maximum height. Determine the velocity at this point.

You will now consider the motion from the highest point back to the ground.

- e) Repeat the problem set-up with the “earlier” instant being that at which the skier is at maximum height and the “later” instant being the moment *just before* she reaches hits the ground. Determine the time taken for this portion of the motion and use it to determine the horizontal distance from the base of the ramp to the skier’s landing point.

116 Launching off a ski ramp, 2

A ski ramp is arranged as illustrated. A skier launches off the top of the ramp with a speed of 28 m/s. Determine the horizontal distance at which the skier lands from the bottom of the ramp. (131F2024)



117 Jumping grasshopper

A grasshopper jumps, leaving the ground at an angle of 70° from the horizontal. The grasshopper reaches a height of 0.60 m. (131F2024)

- Determine the speed with which the grasshopper leaves the ground.
- Determine how far the grasshopper lands from where it jumped.

118 Jumping lemur

A lemur (a type of primate) jumps, leaving the ground at an angle of 40° from the horizontal with speed 6.0 m/s. (131F2024)

- Determine the maximum height reached by the lemur.
- Determine the amount of time for which the lemur is airborne.
- Determine the horizontal distance traveled by the lemur from the moment it leaves the ground until it returns to the ground.

119 Diver splashdown

A diver launches off a platform at an angle of 55° above the horizontal and with speed 8.0 m/s. The platform is 3.0 m above the surface of a pool. (131F2024)

- Determine the diver's speed just before hitting the water.
- Determine the time taken by the diver to hit the water.

120 Stone thrown from a bridge

A person stands on a bridge over a small lake. The person throws a stone with speed 18.0 m/s at an angle of 40° above the horizontal. The stone leaves the hand at a height 3.0 m above the surface of the water. (131F2024)

- Determine the time taken for the stone to reach the highest point in its trajectory.
- Determine the time taken for the stone to hit the water.
- Determine the horizontal distance traveled by the stone between the throw and when it hits the water.

121 Cannonball range

A cannonball is fired from the ground (assume that the ball leaves from the ground level) and must hit a target on the ground a distance of 500 m away. Assume that the ground is horizontal and ignore air resistance. (131F2024)

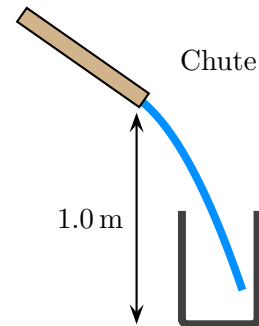
- Suppose that the cannonball leaves at an angle of 50° from the horizontal. Determine the speed with which it must be fired to hit the target.
- Suppose that the cannonball is fired with speed 150 m/s. Determine the angle above the horizontal at which it must be fired so that it hits the target.
- (Challenging) Determine the minimum speed with which the cannonball must be fired so that it can hit the target if the angle is adjusted correctly.

122 Angry tennis player

An angry tennis player hits a tennis ball into the air toward the fence that surrounds the court. The tennis player is a horizontal distance of 20 m from the fence, which is 10 m high. The tennis player hits the ball with speed 28 m/s at an angle of 70° above the horizontal from a height of 1.2 m above the ground. Does the ball reach the fence? Does it pass over the top of the fence? Explain your answer. (131F2024)

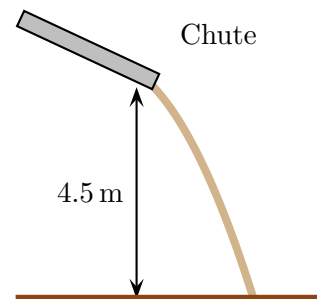
123 Water chute and bucket

Water slides down a chute that is at an angle of 35° above the horizontal. It leaves the end of the chute with speed 3.0 m/s. A bucket with sides 30 cm high and diameter 20 cm is placed beyond the chute so that it catches the water. Determine where the left edge of the bucket should be placed so that the water will land in the bucket. Provide the entire range of possible locations. (131F2024)



124 Grain chute

Grain pours off a chute that is at an angle of 25° above the horizontal. It leaves the end of the chute with speed 5.0 m/s. Determine how far it travels horizontally. (131F2024)



125 Merry-go-round

A merry-go-round is a large flat disk that spins around a vertical axis through its center. A child is at the edge of a merry-go-round with radius 3.0 m. The merry-go-round spins so that the child's acceleration is $1.5g$. Determine the period and frequency of orbit for this to occur. (131F2024)

126 Rotating mountain bike wheel

One standard mountain bike wheel has rims with diameter 29in. A cyclist rides the bike with a constant speed of 15 mph. Determine the speed and centripetal acceleration of a point on the rim in metric units. (131F2024)

127 Vinyl turntable

A turntable that plays vinyl records rotates at 33 rpm. The radius of the turntable is 0.15 m. (131F2024)

- a) Determine the frequency of the turntable.
- b) Determine the period of the turntable.
- c) Determine the speed of a point on the edge of the turntable.

128 Ball on a string

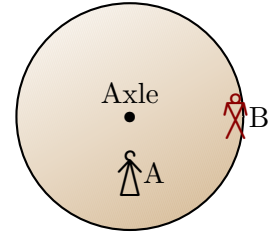
A ball swings with constant speed on the end of a 50 cm long string. Determine the period of orbit (time for one circle) such that the ball's acceleration is $2g$. (131F2024)

129 Orbiting object

An object orbits in a circle with constant speed. The radius of the orbit is r and the object completes one orbit in time T (the period of orbit). Determine an expression for the object's acceleration in terms of T and r . (131F2024)

130 Rotating dolls

Two dolls sit in fixed places on a disk that rotates about its center. Doll A is halfway from the axle to the rim. Doll B is at the rim. The wheel rotates at a uniform constant rate. Let v_A be the speed of A, v_B be the speed of B, a_A the acceleration of A, etc. In each of the following, explain your answer. (131F2024)



a) Which of the following is true?

i) $v_A = \frac{1}{4} v_B$

ii) $v_A = \frac{1}{2} v_B$

iii) $v_A = v_B$

iv) $v_A = 2v_B$

v) $v_A = 4v_B$

b) Which of the following is true?

i) $a_A = \frac{1}{4} a_B$

ii) $a_A = \frac{1}{2} a_B$

iii) $a_A = a_B$

iv) $a_A = 2a_B$

v) $a_A = 4a_B$

131 Acceleration on Grand Mesa versus Grand Junction

Consider standing in two places: one in Grand Junction and the other on top of Grand Mesa. The altitude of Grand Junction is roughly 1380 m and Grand Mesa 3200 m. The aim of this exercise is to determine the difference in velocity and acceleration between the two locations. Grand Junction's location latitude is about 39° North. Note that all objects rotate about an axis through Earth's poles. (131F2024)

- Determine the speed with which a person in Grand Junction moves as Earth (radius 6400 km at sea level) rotates.
- Determine the speed with which a person on top of Grand Mesa moves.
- Determine the acceleration of a person in Grand Junction.
- Determine the acceleration of a person on the Grand Mesa. Are the accelerations noticeably different?

132 Acceleration on Earth's surface

People stand on Earth's surface and are at rest *relative to Earth*. Earth has a radius of 6.4×10^3 km and spins about its poles at a rate of one revolution every 24 hrs. (131F2024)

- Determine the acceleration of a person at Earth's equator.
- Another person stands at a location much closer to the North pole. Is this person's acceleration the same as, larger than or smaller than that of a person at the equator? Explain your answer.

133 Centripetal acceleration

The formula for centripetal acceleration is

$$a = \frac{v_t^2}{r}.$$

In this exercise you will use calculus to prove that this is true. (131F2024)

- Consider an object whose position is represented by the vector (from the origin)

$$\vec{\mathbf{r}} = r \cos(\omega t)\hat{\mathbf{i}} + r \sin(\omega t)\hat{\mathbf{j}}.$$

By considering this at various times, show that it represents the position of a particle traveling in a circle of radius r and with angular velocity ω .

- Determine the tangential velocity vector

$$\vec{\mathbf{v}}_t = \frac{d\vec{\mathbf{r}}}{dt}$$

and the acceleration vector

$$\vec{\mathbf{a}} = \frac{d\vec{\mathbf{v}}_t}{dt}.$$

- Show that

$$\vec{\mathbf{a}} = -\omega^2\vec{\mathbf{r}}$$

and use this to show that

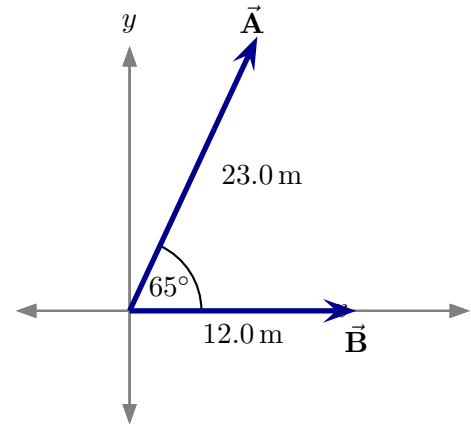
$$a = \omega^2 r = \frac{v_t^2}{r}.$$

Additional Questions

134 Vector subtraction, 1

Two displacement vectors, \vec{A} and \vec{B} are illustrated.
(131F2024)

- Determine the components of $\vec{C} = \vec{A} - \vec{B}$.
- Determine the magnitude of \vec{C} .



135 Ball launched from a cliff

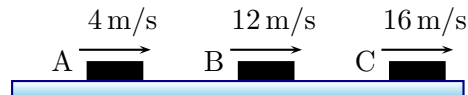
A person hits a ball from the top of a cliff. The ball leaves at a height of 12 m above the surface of the water at an angle 30° above the horizontal. It hits the water 2.5 s later. (131F2024)

- Determine the speed with which the ball is launched.
- Determine the horizontal distance traveled by the ball.

Dynamics

136 Hockey pucks sliding horizontally

Three identical hockey pucks slide horizontally across a frictionless sheet of ice and they maintain the indicated speeds during a particular period. Let F_A be the magnitude of the force acting on A during this period, F_B be the magnitude of the force acting on B, etc. . . . Which of the following is true? Explain your answer. (131F2024)

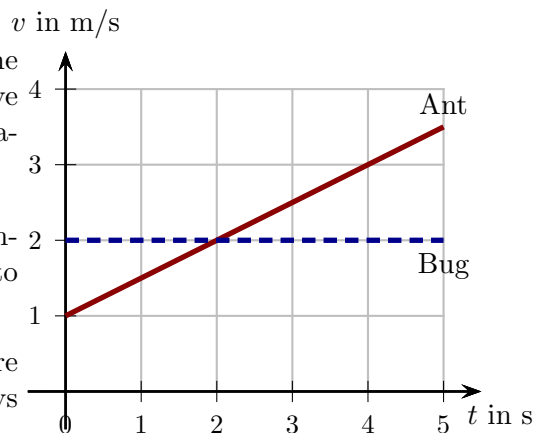


- i) $F_B = 2F_A$ and $F_C = 4F_A$
- ii) $F_A = 2F_B$ and $F_A = 4F_C$
- iii) $F_A = F_B = F_C \neq 0$
- iv) $F_A = F_B = F_C = 0$

137 Forces on an ant and a bug

An ant and a bug walk along straight wires. The graph illustrates their velocities vs. time. They have the same mass. Answer the following, giving explanations for each answer. (131F2024)

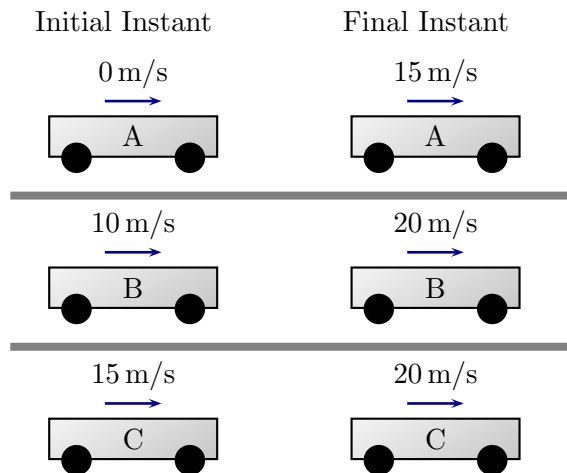
- a) How does the net force on the ant at 2.0s compare (same, larger, four times as large, etc.,) to the net force on the ant at 4.0s?
- b) How does the net force on the ant compare (same, always larger, sometimes larger, always smaller,...) to the net force on the bug?
- c) Is the net force exerted on the ant ever the same as that exerted on the bug? If so, explain when.



138 Moving carts

Three identical carts move horizontally along tracks. Their speeds at two instants 5.0 s apart are indicated. Let F_A be the magnitude of the net force acting on A during this interval, F_B be the magnitude of the net force acting on B, etc. . . . Which of the following is true? Explain your answer. (131F2024)

- i) $F_A > F_B > F_C$.
- ii) $F_B = F_C > F_A$.
- iii) $F_B = F_C < F_A$.
- iv) $F_A = F_B = F_C \neq 0$



139 Riding a bicycle

Suppose that you ride a bicycle at constant speed in straight lines across various surfaces. In each of the following, explain your answers. (131F2024)

- a) If you ride at a constant speed along a flat horizontal surface, which of the following is true?
 - i) The net force on the bicycle is zero.
 - ii) The net force on the bicycle is in the direction of motion.
 - iii) The net force on the bicycle is opposite to the direction of motion.
 - iv) Whether the net force is zero or not depends on the speed.
- b) If you ride at a constant speed along a slanted flat uphill surface, which of the following is true?
 - i) The net force on the bicycle is zero.
 - ii) The net force on the bicycle is in the direction of motion.
 - iii) The net force on the bicycle is opposite to the direction of motion.
 - iv) Whether the net force is zero or not depends on the speed.

140 Pushing carts

Zog and Geraldine (his wife) each push a cart along a horizontal surface where friction is negligible. Both carts are initially at rest. Zog takes the cart with mass 25 kg and exerts a force of 400 N on it for a period of 4.0 s and he then collapses and stops pushing. Geraldine has to push a cart of mass 50 kg and she is also able to exert a force of 400 N on it. Geraldine claims that it is possible for the speed of her cart to eventually reach the speed of Zog's cart. Is this true? Explain your answer. (131F2024)

141 Forces on a cart sliding horizontally

A cart can slide horizontally left or right. (131F2024)

a) Suppose that the cart slides to the left with increasing speed. Which of the following is true about the net force, \vec{F}_{net} , acting on the cart? Explain your choice.

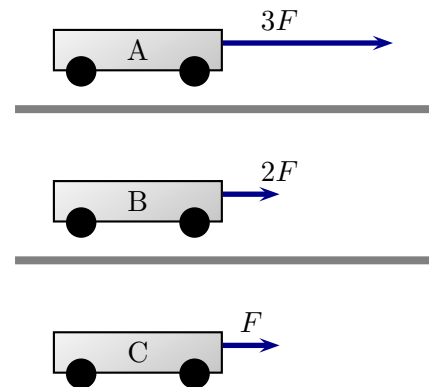
- i) $\vec{F}_{\text{net}} = 0$
- ii) $\vec{F}_{\text{net}} \neq 0$ and points right.
- iii) $\vec{F}_{\text{net}} \neq 0$ and points left.

b) Suppose that the cart slides to the left with decreasing speed. Which of the following is true about the net force acting on the cart? Explain your choice.

- i) $\vec{F}_{\text{net}} = 0$
- ii) $\vec{F}_{\text{net}} \neq 0$ and points right.
- iii) $\vec{F}_{\text{net}} \neq 0$ and points left.

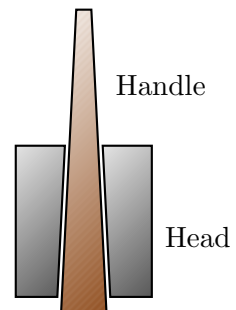
142 Moving carts

Consider three carts with different masses. Different forces act on each cart and each is expressed as a multiple of some basic force, F . The mass of cart A is 8 times the mass of cart C. The mass of cart B is 3 times the mass of cart C. Rank the carts in order of increasing acceleration. Explain your answer. (131F2024)



143 Pickax

A pickax consists of a tapered wooden handle and a metal head that slides over the wooden handle. An exaggerated simplified cross section view is illustrated. The head sometimes works loose. One way to secure it is to hold the handle vertical with the head at the bottom and then drop the handle. Use Newton's laws to explain why even letting the handle hit the ground gently is likely to secure the head more tightly than hitting the head from above. (131F2024)



144 Force, acceleration and velocity

The exercise will investigate the assertion:

“The larger the force on an object, the larger the object’s velocity.”

Consider two carts, labeled A and B, each with mass 3.0 kg that can slide left or right. The net force on cart A is 12.0 N to the right and the net force on cart B is 6.0 N to the right. It would appear that the velocity of A is twice the velocity of B. Consider the situation where the velocity of each cart at 0.0 s is exactly 0.0 m/s and that the forces act on the carts at all later times. Explain your answers in the following. (131F2024)

- Determine the velocity of each cart at 1.0 s, 2.0 s, 3.0 s, 4.0 s and 5.0 s.
- Is the “velocity of A twice the velocity of B?”
- Is the “velocity of A at 2.0 s twice the velocity of B at 4.0 s?” Does this suggest a need to be specific with statements about velocity?

Now consider a different situation where the velocity of A at 0.0 s is exactly -8.0 m/s and the velocity of B is 0 m/s.

- Determine the velocity of each cart at 1.0 s, 2.0 s, 3.0 s, 4.0 s and 5.0 s.
- Is the “velocity of A twice the velocity of B?”
- Is the “velocity of A at 3.0 s twice the velocity of B at 3.0 s?”
- Is the statement: “The larger the force on an object, the larger the object’s velocity” true or not in general?

145 Lowering a bunch of bananas

A bunch of bananas has mass 40 kg. Starting from rest, they are lowered from a ship by a rope through a distance of 12.0 m. The rope pulls vertically upward with tension 305 N and there is an upward air resistance force of 60 N. Determine the time taken to lower the bananas. (131F2024)

146 Sledding dog

A 37.0 kg dog sits on a 3.0 kg sled that can slide along a horizontal frictionless sheet of ice. At one instant the dog and sled are moving right with speed 6.0 m/s. For the next 4.0 s a person exerts a constant 80 N force on the dog/sled in the same direction as they move. (131F2024)

- Determine the velocity of the dog/sled at the end of the 4.0 s period.
- Determine the distance traveled by the dog/sled during the 4.0 s period.

147 Force on an accelerating box

A 50 kg box is initially at rest on a horizontal sheet of ice. Subsequently someone pulls on the box with a constant horizontal force. The box moves 15 m during the first 10 s while the force is applied. Determine the magnitude of the force. (131F2024)

148 Moving a sleepy dog

A sleepy 30 kg dog lies on a horizontal surface. The dog is initially at rest. Subsequently a child pushes horizontally on the dog with a 150 N force. There is no friction between the dog and the surface and forces in the vertical direction are irrelevant to its motion. (131F2024)

- Determine how far the dog moves after the child has pushed for 3.0 s.
- Determine the speed of the dog after the child has pushed for 3.0 s.
- If the child continues pushing for twice as long will the speed be twice as much? Will the distance traveled be twice as much? Explain your answers.

149 Citrus fruit microjets

Citrus fruit emit aromatic oils when the peel is disrupted. This can occur in the form of microscopic scale jets of fluid, that had been stored in small glands within the fruit's peel. Recent studies have shown that the jets emitted from lemons leave with speed about 7.0 m/s. The aim of this problem is to estimate the force exerted on the jet emitted from a single gland. The volume of a typical gland in a lemon is $6.0 \times 10^{-10} \text{ m}^3$. Assuming the fluid has a similar density to water, this would imply that the fluid has mass $6.0 \times 10^{-7} \text{ kg}$. (131F2024)

- Suppose that this entire fluid was accelerated from rest to 7.0 m/s across a distance equal to the radius of a sphere with the volume of the gland. Show that the radius is $5.2 \times 10^{-4} \text{ m}$. Determine the acceleration of the fluid.
- Determine the net force on the fluid. How does this compare to the gravitational force on the fluid? Does gravity matter in this case?

Detailed analyses of this are much more sophisticated and eventually consider pressure within the glands. More details can be found in Smith, et.al. PNAS, **115** (26), E5887–E5895 (2018).

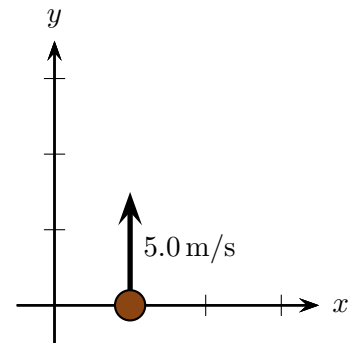
150 Pushing a curling stone across a frozen ice sheet

A 20 kg curling (a type of sport) stone lies at rest on a horizontal sheet of ice. The stone is pushed by a player in a straight line with a constant horizontal force for a distance of 3.0 m and is then released. It travels in a straight line for a further distance of 42 m in 20 s. There is no spin on the stone. Determine the magnitude of the force exerted by the player on the stone. (131F2024)

151 Forces and two dimensional motion

A 2.0 kg rock slides along a horizontal surface. At the moment that it passes the $x = 1.0\text{ m}$, $y = 0.0\text{ m}$ mark it is moving with the illustrated velocity. During the next 4.0 s a constant force $8.0\text{ N}\hat{i}$ acts on the rock. (131F2024)

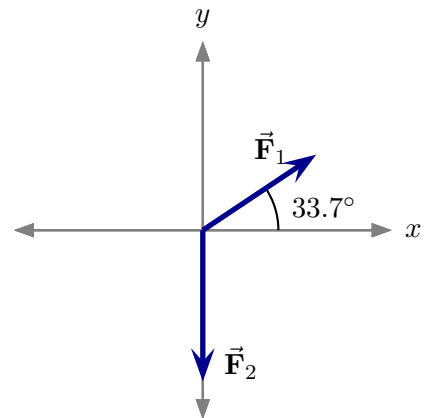
- Determine the rock's position and velocity at the instant 4.0 s later.
- Describe and sketch, as accurately as possible, the trajectory of the rock while the force acts on it.



152 Net force vector: two forces

Two forces, \vec{F}_1 and \vec{F}_2 act on an object and are as illustrated. Their magnitudes are $F_1 = 400\text{ N}$ and $F_2 = 500\text{ N}$. (131F2024)

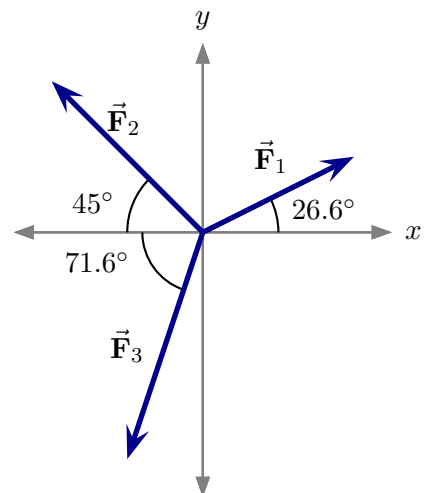
- Determine the x and y components of the net force.
- Determine the magnitude of the net force acting on the object.



153 Net force vector: three forces

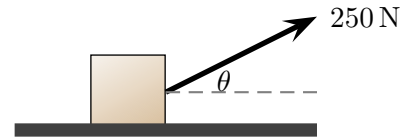
Three forces, \vec{F}_1 , \vec{F}_2 and \vec{F}_3 act on an object and are as illustrated. Their magnitudes are $F_1 = 40\text{ N}$, $F_2 = 25\text{ N}$ and $F_3 = 15\text{ N}$. (131F2024)

- Determine the x and y components of the net force.
- Determine the magnitude of the net force acting on the object.



154 Pulling a box

A 20.0 kg box can move along a frictionless horizontal surface. A person pulls on the box with the indicated force. Initially assume that the force pulls horizontally; later we will consider a general angle. (131F2024)



- Draw a free body diagram for the box.
- Write Newton's Second Law in its component form, i.e. write

$$F_{\text{net } x} = \Sigma F_x = \dots$$

$$F_{\text{net } y} = \Sigma F_y = \dots$$

Insert as much information as possible about the components of acceleration at this stage. You will return to these equations soon; they will generate the algebra that eventually gives you the acceleration and the normal force.

- List all the components of all the forces, using one of the two formats below.

$$F_{gx} = \dots$$

$$F_{gy} = \dots$$

$$n_x = \dots$$

$$n_y = \dots$$

$$F_{\text{person } x} = \dots$$

$$F_{\text{person } y} = \dots$$

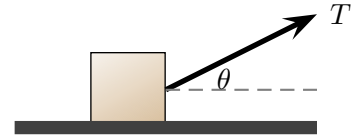
Force	x comp	y comp
\vec{F}_g		
\vec{n}		
\vec{F}_{person}		

Use these to determine expressions for the components of the net force.

- Use Newton's second law in component form to relate the acceleration components to the forces.
- Determine the normal force on the box.
- Determine the acceleration of the box.
- Now suppose that $\theta = 25^\circ$ and repeat the entire exercise to determine the normal force and the acceleration.

155 Rope pulling a box

A 45.0 kg box can move along a frictionless horizontal surface. A rope pulls on the box with tension T at angle θ . (131F2024)



- Do you expect that the normal force exerted by the floor depends on the tension and angle of rope pull?
- Draw a free body diagram for the box.
- Write Newton's Second Law in its component form and insert as much information as possible about the components of acceleration at this stage. These equations will generate the algebra that eventually gives you the acceleration and the normal force.
- List all the components of all the forces.
- Use Newton's second law in component form to relate the acceleration components to the forces.
- Determine an expression for the acceleration of the box.
- Determine an expression for the normal force on the box. Does the expression support your expectations about how normal force depends on the tension and the angle of pull?
- Determine the acceleration and normal force when the rope pulls horizontally with force 275 N.
- Determine the acceleration and normal force when the rope pulls midway between horizontally and vertically 275 N.

156 Dynamics of a single object

A 5.0 kg box can move along a frictionless horizontal surface. A person exerts a force at the illustrated angle. The aim of this exercise is to use Newton’s laws to determine the acceleration of the box and the normal force exerted by the surface (provided that the box stays on the surface). (131F2024)



The *entire collection* of these steps is called “applying Newton’s laws of mechanics to determine the acceleration of the block.”

- Draw a free body diagram for the block.
- Write Newton’s Second Law in its component form, i.e. write

$$F_{\text{net } x} = \Sigma F_x = \dots \quad (1)$$

$$F_{\text{net } y} = \Sigma F_y = \dots \quad (2)$$

Insert as much information as possible about the components of acceleration at this stage. You will return to these equations shortly; they will generate the algebra that eventually gives you the acceleration and the normal force.

- Determine the magnitude of the gravitational force. Let n be the *magnitude* of the normal force. Do you think that $n = mg$?
- List all the components of all the forces, using one of the two formats below.

$$F_{gx} = \dots$$

$$F_{gy} = \dots$$

$$n_x = \dots$$

$$n_y = \dots$$

⋮

Force	x comp	y comp
$\vec{\mathbf{F}}_g$		
$\vec{\mathbf{n}}$		
⋮		

- Use Eq. (1) to obtain an equation relating various quantities that appear in this problem. Do the same with Eq. (2). Solve these for the acceleration and the magnitude of the normal force. Is $n = mg$?
- Suppose that rather than pull up, the person pushed down on the box at the same angle from the left and with the same force. Would the acceleration and normal forces differ from the case where the person pulled up?
- You may have noticed that the acceleration does not depend on the normal force. This is only true if there is no friction. It turns out that when friction is present, the magnitude of friction force increases as the normal force increases. Knowing this, would pulling up or pushing down give a larger acceleration?

157 Multiple forces and motion

A 20.0 kg box can move along a frictionless horizontal surface. Three people pull horizontally on the box with the indicated forces. (131F2024)



- Draw a free body diagram for the box.
- Write Newton's Second Law in its component form, i.e. write

$$F_{\text{net } x} = \Sigma F_x = \dots$$

$$F_{\text{net } y} = \Sigma F_y = \dots$$

Insert as much information as possible about the components of acceleration at this stage. You will return to these equations soon; they will generate the algebra that eventually gives you the acceleration and the normal force.

- List all the components of all the forces, using one of the two formats below.

$$F_{1x} = \dots$$

$$F_{1y} = \dots$$

$$F_{2x} = \dots$$

$$F_{2y} = \dots$$

$$F_{3x} = \dots$$

$$F_{3y} = \dots$$

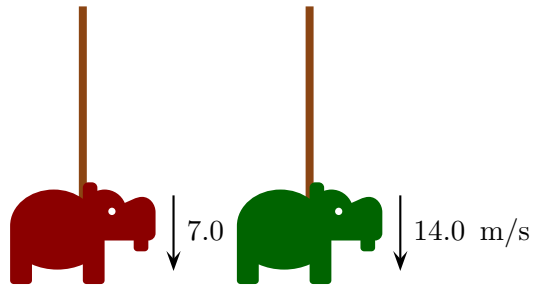
Force	x comp	y comp
\vec{F}_1		
\vec{F}_2		
\vec{F}_3		

Use these to determine the components of the net force.

- Use Newton's second law, which you wrote earlier, to determine the components of the acceleration of the box.
- If the box were moving right while these forces act would it slow down or speed up?
- If the box were moving left while these forces act would it slow down or speed up?

158 Lowering hippos

Two hippos are lowered vertically by cables. The hippos have the same mass. The red hippo is lowered at a constant speed of 7.0 m/s and the green hippo at a constant speed of 14.0 m/s . Let T_{red} be the tension in the rope lowering the red hippo and T_{green} be the tension in the rope lowering the green hippo. Which of the following is true? Explain your answer. (131F2024)



- i) $T_{\text{green}} < T_{\text{red}}/2$
- ii) $T_{\text{green}} = T_{\text{red}}/2$
- iii) $T_{\text{red}}/2 < T_{\text{green}} < T_{\text{red}}$
- iv) $T_{\text{green}} = T_{\text{red}}$
- v) $T_{\text{red}} < T_{\text{green}} < 2T_{\text{red}}$
- vi) $T_{\text{green}} = 2T_{\text{red}}$
- vii) $T_{\text{green}} > 2T_{\text{red}}$

159 Lowering a box with ropes

Two ropes are attached to the top of a 75 kg box. They both pull vertically up with the same tension. The box moves downwards with constant downward acceleration of 2.5 m/s^2 . Determine the tension in each rope. (131F2024)

160 Lowering a bunch of bananas

A bunch of bananas has mass 40 kg . Starting from rest, they are lowered from a ship by a rope through a distance of 12.0 m . The rope pulls vertically upward with tension 305 N and there is an upward air resistance force of 60 N . Determine the time taken to lower the bananas. (131F2024)

161 Box on the floor of an elevator

A 20 kg box sits on the floor of an elevator. (131F2024)

- a) During an initial 1.5 s period the elevator moves upward with a speed that increases steadily from 0 m/s to 7.5 m/s . Determine the normal force exerted by the floor on the box during this period.
- b) During a later 2.5 s period the elevator moves upward with a speed that decreases steadily from 7.5 m/s to 0 m/s . Determine the normal force exerted by the floor on the box during this period.

162 Alice in an elevator

Alice, with mass 58 kg stands in an elevator, which moves vertically. The elevator accelerates (at a constant rate) from rest to speed of 4.0 m/s during a period of 1.5 s. It then moves with a constant velocity for a period of 10.0 s. Finally it slows to a stop at a constant rate during a period of 2.0 s. (131F2024)

- a) Determine the total distance traveled by Alice during the entire journey.
- b) Determine the normal force exerted by the elevator floor on Alice during each of the three stages of motion.

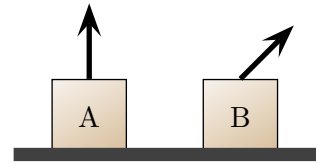
163 Free fall in an elevator

A phone with mass m sits on the floor of an elevator, which is initially at rest. The elevator cable snaps and the elevator and phone then undergo free fall (i.e. both move downward with an acceleration g). While they do this which is true of the magnitude of the normal force, n , acting on the phone? Explain your choice. (131F2024)

- i) $n = 0$.
- ii) $mg > n > 0$.
- iii) $n = mg$.
- iv) $n > mg$

164 Normal forces

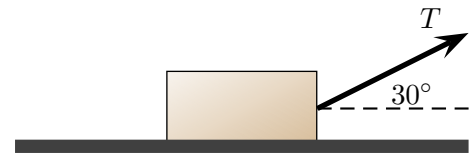
Two identical boxes are at rest on a rough horizontal surface. A person pulls on each with the same force but in different directions on the boxes. On box A it pulls vertically and on box B it pulls at an angle of 45° from the vertical. Which of the following is true regarding the normal force exerted on A and that exerted on B? Explain your choice. (131F2024)



- i) $n_A = n_B$.
- ii) $n_A < n_B$.
- iii) $n_A > n_B$.

165 Accelerating box

A rope pulls on a 30 kg box which slides across a frictionless horizontal surface. The box moves along the surface to the right with acceleration 3.5 m/s^2 . (131F2024)



- a) Determine the tension in the rope.
- b) Determine the normal force exerted by the floor on the box.

166 Multiple forces and motion

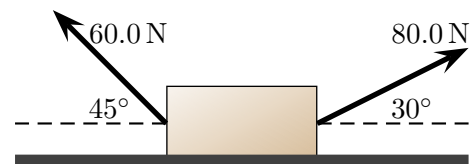
The illustrated forces act on an 80.0 kg box that slides along a frictionless horizontal surface. (131F2024)



- a) Determine the acceleration of the box.
- b) Suppose that at one moment the box was moving left. Will it slow down, speed up or move with constant velocity in the next moments? Explain your answer.
- c) Suppose that at one moment the box was moving right. Will it slow down, speed up or move with constant velocity in the next moments? Explain your answer.

167 Multiple forces on a single object

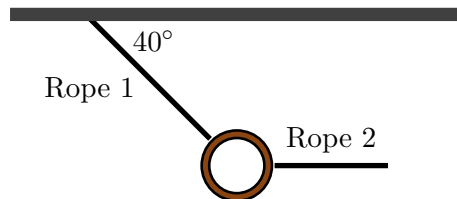
A 25.0 kg crate can move along a frictionless horizontal surface. Two people exert forces on the crate as illustrated. (131F2024)



- a) Determine the acceleration of the crate.
- b) Determine the normal force on the crate.

168 Suspended ring in equilibrium, 1

A 2.50 kg ring is suspended from the ceiling and is held at rest by two ropes as illustrated. Rope 2 pulls horizontally. The aim of this exercise is to use Newton's 2nd Law to determine the tension in each rope. One piece of background information that you will need to answer this is that the magnitude of the gravitational force on an object of mass m is $F_g = mg$. (131F2024)



- Draw a free body diagram for the ring. Label the tension forces \vec{T}_1 and \vec{T}_2 .
- Write Newton's 2nd Law in its component form, i.e. write

$$F_{\text{net } x} = \Sigma F_{ix} = ma_x \quad (3)$$

$$F_{\text{net } y} = \Sigma F_{iy} = ma_y \quad (4)$$

Insert as much information as possible about the acceleration. You will return to these equations shortly; they will generate the algebra that eventually gives you the tensions.

- These equations require all components of all forces, including the two unknown tension forces. In order to manage these, you should express the components of each tension force in terms of its magnitude. When doing this denote the magnitude of the tension in rope 1 by T_1 and for rope 2, by T_2 .
- List as much information as possible about each component for each force; each could be a number or an algebraic expression. Use one of the two formats below.

$$F_{gx} = \dots$$

$$F_{gy} = \dots$$

$$T_{1x} = \dots$$

$$T_{1y} = \dots$$

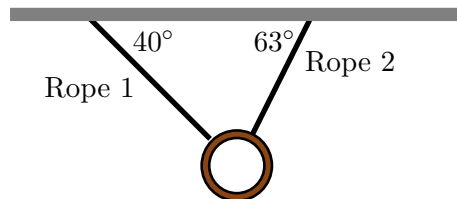
$$\vdots$$

Force	x comp	y comp
\vec{F}_g		
\vec{T}_1		
\vdots		

- Use Eq. (3) to obtain an equation relating various quantities that appear in this problem. Do the same with Eq. (4). You should get two expressions that contain the two unknowns T_1 and T_2 . Solve them for the unknowns.
- If you had one rope that is rated to break when the tension exceeds 30 N and another rated to break when the tension exceeds 40 N, which one would you use to suspend the object as illustrated above?

169 Suspended ring in equilibrium, 2

A 2.50 kg ring is suspended from the ceiling and by two ropes as illustrated. The aim of this exercise is to use Newton's 2nd Law to determine the tension in each rope. (131F2024)



- Draw a free body diagram for the ring. Label the tension forces \vec{T}_1 and \vec{T}_2 .
- Write Newton's 2nd Law in its component form, i.e. write

$$F_{\text{net } x} = \Sigma F_{ix} = ma_x \quad (5)$$

$$F_{\text{net } y} = \Sigma F_{iy} = ma_y \quad (6)$$

Insert as much information as possible about the acceleration. You will return to these equations shortly; they will generate the algebra that eventually gives you the tensions.

- These equations require all components of all forces, including the two unknown tension forces. In order to manage these, you should express the components of each tension force in terms of its magnitude. When doing this denote the magnitude of the tension in rope 1 by T_1 and for rope 2, by T_2 .
- List as much information as possible about each component for each force; each could be a number or an algebraic expression. Use one of the two formats below.

$$F_{gx} = \dots$$

$$F_{gy} = \dots$$

$$T_{1x} = \dots$$

$$T_{1y} = \dots$$

⋮

Force	x comp	y comp
\vec{F}_g		
\vec{T}_1		
⋮		

- Use Eq. (5) to obtain an equation relating various quantities that appear in this problem. Do the same with Eq. (6). You should get two expressions that contain the two unknowns T_1 and T_2 . Solve them for the unknowns.

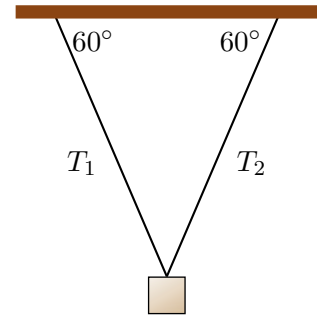
170 Suspended ball

A 6.0 kg ball is suspended from the ceiling. A person pushes horizontally on the ball so that the ball is at rest with the rope at an angle of 70° from the horizontal. Determine the force exerted by the person. (131F2024)

171 Object suspended at rest

A 0.20 kg block is suspended by two ropes as illustrated. The tension in the left rope is T_1 and in the right rope it is T_2 . (131F2024)

- Is T_1 the same as or different to T_2 ? Explain your answer.
- Determine the magnitude of the tension in each rope.



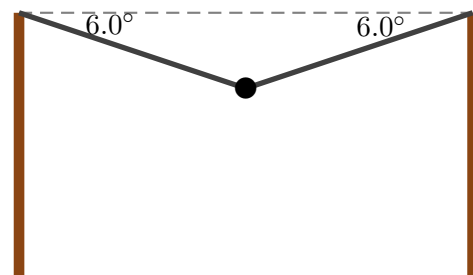
172 Stretched clothesline

A clothesline (rope) initially droops approximately horizontally between two poles. A hanger is suspended at rest from the midpoint of the clothesline. (131F2024)

- Is it possible to stretch the clothes line so that it is perfectly horizontally while the hanger is suspended? Explain your answer using Newton's laws.
- The hanger is removed. Is it possible to stretch the clothes line so that it is perfectly horizontally? Explain your answer using Newton's laws.

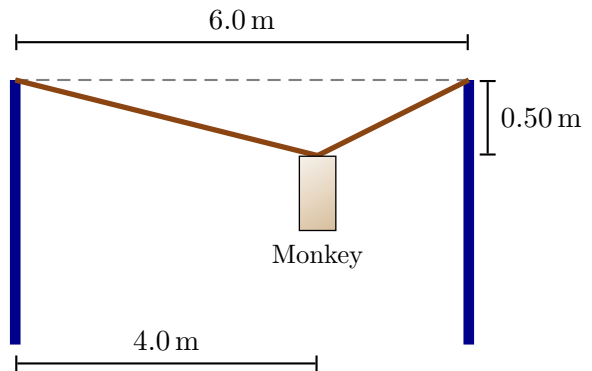
173 Slackline

A 75.0 kg person is at the midpoint of a slackline (cord stretched between two supports). The angle between the slackline and the horizontal is 6.0° . Determine the tension in the slackline. (131F2024)



174 Dangling monkey

A 15.0 kg monkey hangs from a rope stretched between the tops of two equally tall supports in the illustrated configuration. Determine the tension in the rope left of the monkey and the tension in the rope right of the monkey. (131F2024)



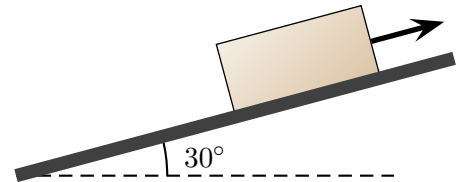
175 Sledding down a slope

A sled and person, with combined mass 100 kg slide down a flat frictionless surface that is angled at 20° above the horizontal. (131F2024)

- Determine the acceleration of the sled and person.
- Determine the normal force on the sled and person.

176 Object pulled along a ramp

A 4.0 kg box can move along a frictionless ramp angled 30° from the horizontal. A person pulls on a rope which exerts a force of 15 N up the ramp parallel to its surface. The object of this exercise is to determine the acceleration of the box. (131F2024)



- Draw a free body diagram for the box.
- Describe the x and y axes that you will use.
- Write Newton's Second Law in vector form and also in its component form, i.e. write

$$F_{\text{net } x} = \Sigma F_{ix} = \dots \quad (7)$$

$$F_{\text{net } y} = \Sigma F_{iy} = \dots \quad (8)$$

Insert as much information as possible about the components of acceleration at this stage. The resulting equations will generate much of the algebra that follows.

- Determine the magnitude of the gravitational force.
- List all the components of all the forces, using one of the two formats below.

$$F_{gx} = \dots$$

$$F_{gy} = \dots$$

$$n_x = \dots$$

$$n_y = \dots$$

$$\vdots$$

Force	x comp	y comp
$\vec{\mathbf{F}}_g$		
$\vec{\mathbf{n}}$		
\vdots		

- f) Use Eq. (7) to obtain an equation relating various quantities that appear in this problem. Do the same with Eq. (8). Use the resulting equations to determine the acceleration of the box.
- g) Is it possible to say with certainty whether the box is moving up the ramp or down the ramp? Is either direction possible in this situation? If only one direction is possible, which is it?
- h) Suppose that the box is initially at rest. With the indicated applied force, how long will it take for the box to slide a distance of 2.0 m along the ramp?

177 Bathroom scale in a train

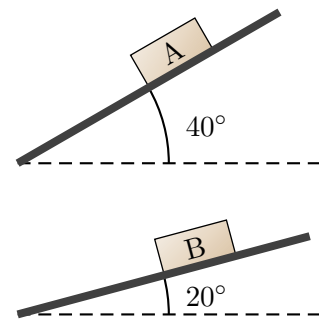
A bathroom scale reads the normal force that it exerts on the object on the scale. Suppose that a bathroom scale is fixed to the floor of a train and a person stands on the scale. The train travels at a constant speed along horizontal sections of track and then up and down inclined sections (the slopes of the inclined sections are the same). In the following explain your answers. (131F2024)

- Is the reading on the scale when traveling up the inclined section the same as, smaller than or larger than when traveling along the horizontal section?
- Is the reading on the scale when traveling down the inclined section the same as, smaller than or larger than when traveling up the inclined section?
- Would the reading on the scale while the train travels up the inclined section depend on the speed with which it travels?

178 Blocks on ramps

Two blocks with the same mass are at rest on the illustrated inclined surfaces. Which of the following is true about the magnitudes of the normal forces that act on the blocks? Explain your answer. (131F2024)

- $n_A > n_B$.
- $n_A = n_B$.
- $\frac{1}{2} n_B < n_A < n_B$.
- $n_A = \frac{1}{2} n_B$.
- $n_A < \frac{1}{2} n_B$.



179 Different boxes on the same ramp

Two boxes, one red (mass 3.0 kg) and the other blue (mass 6.0 kg) are each on frictionless ramps inclined at an angle of 10° from the horizontal. They are released from rest. Let a_{red} be the acceleration of the red box and a_{blue} that of the blue box. Which of the following is true? Explain your answer. (131F2024)

- $a_{\text{blue}} = \frac{1}{2} a_{\text{red}}$
- $a_{\text{blue}} = a_{\text{red}}$
- $a_{\text{blue}} = 2a_{\text{red}}$
- $a_{\text{blue}} > 2a_{\text{red}}$

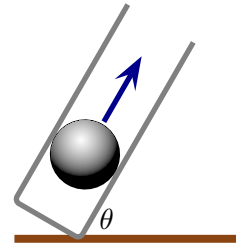
180 Snowboarder sliding up a ramp

An 80 kg snowboarder slides along a horizontal surface with a speed of 25 m/s and approaches a surface inclined at 10° above the horizontal. Both surfaces are frictionless and you can ignore air resistance. (131F2024)

- Determine the acceleration of the snowboarder along the inclined surface.
- Determine how far along the inclined surface the snowboarder slides before stopping.
- Determine the maximum vertical height ascended by the snowboarder along the ramp.

181 Launch force on a cannonball

A 40 kg cannonball is launched at an angle of θ above the horizontal. The cannonball is initially at rest at the ground level and reaches height h . The length of the cannon tube is L . The aim of this exercise is to determine the launching force exerted on the cannonball, assuming that it is constant and that it points parallel to the tube. (131F2024)



- Show that the launch speed is

$$v_{\text{launch}} = \frac{\sqrt{2g(h - L \sin \theta)}}{\sin \theta}.$$

- Show that the magnitude of the acceleration of the cannonball, while in the cannon tube is

$$a = g \frac{h - L \sin \theta}{L \sin^2 \theta}.$$

- Show that the launching force exerted by the cannon has magnitude

$$F = \frac{mg}{L \sin^2 \theta} (h - L \sin \theta \cos^2 \theta).$$

182 Blow dart

A toy blow dart gun fires a 60 g dart through a tube. The tube has length 40 cm and is angled at 75° from the horizontal. The dart is propelled from rest by air which exerts a constant force and it reaches a height of 6.0 m above the upper end of the tube. Ignore air resistance. (131F2024)

- Determine the launch velocity of the dart.
- Determine the magnitude of the acceleration of the dart while in the tube.
- Determine the force exerted by the propelling air while the dart is in the tube.

183 Towing a car up a ramp, 1

A rope pulls a 2200 kg car up an icy ramp which is tilted at angle 10° above the horizontal. The rope pulls parallel to the ramp with tension 4500 N. Determine the acceleration of the car, ignoring friction and air resistance. (131F2024)

184 Towing a car up a ramp, 2

A rope pulls a 2500 kg car up an icy ramp which is tilted at angle 15° above the horizontal. The rope pulls with a constant force parallel to the ramp. The car is at rest at the bottom of the ramp and while the rope pulls it travels a distance 54 m along the ramp in 6.0 s. Determine the tension in the rope. Ignore friction and air resistance. (131F2024)

185 Sledding on a slope

A person in a sled is at rest at the top of a slope that is angled 15° above the horizontal. The length of the slope is an 800 m (about 0.5 mi). The combined mass of the person and sled is 90 kg. They are released from the top of the slope and slide straight down without pushing. While this happens the air exerts a constant force of 150 N exactly opposite to the direction in which they move. Ignore friction between the sled and the slope. (131F2024)

- a) Determine the acceleration of the person and sled. *You must solve this by starting with a FBD, using Newton's second law, finding components, Simply looking up a formula is not adequate.*
- b) Determine the time taken for the person and sled to reach the bottom of the slope.
- c) Determine the speed of the person and sled at the bottom of the slope.

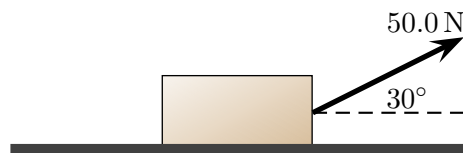
186 Dragging a box horizontally

A 25 kg box can move along a rough horizontal surface. The coefficient of static friction between the box and surface is 0.30. A rope pulls horizontally on the box. Determine the minimum force needed to move the box from rest. (131F2024)



187 Dynamics of a single object with friction, 1

A 15.0 kg box moves rightward along a horizontal surface. A rope pulls with a force at the illustrated angle. The coefficient of kinetic friction is 0.250. The primary aim of this exercise will be to determine the acceleration of the box. (131F2024)



- Draw a free body diagram for the block.
- Write Newton's Second Law in its component form, i.e. write

$$F_{\text{net } x} = \Sigma F_x = \dots \quad (9)$$

$$F_{\text{net } y} = \Sigma F_y = \dots \quad (10)$$

Insert as much information as possible about the components of acceleration at this stage. Can you describe in words what these equations are telling you to do?

- Determine the magnitude of the gravitational force. Let n be the *magnitude* of the normal force. Using this write an expression for the magnitude of the friction force. Do you know the exact number for the friction force at this point?
- List all the components of all the forces, using one of the two formats below.

$$F_{gx} = \dots$$

$$F_{gy} = \dots$$

$$n_x = \dots$$

$$n_y = \dots$$

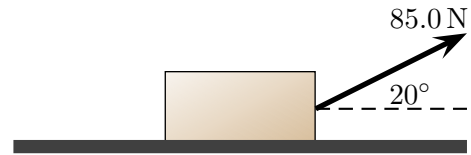
$$\vdots$$

Force	x comp	y comp
\vec{w}		
\vec{n}		
\vdots		

- Use Eq. (9) to obtain an equation relating various quantities that appear in this problem. Do the same with Eq. (10). Does either give the acceleration immediately? Can one of them at least give the normal force immediately?
- Determine the normal force and use this result to find the acceleration.
- What tension would be required for the box to have acceleration 4.00 m/s^2 to the right?

188 Dynamics of a single object with friction, 2

A 15.0 kg box moves rightward along a horizontal surface. A rope pulls with a force at the illustrated angle. The coefficient of kinetic friction is 0.350. Use *all of the steps of Newton's Second Law* to determine the acceleration of the box. (131F2024)



189 Pushing a box at constant speed

A 10 kg box can move along a rough horizontal surface. You discover that when you push horizontally with a force of 50 N that the box moves with constant speed. Which of the following is true about the magnitude of the friction force? Explain your answer. (131F2024)

- i) $f = 10 \text{ N}$
- ii) $f = 50 \text{ N}$
- iii) $f = 98 \text{ N}$
- iv) f is more than 50 N but less than 98 N.
- v) f is more than 98 N.

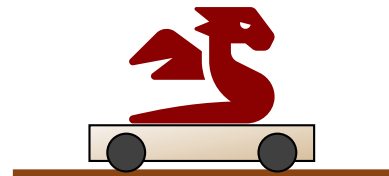
190 Pushing crates along a rough surface

Two people, Jordi and Agnes, each push horizontally on identical crates, which move at constant speeds along the same rough horizontal surface. Jordi pushes his crate faster than Agnes pushes her crate. Which of the following is true? Explain your answer. (131F2024)

- i) Jordi exerts a larger force than Agnes.
- ii) Jordi exerts a smaller force than Agnes.
- iii) Jordi exerts the same force as Agnes.

191 Dragon on a cart

A dragon rides on the horizontal surface of a wooden cart. The cart moves along another horizontal surface. The dragon does not slip on the cart's surface. (131F2024)



- a) The cart moves with increasing speed to the right and so does the dragon. Use this to determine the direction of the dragon's acceleration and use that to determine the direction of the friction force exerted by the cart on the dragon.
- b) The cart moves with decreasing speed to the left and so does the dragon. Use this to determine the direction of the dragon's acceleration and use that to determine the direction of the friction force exerted by the cart on the dragon.

192 Crate on a truck

A crate lies on the flat, horizontal bed of a truck. The truck moves along a horizontal surface. In the following the truck moves so that the crate does not slip along the bed of the truck. (131F2024)

- Is there a friction force between the crate and the truck when they both move north at the same constant speed along a straight horizontal road? Explain your answer.
- Suppose that the truck speeds up while heading North. In which direction does the friction force point when it does this? Explain your answer.
- Suppose that the truck slows down while heading North. In which direction does the friction force point when it does this? Explain your answer.

Explain your answers. *Hint: Consider applying Newton's second law to the crate.*

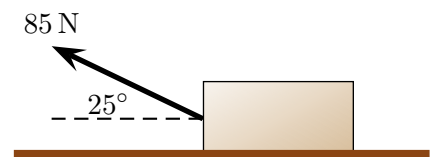
193 Braking car

A 2000 kg car moves along a horizontal concrete road with speed 24 m/s. At one moment the driver applies the brakes sharply and the tires slide without rotating across the concrete surface. The coefficient of static friction in this case is 0.90 and the coefficient of kinetic friction is 0.60. Air resistance provides a 3500 N force acting opposite to the car's motion. (131F2024)

- Determine the acceleration of the car.
- Determine the distance which the car travels while the brakes are applied and it skids.

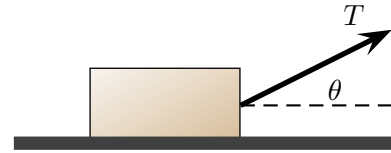
194 Sliding oak box

A person tries to control a 16 kg wooden (oak) box that slides down a ramp by pulling on a rope attached to the box. The box speeds up as it slides down the ramp and then moves right along a horizontal wooden (oak) surface. At this stage the rope is at an angle of 25° above the horizontal and the tension in the rope is 85 N. The coefficient of kinetic friction between two oak surfaces is 0.32. Determine the acceleration of the oak box as it slides along the horizontal surface. (131F2024)



195 Dragging a crate

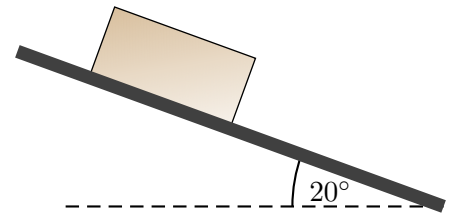
A crate with mass m is dragged along a rough horizontal surface by a rope that pulls with tension T . The rope can pull at various angles. Suppose that you want to produce the largest acceleration for a given tension. (131F2024)



- a) Explain what the drawbacks, in terms of attaining higher acceleration, would be of pulling at a larger angle.
- b) Explain what the benefits, in terms of attaining higher acceleration, would be of pulling at a larger angle.
- c) Determine an expression for the acceleration in terms of T, m, g, θ , and the coefficient of kinetic friction. Explain how you could use this to find the angle that would give the optimal acceleration.

196 Speed at the bottom of a rough ramp, 1

A 10 kg box can move along a 4.0 m long rough ramp angled 20° from the horizontal. The coefficient of kinetic friction between the box and the ramp is 0.25 and the coefficient of static friction is 0.30. The box is released from rest at the top of the ramp and moves down the ramp. The aim of this exercise is to determine the speed of the box at the bottom of the ramp. (131F2024)



- a) Draw a free body diagram for the box.
- b) Describe your choice of x and y axes.
- c) Write Newton's Second Law in vector form and also in its component form, i.e. write

$$F_{\text{net } x} = \Sigma F_x = \dots \tag{11}$$

$$F_{\text{net } y} = \Sigma F_y = \dots \tag{12}$$

where x and y refer to your specially chosen axes. Insert as much information as possible about the components of acceleration at this stage. The resulting equations will generate much of the algebra that follows.

- d) Determine expressions for the *magnitudes* of the gravitational and the friction forces.
- e) List all the components of all the forces, using one of the two formats below.

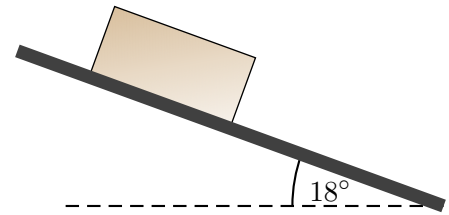
$$\begin{aligned}
 F_{gx} &= \dots \\
 F_{gy} &= \dots \\
 n_x &= \dots \\
 n_y &= \dots \\
 &\vdots
 \end{aligned}$$

Force	x comp	y comp
\vec{F}_g		
\vec{n}		
\vdots		

- f) Use Eq. (11) to obtain an equation relating various quantities that appear in this problem. Do the same with Eq. (12). Use the resulting equations to find the acceleration of the box.
- g) Determine the speed of the box when it reaches the bottom of the ramp.
- h) Do these results depend on the mass of the box?
- i) What would be the minimum force, pushing parallel to the ramp, required to keep the box at rest?

197 Speed at the bottom of a rough ramp, 2

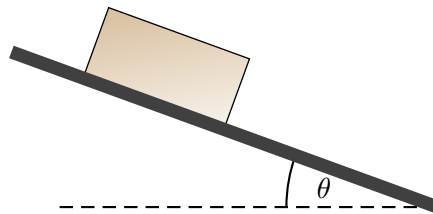
A 25 kg box can move along a 6.0 m long rough ramp angled 18° from the horizontal. The coefficient of kinetic friction between the box and the ramp is 0.25 and the coefficient of static friction is 0.30. The box is released from rest at the top of the ramp and moves down the ramp. (131F2024)



- a) Determine the speed of the box when it reaches the bottom of the ramp.
- b) Does the speed depend on the mass of the box?
- c) What would be the minimum force, pushing parallel to the ramp, required to keep the box at rest?

198 Perfect friction on a rough ramp

A box can move along a rough ramp which makes an angle θ from the horizontal and which has length L . The box is launched with speed v from the top of the ramp. The aim of this exercise is to determine the coefficient of friction needed to bring the box to a stop at the bottom of the ramp. (131F2024)



- The first part of the solution uses kinematics to assess the acceleration of the box. Using kinematics, and eventually dynamics, is greatly simplified by choosing an appropriate “ x ” and a “ y ” axis. These do not have to be along the usual vertical and horizontal directions. Regardless of the axes that you choose, the usual general kinematics and dynamics equations will be valid. Describe the “ x ” and “ y ” axes that you will use.
- Determine an expression for the magnitude of the acceleration of the box, a , in terms of variables relevant to this problem, such as L, θ, v , and possibly the mass of the box, m .
- Draw a free body diagram for the box.
- Write Newton’s Second Law in vector form and also in its component form, i.e. write

$$F_{\text{net } x} = \Sigma F_{ix} = \dots \quad (13)$$

$$F_{\text{net } y} = \Sigma F_{iy} = \dots \quad (14)$$

where x and y refer to your specially chosen axes. Insert as much information as possible about the components of acceleration at this stage. The resulting equations will generate much of the algebra that follows.

- Determine expressions for the *magnitudes* of the gravitational and the friction forces.
- List all the components of all the forces, using one of the two formats below.

$$F_{gx} = \dots$$

$$F_{gy} = \dots$$

$$n_x = \dots$$

$$n_y = \dots$$

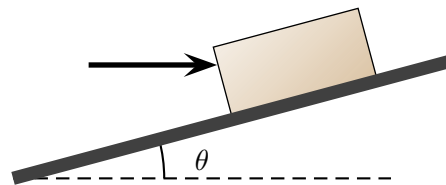
$$\vdots$$

Force	x comp	y comp
$\vec{\mathbf{F}}_g$		
$\vec{\mathbf{n}}$		
\vdots		

- Use Eq. (13) to obtain an equation relating various quantities that appear in this problem. Do the same with Eq. (14). Use the resulting equations to find an expression for the coefficient of kinetic friction. Does the result depend on the mass of the box?
- Suppose that the ramp is a roof whose length is 5.0 m and which is angled at 15° from the horizontal. If the box is pushed with speed 4.0 m/s, determine the coefficient of friction needed to stop the box at the bottom of the roof.

199 Box pushed up a rough ramp

A block with mass m slides up a ramp at angle θ from the horizontal. The coefficient of kinetic friction between the surfaces is μ_k . A person pushes horizontally with a force F_p . The object of this exercise is to determine an expression for the acceleration of the block



$a =$ formula involving m, θ, F_p, μ_k, g and constants.

The *entire collection* of these steps is called “applying Newton’s laws of mechanics to determine the acceleration of the block.” (131Sp2023)

- Draw a free body diagram for the block.
- Describe the x and y axes that you will use.
- Write Newton’s Second Law in vector form and also in its component form, i.e. write

$$F_{\text{net } x} = \dots \quad (15)$$

$$F_{\text{net } y} = \dots \quad (16)$$

Insert as much information as possible about the components of acceleration at this stage. The resulting equations will generate much of the algebra that follows.

- Determine expressions for the magnitudes of all the friction and gravitational forces. Do you think that $n = mg$?
- List all the components of all the forces, using one of the two formats below.

$$F_{gx} = \dots$$

$$F_{gy} = \dots$$

$$n_x = \dots$$

$$n_y = \dots$$

\vdots

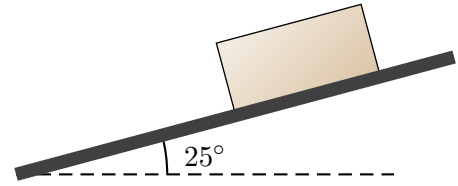
Force	x comp	y comp
\vec{n}		
\vdots		
\vdots		

- Note that $F_{\text{net } x} = \sum_i F_{ix}$ (i.e. the x component of the net force is the sum of the x components of individual forces). Use this and Eq. (15) to obtain an equation relating various quantities that appear in this problem. Do the same with Eq. (16). Use the resulting algebraic expressions to get an expression for n . Is this mg ? Use the resulting equations to get an expression for a .

200 Crate sliding along a rough ramp

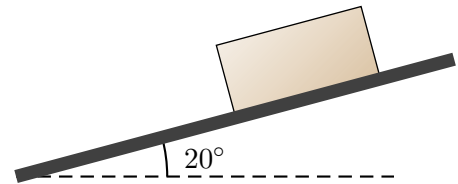
A 15 kg crate can slide up or down a ramp at angle 25° from the horizontal. The coefficient of kinetic friction between the crate and the ramp is 0.30. (131F2024)

- Determine the crate's acceleration if it slides up the ramp.
- Determine the crate's acceleration if it slides down the ramp.



201 Crate pushed up a rough ramp

A 12 kg crate can slide up or down a ramp at angle 20° from the horizontal. The coefficient of kinetic friction between the crate and the ramp is 0.40. A hand pushes with a 120 N force parallel to the ramp in the uphill direction. Determine the crate's acceleration if it slides up the ramp. (131F2024)



202 Box on a ramp

A 100 kg box is at rest on a ramp that is inclined at angle 25° from the horizontal. (131F2024)

- Determine the friction force on the box.
- Determine the minimum coefficient of static friction between the box and ramp that allows the box to stay at rest.

203 Brick on a slope

A 1.50 kg brick is at rest on an inclined surface. The angle between the surface and the horizontal is 10° . (131F2024)

- Determine the friction force that the surface exerts on the brick.
- Determine the minimum coefficient of static friction between these surfaces that allows for the brick to stay at rest.

204 Car on a slope

A car with mass 1500 kg is at rest on an incline. The coefficient of static friction between dry asphalt and rubber here is 0.90 and the coefficient of static friction between wet asphalt and rubber is 0.40. (131F2024)

- Determine the maximum angle of the incline so that the car will stay at rest when the surface is dry.
- Determine the maximum angle of the incline so that the car will stay at rest when the surface is wet.

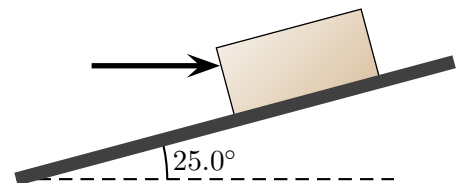
205 Launching a box up a ramp

A ramp with length 2.0 m is inclined at an angle of 15° above the horizontal. A tiny box with mass 0.25 kg is launched with a very brief kick imparting speed 2.75 m/s at the base of the ramp. The coefficient of kinetic friction between the box and slope is 0.20. (131F2024)

- Determine the acceleration of the box as it slides up the ramp.
- Determine whether the box reaches the other end of the ramp or not.

206 Box pushed up a rough ramp

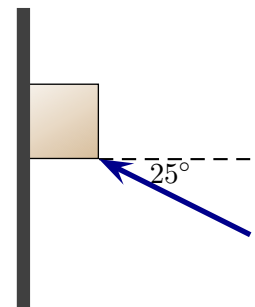
A 6.00 kg box slides up a ramp at angle 25.0° from the horizontal. The coefficient of kinetic friction between the surfaces is 0.450. A person pushes horizontally with a 80.0 N force as illustrated. Determine the acceleration of the box. (131F2024)



207 Book held against a wall

A 10.0 kg book lies against a vertical wall and a hand pushes on it as illustrated. The coefficient of static friction between the book and wall is 0.30. The coefficient of kinetic friction between the book and wall is 0.20. (131F2024)

- Determine the minimum force needed so that the book stays at rest on the wall.
- Determine the magnitude of the force exerted by the hand so that the book slides down the wall with constant speed.



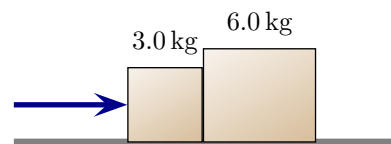
208 Ice skaters

Two ice skaters, Alice with mass 55 kg and Bob with mass 85 kg, are initially at rest on a frictionless horizontal surface. Bob gently pushes on Alice and while he does this she has a constant acceleration of 0.020 m/s^2 . (131F2024)

- Determine the force exerted by Bob on Alice while she accelerates.
- Determine the force exerted by Alice on Bob while Alice accelerates.
- Determine the acceleration of Bob during the period while he is pushing on Alice.

209 Pushing crates in contact

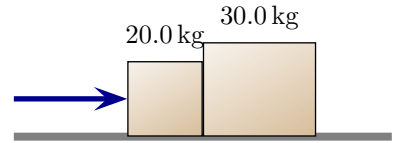
Two crates can move along a horizontal surface. There is no friction between either crate and the surface. The crates maintain contact with each other. A hand pushes on the crate at the left. (131F2024)



- Which of the following is true? Explain your answer.
 - The force exerted by the smaller crate on the larger crate is less than the force exerted by the hand.
 - The force exerted by the smaller crate on the larger crate is more than the force exerted by the hand.
 - The force exerted by the smaller crate on the larger crate is the same as the force exerted by the hand.
- The positions of the two crates are interchanged so that the larger crate is to the left of the smaller crate. Which of the following is true? Explain your answer.
 - The force exerted by the larger crate on the smaller crate is less than the force exerted by the hand.
 - The force exerted by the larger crate on the smaller crate is more than the force exerted by the hand.
 - The force exerted by the larger crate on the smaller crate is the same as the force exerted by the hand.
- For either of these arrangements, the crates could either be moving left or else right. Does the direction of motion affect the answers to the previous questions? Explain your answer.

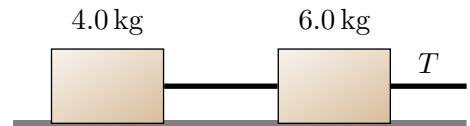
210 Pushing boxes in contact

Two boxes can move along a frictionless horizontal surface. The boxes maintain contact with each other. A person pushes with a 100 N force on the box at the left. Determine the force that the box at the left exerts on the box at the right. (131F2024)



211 Dragging connected crates

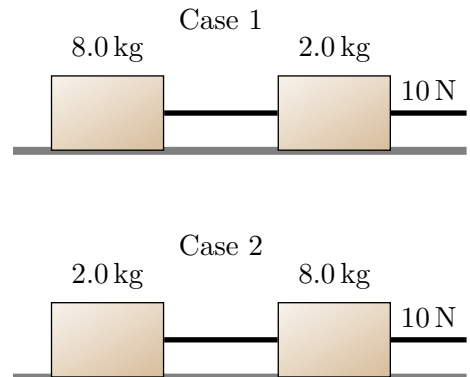
Two different crates can move along a horizontal surface. There is no friction between either crate and the surface. The crates are connected by a rope. A hand pulls on the other rope with tension T . (131F2024)



- a) Which of the following is true? Explain your answer.
 - i) The tension in the connecting rope is the same as T .
 - ii) The tension in the connecting rope is more than T .
 - iii) The tension in the connecting rope is less than T .
- b) Which of the following is true? Explain your answer.
 - i) The tension in the connecting rope depends on whether the heavier crate is to the left.
 - ii) The tension in the connecting rope does not depend on whether the heavier crate is to the left.

212 Connected objects: tension and acceleration

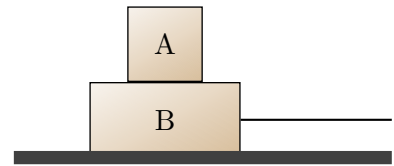
Two objects on a frictionless horizontal surface are connected by a massless rope. They are connected in different orders and pulled by another massless rope. (131F2024)



- a) Let a_1 be the acceleration for case 1 and a_2 the acceleration for case 2. Which of the following is true? Explain your choice.
 - i) $a_1 = a_2$
 - ii) $a_1 > a_2$
 - iii) $a_1 < a_2$
- b) Let T_1 be the tension in the rope connecting the blocks for case 1 and T_2 that for case 2. Which of the following is true? Explain your choice.
 - i) $T_1 = T_2 = 10 \text{ N}$
 - ii) $T_1 < T_2 < 10 \text{ N}$
 - iii) $T_2 < T_1 < 10 \text{ N}$
 - iv) $T_1 = T_2 < 10 \text{ N}$

213 Slipping stacked objects

Two boxes are stacked and move along a frictionless horizontal surface as illustrated. Block A has mass 2.0 kg and block B has mass 3.0 kg. A rope is attached to block B and pulls horizontally with a 50 N force. The coefficient of friction between block A and block B is 0.25. Determine the acceleration of each block, assuming that they both move right and that B moves faster than A. (131F2024)



214 Connected objects: tension and acceleration

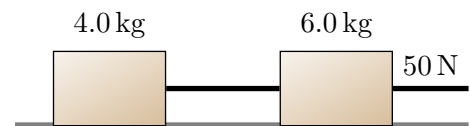
Two boxes can move along a horizontal surface. There is no friction between either box and the surface. The boxes are connected by a rope. A hand pulls on the other rope with force 50 N. (131F2024)



- Determine the acceleration of each box.
- Determine the tension in the rope connecting the boxes.

215 Connected objects: friction

Two boxes can move along a horizontal surface. There is no friction between the 6.0 kg box and the surface. There is friction for the other box: the coefficient of static friction is 0.70 and the coefficient of kinetic friction is 0.50. The boxes are connected by a rope. A hand pulls on the other rope with force 50 N. (131F2024)



- Determine the acceleration of each box.
- Determine the tension in the rope connecting the boxes.

216 Connected objects separated by a spring

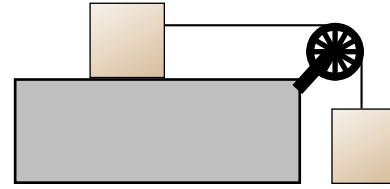
Two boxes can move along a horizontal surface. They are connected by a spring with spring constant 600 N/m. A person exerts a constant 100 N force pointing right on the box on the left. The boxes eventually move so that they maintain a fixed distance between them. (131F2024)



- Determine the distance by which the spring is compressed when they move like this.
- Suppose that the situation is repeated but a constant 100 N force is now exerted on the box on the right; this force points left. Is the compression of the spring the same as before? Explain your answer.

217 Level/suspended blocks without friction

Two blocks are connected by a string, which runs over a massless pulley. One block, with mass 3.0 kg is suspended and the other block, with mass 7.0 kg can move along a frictionless horizontal surface. The string connected to the block on the surface runs horizontally. (131F2024)



- Draw a free body diagram for the *box on the surface*.
- Write Newton's Second Law in component form for the *box on the surface*, i.e. write

$$F_{\text{net } x} = \Sigma F_x = \dots \quad (17)$$

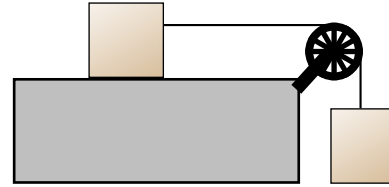
$$F_{\text{net } y} = \Sigma F_y = \dots \quad (18)$$

Insert as much information as possible about the components of acceleration at this stage. The resulting equations will generate much of the algebra that follows.

- List all the components of all the forces for the box on the surface.
- Use Eqs. (21) and (22) and the components to obtain an equation relating the tension in the rope and the acceleration of the box. Can you solve this for acceleration at this stage?
- Repeat parts a) to d) for the *suspended crate*. Be careful about the acceleration!
- Combine the equations for the two objects to obtain the acceleration and the tension in the rope.

218 Level/suspended blocks without friction and a hand

Two blocks are connected by a string, which runs over a massless pulley. One block, with mass m_1 , is suspended and the other block, with mass m_2 , can move along a frictionless horizontal surface. The string connected to the block on the surface runs horizontally. A hand exerts a constant force on the block on the surface. This force has magnitude F_{hand} and points horizontally to the left. (131F2024)



- Draw a free body diagram for the *box on the surface*.
- Write Newton's Second Law in component form for the *box on the surface*, i.e. write

$$F_{\text{net } x} = \Sigma F_x = \dots \quad (19)$$

$$F_{\text{net } y} = \Sigma F_y = \dots \quad (20)$$

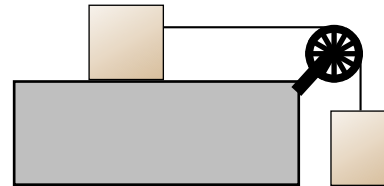
Insert as much information as possible about the components of acceleration at this stage. The resulting equations will generate much of the algebra that follows.

- List all the components of all the forces for the box on the surface.
- Use Eqs. (21) and (22) and the components to obtain an equation relating the tension in the rope and the acceleration of the box. Can you solve this for acceleration at this stage?
- Repeat parts a) to d) for the *suspended crate*. Be careful about the acceleration!
- Combine the equations for the two objects to obtain the acceleration and the tension in the rope. These should be of the form

$$a = \text{expression with only } m_1, m_2, g, F_{\text{hand}} \text{ and constants.}$$

219 Connected objects: Level/suspended blocks with friction

A 6.0 kg crate is suspended from a rope which runs over a massless pulley and is connected to a 4.0 kg box, which moves right along a rough horizontal surface. The coefficient of friction between the box and surface is 0.25. The string connected to the box on the surface runs horizontally. The aim of this exercise will be to determine the acceleration of the objects, ignoring any air resistance.



(131F2024)

- Draw a free body diagram for the *box on the surface*.
- Write Newton's Second Law in component form for the *box on the surface*, i.e. write

$$F_{\text{net } x} = \Sigma F_x = \dots \quad (21)$$

$$F_{\text{net } y} = \Sigma F_y = \dots \quad (22)$$

Insert as much information as possible about the components of acceleration at this stage. The resulting equations will generate much of the algebra that follows.

- List all the components of all the forces for the box on the surface.
- Use Eqs. (21) and (22) and the components to obtain an equation relating the tension in the rope and the acceleration of the box. Can you solve this for acceleration at this stage?
- Repeat parts a) to d) for the *suspended crate*. Be careful about the acceleration!
- Combine the equations for the two objects to obtain the acceleration and the tension in the rope.

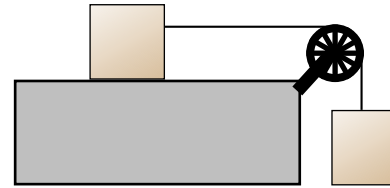
The analysis can be performed for objects of any mass. Let m_1 be the mass of the block on the surface, m_2 the mass of the suspended block and μ_k be the coefficient of friction between the block and the surface.

- Determine an expression for the magnitude of acceleration of the blocks, a . This should be of the form

$$a = \text{formula with only } m_1, m_2, g, \mu_k \text{ and constants.}$$

220 Connected objects: Level/suspended blocks with friction, 2

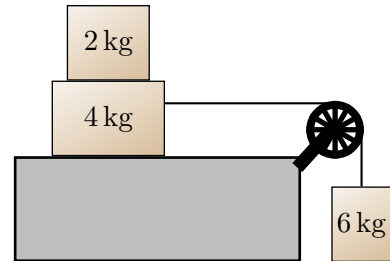
A 3.0 kg box is suspended from a rope which runs over a massless pulley and is connected to a 5.0 kg crate, which moves right along a rough horizontal surface. The coefficient of kinetic friction between the crate and surface is 0.35. The string connected to the crate runs horizontally. (131F2024)



- a) Determine the acceleration of the objects, ignoring any air resistance.
- b) Determine the tension in the rope.
- c) Suppose that the surface is polished and this reduces the friction to the point that the acceleration is $0.25g$. Determine the coefficient of friction in this case.

221 Stacked objects on a surface connected to a suspended object

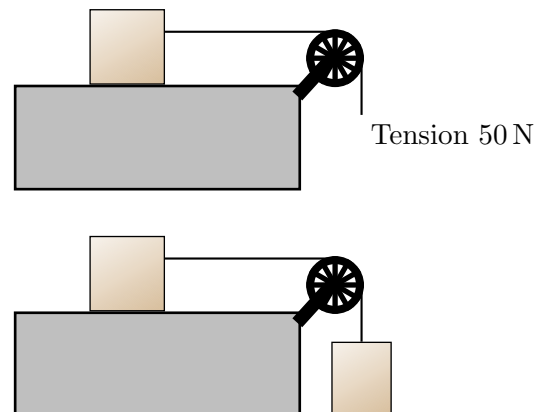
Various boxes are arranged as illustrated. The 4 kg block lies on a table with a horizontal surface. (131F2024)



- a) Suppose that all the blocks are at rest. Is there a friction force between the 4 kg block and the table? If so, in which direction? Explain your answer.
- b) Suppose that all the blocks are at rest. Is there a friction force between the 2 kg block and the 4 kg block? If so, in which direction? Explain your answer.
- c) Suppose that the block on the table accelerates to the right and the 2 kg block does not slip relative to the 4 kg block. Is there a friction force between the 2 kg block and the 4 kg block? If so, in which direction? Explain your answer.

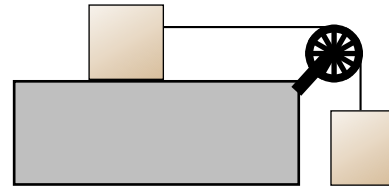
222 Connected objects: block dragged along a surface

A block on a surface can be dragged along a frictionless surface by a connecting rope in two ways. Either a person exerts a force on the rope or else a suspended block exerts a force on the rope. Suppose that the person exerts a 50 N tension force on the rope and that the weight (gravitational force on) of the suspended block is exactly 50 N. In which case is the acceleration of the block on the horizontal surface largest? Explain your answer. (131F2024)



223 Level/suspended blocks without friction

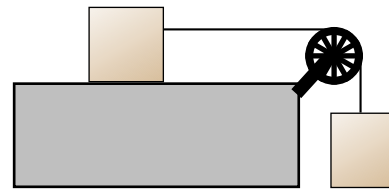
Two blocks are connected by a string, which runs over a massless pulley. A 10 kg block is suspended and a 5.0 kg block can slide along a frictionless horizontal surface. The string connected to the block on the surface runs horizontally. The blocks held at rest and then released. They move, constantly speeding up. Which of the following is true regarding the tension in the connecting string, T , while they move? Explain your choice. (131F2024)



- i) $T = 0$.
- ii) $98 \text{ N} > T > 0$.
- iii) $T = 98 \text{ N}$.
- iv) $T > 98 \text{ N}$

224 Box dragged along a rough surface

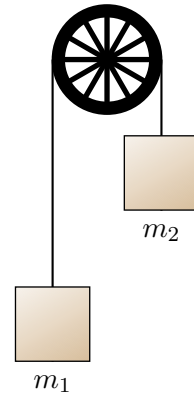
A small box, with mass m , is connected to a suspended mass, with mass $3m$, by a string, which runs over a massless pulley. The box can slide along a rough horizontal surface. The coefficient of static friction is μ_s and the coefficient of kinetic friction is μ_k , where $\mu_k < \mu_s$. In the following the only variables that are allowed to appear in expressions are: m, g, μ_k and μ_s . (131F2024)



- a) Determine an expression for the minimum coefficient of static friction such that the box does not move.
- b) The box is given a brief push to the left and it begins to move. Determine an expression for the acceleration of the box *after* this brief push and while it is moving.

225 Atwood's machine

Two blocks, with masses indicated, are connected by a string which runs over a massless pulley. The aim of this exercise is to determine the acceleration of the blocks. (131F2024)



- a) Draw a free body diagram for the *block on the left*.
- b) Write Newton's Second Law in component form for the *block on the left*, i.e. write

$$F_{\text{net } x} = \Sigma F_x = \dots \tag{23}$$

$$F_{\text{net } y} = \Sigma F_y = \dots \tag{24}$$

Insert as much information as possible about the components of acceleration at this stage. The resulting equations will generate much of the algebra that follows.

- c) List all the components of all the forces for the block on the left.

$$F_{gx} = \dots$$

$$F_{gy} = \dots$$

$$n_x = \dots$$

$$n_y = \dots$$

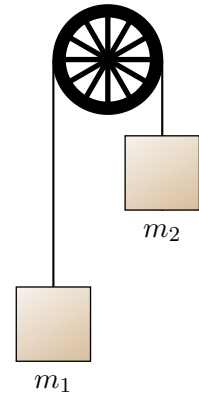
⋮

Force	<i>x</i> comp	<i>y</i> comp
\vec{F}_g		
\vec{n}		
⋮		

- d) Use Eqs. (23) and (24) and the components to obtain an equation relating the tension in the rope and the acceleration of the block on the left. Can you solve this for acceleration at this stage?
- e) Repeat parts a) to d) for the *block on the right*. Be careful about the acceleration!
- f) Combine the equations for the two objects to obtain the acceleration and the tension in the rope.

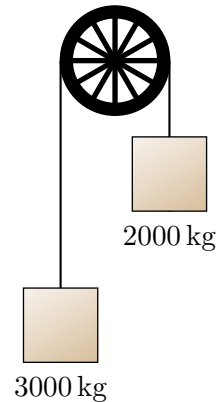
226 Atwood's machine variation

Two blocks, with masses $m_1 > m_2$, are connected by a string which runs over a massless pulley. A hand exerts a constant downward force with magnitude F_{hand} on the block on the left. Determine an expression for the magnitude of acceleration of the blocks, a . The only variables allowed in the expression are m_1, m_2, g and F_{hand} . (131F2024)



227 Counterbalanced elevator

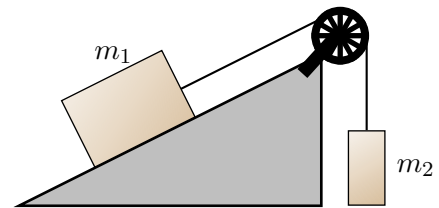
A 3000 kg elevator is connected to a 2000 kg block by a rope that runs over a pulley. Determine the acceleration of the elevator. (131F2024)



228 Connected objects: ramp

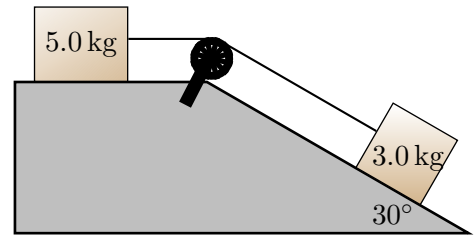
A block on a rough ramp is connected via a string to another suspended block. At one instant the block on the ramp is sliding up the ramp. (131F2024)

- Determine an expression for the acceleration of the block on the ramp if the ramp is inclined at an angle of 65° above the horizontal, the mass of the block on the ramp is 8.0 kg, the mass of the suspended block is 9.0 kg and the coefficient of kinetic friction is 1.2. Does the block on the ramp speed up or slow down?
- Determine a general expression for the acceleration of the block moving up the ramp if the ramp is inclined at an angle of θ above the horizontal and the coefficient of kinetic friction is μ_k .
- What condition must the masses satisfy for the block to speed up as it ascends the ramp?



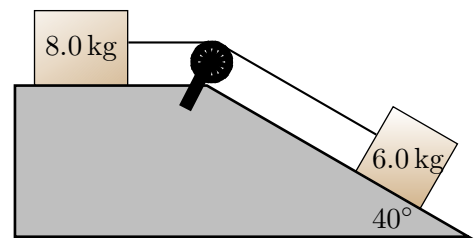
229 Connected objects: horizontal and a ramp

Blocks connected by a massless string are able to slide on the illustrated frictionless surfaces. The strings run parallel to the surfaces. Determine the acceleration of the block on the ramp. (131F2024)



230 Connected objects: horizontal and a rough ramp

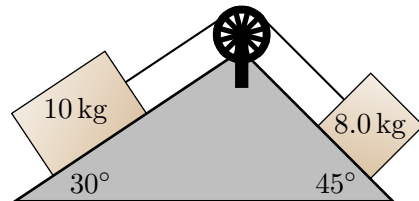
Blocks connected by a massless string are able to slide on the illustrated surfaces. The strings run parallel to the surfaces. The horizontal surface is frictionless and the ramp has coefficient of kinetic friction 0.50. Determine the acceleration of the block on the ramp if it descends. (131F2024)



231 Connected objects: two ramps

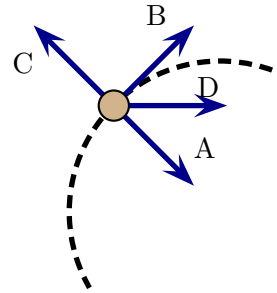
Blocks connected by a massless string are able to slide on two frictionless ramps. (131F2024)

- a) Determine the acceleration of the blocks.
- b) Suppose that the block on the right were replaced by another block so that the two blocks slide with constant speed. What would the mass of the block on the right be?



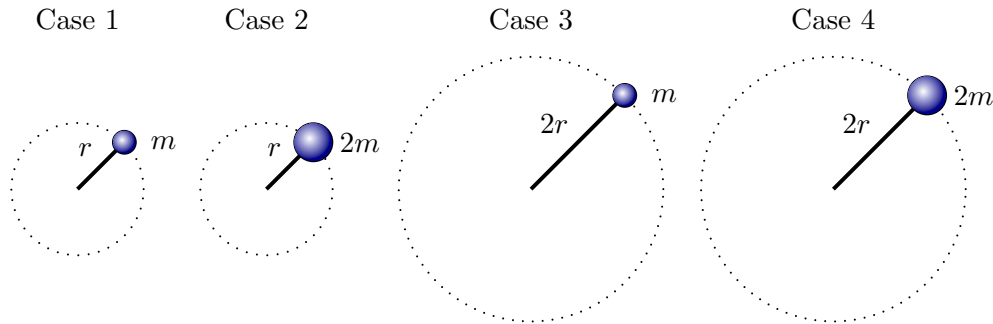
232 Bug walking in a circle

A bug walks at a constant speed in a circular path on a horizontal surface. Which vector best illustrates the net force on the bug at the illustrated moment? Explain your choice. (131F2024)



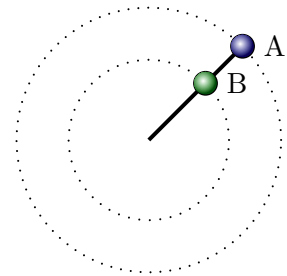
233 Balls swinging on the end of strings

Various balls at the end of strings swing in circles, with the same speed, on a horizontal frictionless surface. The diagram illustrates this as viewed from above. Rank the cases in order of increasing tension in the string. Explain your answer. (131F2024)



234 Connected balls swinging in horizontal circles, 1

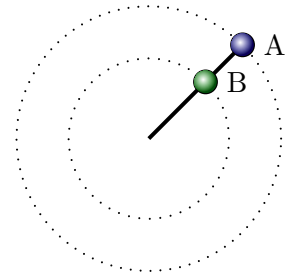
Two balls are connected by strings. They are on a horizontal frictionless surface and swing in horizontal circles about a central point; both strings point radially outward from the center as the balls move. The diagram illustrates this as viewed from above. (131F2024)



- Draw a free body diagram, as viewed from the side, for ball B at the moment when the balls are at the leftmost location of their motion.
- Draw the direction of the net force on the ball B at the moment when the balls are at the leftmost location of their motion.
- How does the tension in the inner string compare (larger, smaller, same) to that in the string connecting the two balls? Explain your answer.

235 Connected balls swinging in horizontal circles, 2

Two balls are connected by strings. They are on a horizontal frictionless surface and swing in horizontal circles about a central point; both strings point radially outward from the center as the balls move. The diagram illustrates this as viewed from above. (131F2024)



- Suppose that A has a larger mass than B. Is the tension in the inner string the same as, larger than or equal to the tension in the outer string? Explain your answer.
- Suppose that A has a smaller mass than B. Is the tension in the inner string the same as, larger than or equal to the tension in the outer string? Explain your answer.
- Now consider two particular balls, one with mass 3.0 kg and the other with mass 5.0 kg. They swing in circles as illustrated in one of the following two arrangements: i) the 3.0 kg is further out and the 5.0 kg is closer in, or ii) the 5.0 kg is further out and the 3.0 kg is closer in. In each case they swing with the same angular velocity. Which arrangement (larger mass on the outside, smaller mass on the outside) results in a larger tension in the outer string? Explain your answer.

236 Merry-go-round dynamics

A 50 kg child sits at the edge of a merry-go-round with radius 2.5 m. The merry-go-round rotates with frequency 15 revolutions per minute. (131F2024)

- Determine the direction of the friction force on the child.
- Determine the magnitude of the friction force on the child.

237 Child swinging on ice

A parent and child are each on a horizontal sheet of ice. The parent is fixed to the ice and swings the child, who is connected by a horizontal rope to the parent. The child slides without any friction with constant velocity. (131F2024)

- Determine an expression for the tension in the rope in terms of the mass of the child, the length of the rope and the speed of the child.
- Determine an expression for the tension in the rope in terms of the mass of the child, the length of the rope and the period of motion of the child.
- Determine an expression for the period of motion of the child so that the tension felt by the parent is larger the force that the parent must exert to hold the child at rest off the ground.

238 Particle accelerator

The Large Hadron Collider (LHC) accelerates subatomic particles and then lets them collide with a target. During this trip the particles move through a circular tunnel with radius 2.80 km. One type of particle use in the LHC is a proton. Suppose that a proton travels this circle at 10% of the speed of light. Determine the net force on the proton. *Note: to be strictly correct this needs to include the effects of special relativity. The approach here will give an approximate force. (131F2024)*

239 Rotating Earth

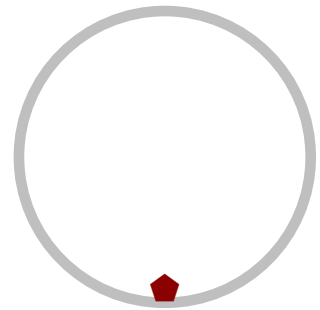
Earth rotates, completing one rotation in a day (86400 s). Suppose that you stand on a scale at sea level at Earth's equator; the scale measures the normal force that it exerts and converts it into a reading in kilograms by dividing by 9.81 m/s^2 . In the following ignore air resistance. (131F2024)

- Explain why the force measured by the scale will not equal the gravitational force exerted by Earth.
- Assuming that your mass is 70.0 kg, determine the normal force and the scale reading.
- Suppose that Earth's rotation were to speed up. How will this affect the scale reading?
- If Earth were able to speed up, determine the time for one rotation such that the scale would read zero. How many hours would it take Earth to complete one rotation? What would happen to you if you were standing on the scale and Earth continued to speed up beyond this threshold?

240 Animals in a space station

A particular space station is essentially a giant circular hoop with radius 600 m. People and objects reside on the inside of the hoop. The hoop rotates at a constant rate. (131F2024)

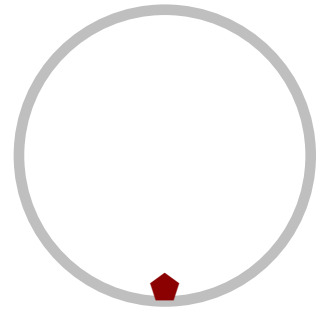
- Consider 30 kg dog on the inside of the space station. The dog is fixed at one point on the floor. Determine the speed of the space station so that the dog feels a normal force the same as it would at rest on Earth.
- Determine the time taken for the space station of the previous part to complete one orbit.
- A 4.0 kg rabbit also sits on the floor of the space station. It would also like to feel the same normal force as it does on Earth. Would the time taken for the space station to complete one orbit have to change to do this? Explain your answer.



241 Science fiction space station

A science fiction movie features a space station which is essentially a giant circular hoop. People and objects reside on the inside of the hoop. The hoop rotates at a constant rate. (131F2024)

- Consider an object with mass m at rest relative to the surface of the space station floor. Determine an expression for the normal force exerted by the floor in terms of the radius and the time taken to complete one orbit.
- The floor will feel the same as Earth's surface if the normal force equals mg . Determine an expression for the time taken to complete one orbit so that this is true.
- Imagine a space station with a radius of 5.0 km. Determine the time it should take to complete one rotation so that the space station floor feels like Earth's surface.



242 Car turning

A 3000 kg car travels along a flat (horizontal) road. The road curves with a radius of 350 m. The maximum speed with which the car can do the turn is 18 m/s. (131F2024)

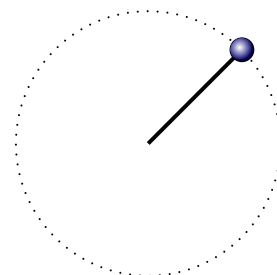
- Determine the net force on the car. What force provides the net force?
- Determine the (minimum) coefficient of static friction that allows the car to follow this curve at the given speed.

243 Coin on a turntable

A coin with mass m sits on a horizontal turntable with radius R . The distance from the turntable axle to the coin is d . The coefficient of static friction between the coin and the turntable is μ_s and the coefficient of kinetic friction is μ_k . The turntable is initially at rest and slowly speeds up. Determine an expression for the maximum angular velocity of the turntable so that the coin does not slip. (131F2024)

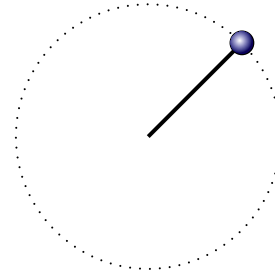
244 Ball swinging in a vertical circle, 1

A 0.80 kg ball swings with in a vertical circle at the end of a 0.50 m long string. The speed of the ball at the highest point in the circle is 3.0 m/s. Determine the tension in the string at this moment. (131F2024)



245 Ball swinging in a vertical circle, 2

A 0.20 kg ball swings with in a vertical circle at the end of a 0.50 m long string. (131F2024)



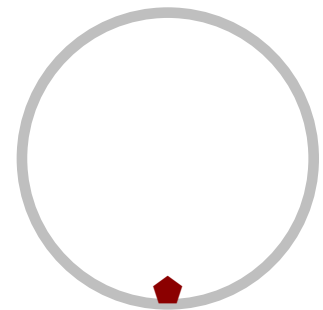
- Draw a free body diagram for the ball at the highest point of the circle. Draw a free body diagram at the lowest point.
- Suppose that the speed of the ball is constant throughout its motion. How does net force at the highest point of the circle compare (larger, smaller, same) to that at the lowest point of the circle? Use your answer to compare (larger, smaller, same) the tension in the string at the lowest point of the circle to the tension at the highest point of the circle.
- Suppose that the string will break if the tension in it exceeds 5.0 N. Use Newton's second law to analyze the situation where the tension is largest (i.e. highest or lowest point) and determine the maximum speed with which the ball can move so that the string does not break.
- In general the speed of the ball can vary as it swings. As the speed decreases does the tension at the top of the circle increase, decrease or stay constant? Determine the minimum speed so that the tension is not zero. Describe what happens if the speed drops beneath this.

246 Monkey on a rope

A monkey with mass m_{monkey} swings from a rope. Is the tension in the rope at the bottom of the swing larger than, smaller than or the same as $m_{\text{monkey}}g$? Explain your answer. (131F2024)

247 Sliding inside a hoop

A small 0.10 kg object slides around the inside a frictionless vertical hoop with radius 0.25 m. (131F2024)



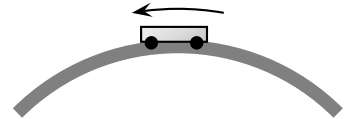
- The speed of the object at the top of the hoop is 2.0 m/s. Determine the normal force exerted by the hoop on the object at the top of the loop.
- The object reaches the bottom of the hoop with speed 3.71 m/s. Determine the normal force exerted by the hoop on the object at the bottom of the loop.
- Determine the minimum speed at the top of the loop so that the object stays on the inside surface of the loop.

248 Loop-the-loop

A 50 g ball slides, without rolling, inside a vertically oriented circular loop with radius 80 cm. In two different situations, the ball is set into motion so that its speed is i) 4.0 m/s and the top of the loop and ii) 4.0 m/s and the bottom of the loop. Determine the normal force exerted by the loop on the ball in each case. (131F2024)

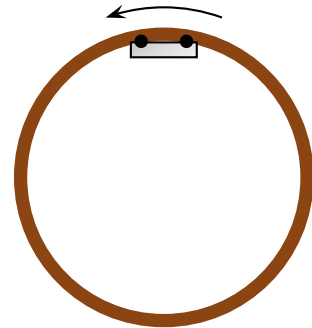
249 Cart sliding over a hill

A 30 kg cart slides over a hill, which has a circular cross-section of radius 12 m. The speed of the cart at the highest point is 4.0 m/s. Determine the normal force exerted by the hill on the cart. (131F2024)



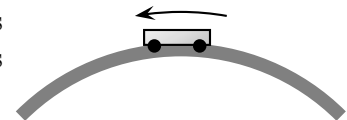
250 Roller coaster inside a circular loop

Two identical roller coaster cars, A and B, each move along a track that makes a vertically oriented circle with radius R . Each passes along the inside of the track at the top and is in contact with the track at this point. The speed of A is larger than the speed of B. Determine whether the normal force on A is the same as, smaller than or larger than the normal force on B. (131F2024)



251 Roller coaster on circular tracks

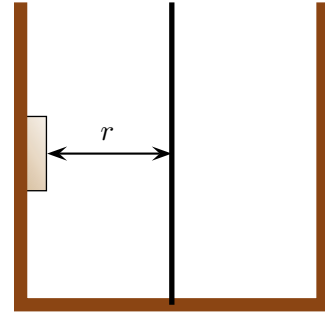
A roller coaster with mass m moves along a track that makes a vertically oriented circle with radius R . At one moment it passes along the outside of the track at the top, as illustrated. The cart's speed at the top of the track is $v > 0$. (131F2024)



- Determine an expression for the normal force exerted by the track on the cart at the top of the loop. Assume that the cart has no mechanism for “grabbing” the track.
- Determine an expression for the maximum speed which the cart can have so that it will stay on the track without a “grabbing” mechanism that keeps it on the track.
- For an actual roller coaster, use this analysis to estimate the maximum speed so that the cart will stay on the track without a “grabbing” mechanism. Provide any data that you need to supply to do the estimate.

252 Block inside a revolving drum

A spinning drum has vertical wooden sides. A wooden block is placed inside the drum and the drum is eventually made to spin with a constant angular velocity. This is done in such a way that the block does not slide relative to the drum; it rotates at the same rate as the drum. In the first part of the problem suppose that the mass of the block is 2.0 kg, the coefficient of static friction is 0.60 and the radius of the drum is 0.80 m. (131F2024)

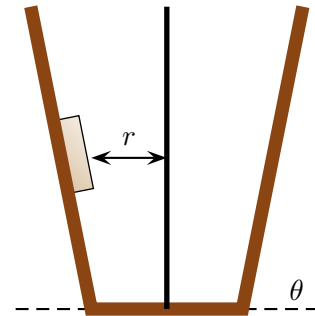


- Draw a free body diagram for the block, identify the direction of the acceleration and write Newton's 2nd law in component form. Is $n = mg$ in this case?
- Determine an expression for the minimum angular velocity with which the drum must rotate so that the block does not slip.

Now consider the more general case where the drum has radius r and the coefficient of static friction is μ_s .

- Determine an expression for the minimum angular velocity (in terms of g , r and μ_s) so that the block does not slip. Does it depend on the mass of the block?

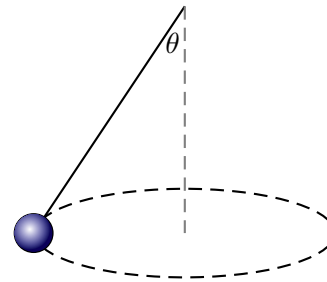
Now suppose that the spinning drum has tilted sides. A block is placed inside the drum and the drum is eventually made to spin with a constant angular velocity ω . This is done in such a way that the block does not slide relative to the drum; it rotates at the same rate as the drum. Let r be the distance from the block to the axle of the drum.



- Determine the minimum angular velocity required for the block not to slip when $\theta = 75^\circ$, $r = 2.5$ m and the coefficient of static friction is 0.2.

253 Conical pendulum

A ball with mass m swings with a constant speed at the end of a string with length L . The angle between the string and the vertical, θ , is constant. Suppose that one can easily measure L , m and θ . We would like to determine expressions for other quantities in terms of these. (131F2024)



- Determine an expression for the speed of the ball in terms of m , θ , L and constants. For a given string length and angle are there multiple possible speeds at which the ball can move? Are there multiple possible masses such that the ball can trace the same path for a given string length and angle?
- Determine an expression for the angular velocity of the ball in terms of m , θ , L and constants.
- Determine an expression for the period of orbit of the ball in terms of m , θ , L and constants.

254 Speed on a banked turn on a road

A road is constructed with a turn of radius 600 m. The surface of the road is banked at an angle of 2.5° from the horizontal. Determine the speed such that a car can complete the turn without sliding sideways on the road. (131F2024)

255 Banked road angle

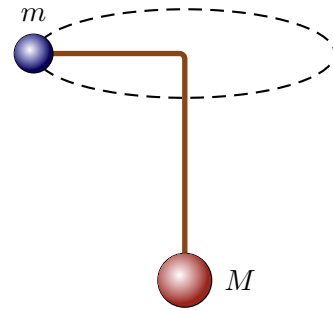
A road is constructed with a turn of radius 300 m. The surface of the road is banked at an angle in order to assist cars to make the turn. A car travels through this turn with constant speed of 35 m/s (about 80 mph). (131F2024)

- Suppose that there is no friction between the tires and road. Determine the angle at which the road must be banked so that the car can complete the turn without slipping.
- There usually is friction between the tires and road. For wet concrete and rubber the coefficient of static friction is 0.30 and the coefficient of kinetic friction is 0.20. Determine the minimum angle at which the road must be banked so that the car completes the turn without slipping.

256 Connected balls

A ball with mass m swings at the end of a string on a horizontal frictionless surface. The string runs through a hole in the surface and another ball with mass M is connected to it. Determine an expression for mass of the suspended ball such that the ball on the table swings in a circle with constant speed v and constant radius R . (131F2024)

- Determine an expression (in terms of m, R, v and g) for mass of the suspended ball such that the ball on the table swings in a circle with constant speed v and constant radius R .
- Determine an expression for the time for one orbit (in terms of m, R, M and g) such that the ball on the table swings in a circle with constant speed and constant radius R .



Work and Energy

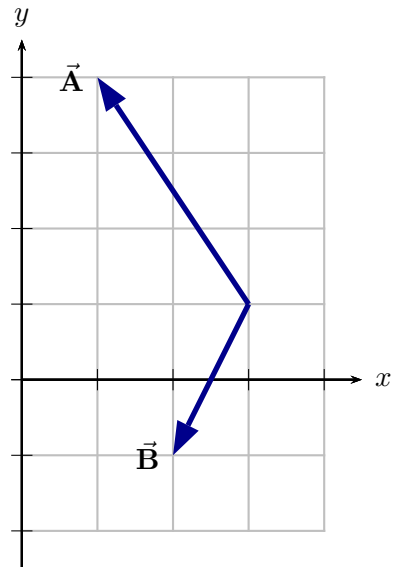
257 Vectors: dot products, algebraic

For each of the following, determine A_x, A_y, B_x, B_y and A_z and B_z (if applicable) and determine the dot product, $\vec{\mathbf{A}} \cdot \vec{\mathbf{B}}$. *Note: If your answer for the dot product contains $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$ then it is very incorrect! (131F2024)*

- a) $\vec{\mathbf{A}} = 2\hat{\mathbf{i}} + 2\hat{\mathbf{j}}$ and $\vec{\mathbf{B}} = 2\hat{\mathbf{i}} + 2\hat{\mathbf{j}}$.
- b) $\vec{\mathbf{A}} = 2\hat{\mathbf{i}} + 2\hat{\mathbf{j}}$ and $\vec{\mathbf{B}} = 2\hat{\mathbf{i}} - 2\hat{\mathbf{j}}$.
- c) $\vec{\mathbf{A}} = 2\hat{\mathbf{i}} - 2\hat{\mathbf{j}}$ and $\vec{\mathbf{B}} = 2\hat{\mathbf{i}} + 2\hat{\mathbf{j}}$.
- d) $\vec{\mathbf{A}} = 2\hat{\mathbf{i}} - 2\hat{\mathbf{j}}$ and $\vec{\mathbf{B}} = 2\hat{\mathbf{i}} - 2\hat{\mathbf{j}}$.
- e) $\vec{\mathbf{A}} = 2\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$ and $\vec{\mathbf{B}} = 2\hat{\mathbf{i}} + 2\hat{\mathbf{j}}$.
- f) $\vec{\mathbf{A}} = 2\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$ and $\vec{\mathbf{B}} = 2\hat{\mathbf{i}} - 2\hat{\mathbf{j}}$.
- g) $\vec{\mathbf{A}} = 2\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$ and $\vec{\mathbf{B}} = 2\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$.
- h) $\vec{\mathbf{A}} = 1\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$ and $\vec{\mathbf{B}} = 2\hat{\mathbf{i}} + 1\hat{\mathbf{j}} - 3\hat{\mathbf{k}}$.

258 Dot product of two vectors, graphical, 1

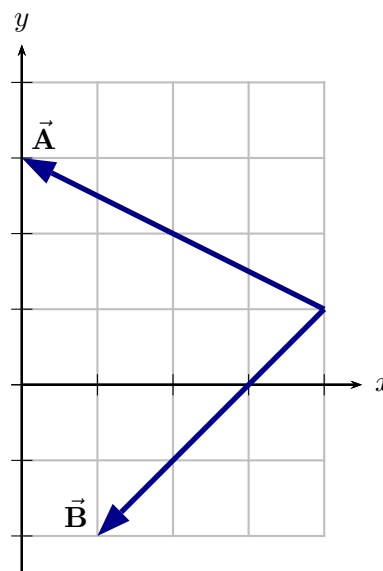
Determine $\vec{\mathbf{A}} \cdot \vec{\mathbf{B}}$ for the illustrated vectors. (131F2024)



259 Dot product of two vectors, graphical, 2

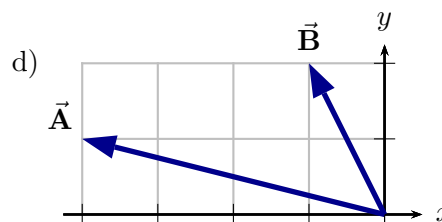
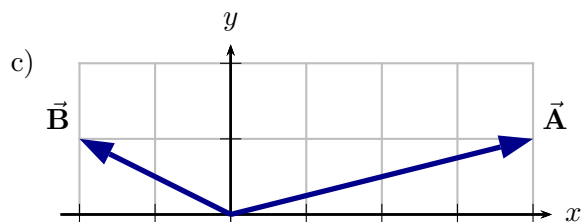
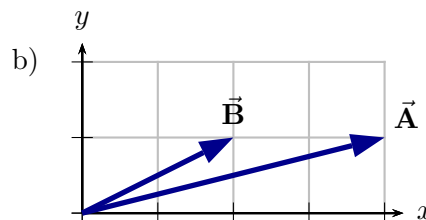
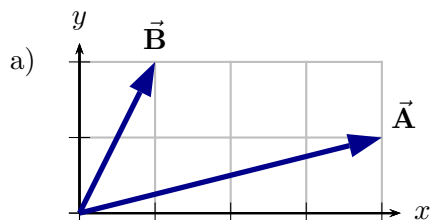
Consider the vectors \vec{A} and \vec{B} . (131F2024)

- Determine the magnitude of each vector.
- Determine the dot product of the vectors.
- Use the dot product to determine the angle between the vectors.



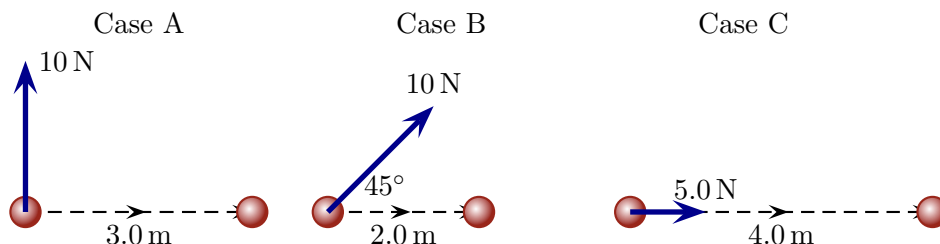
260 Vectors: dot products, graphical

For each of the following, express \vec{A} in component form \vec{B} using unit vectors and determine the dot product, $\vec{A} \cdot \vec{B}$. Note: If your answer for the dot product contains \hat{i} and \hat{j} then it is very incorrect! (131F2024)



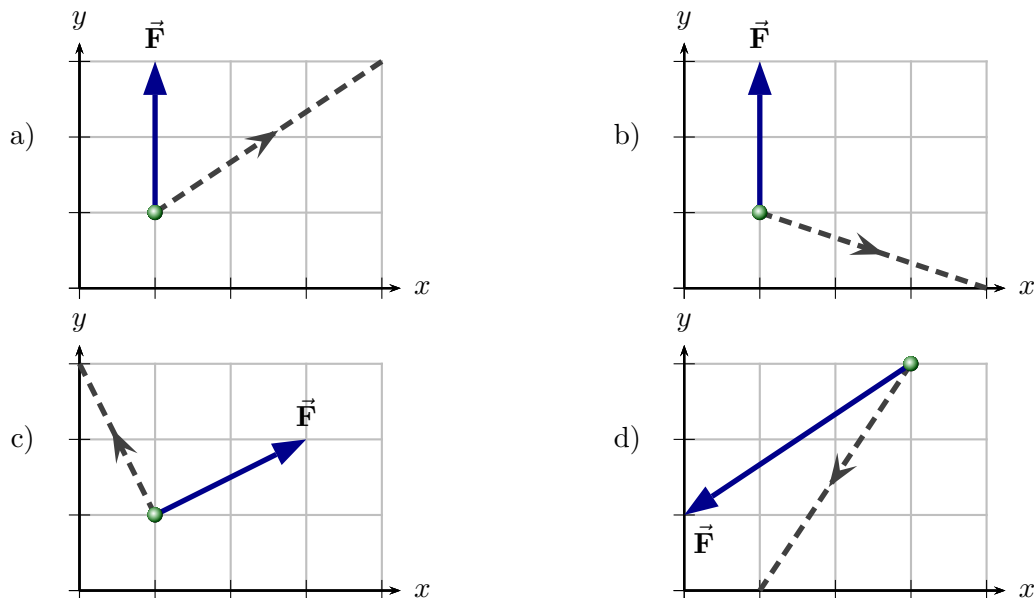
261 Ranking works

Three identical balls move under various forces as illustrated (there are other hidden forces present in each case). Rank these cases in order of increasing work done by the illustrated force. (131F2024)



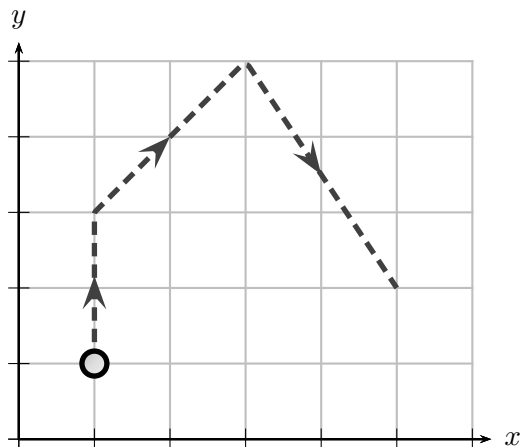
262 Work on sliding balls

Various balls slide along straight trajectories while constant forces act on them as illustrated. The grid units are meters for displacement and Newtons for force. In each case, determine the work done by the force. (131F2024)

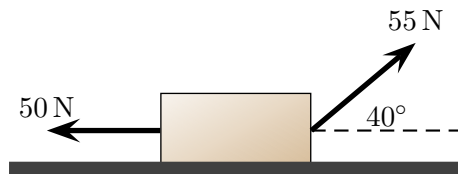


263 Work on a puck

A hockey puck slides along the indicated trajectory. The grid units are meters. During this period various forces act on the puck. One of these is constant and is given by $\vec{F} = 45\text{ N}\hat{i} - 30\text{ N}\hat{j}$. Determine the work done by \vec{F} as the puck slides along the illustrated trajectory. (131F2024)

**264 Tugging on a box**

A 6.0 kg box moves 8.0 m to the right along a horizontal frictionless surface. Alice pulls left on the crate with a 50 N force as illustrated. Bob pulls right with a 55 N force, angled as illustrated. (131F2024)



- Determine the work done by Alice on the box.
- Determine the work done by Bob on the box.
- Determine the net work done on the box.

265 Work done on a crate

A 20 kg crate moves along a horizontal frictionless surface. One person pulls up on the crate from the right and another pulls up on the left as illustrated. The crate is initially at rest and subsequently slides right for 5.0 m. (131F2024)



- Determine the work done by each of the following forces: gravity, normal force, and the forces exerted by the two people.
- Determine the net work done on the crate.

266 Raising an elevator

An elevator is raised by a cable and slows to a stop. (131F2024)

- a) While the elevator slows is the work done by the tension in the cable positive, negative or zero? Explain your answer.
- b) While the elevator slows is the magnitude of the work done by tension larger than, smaller than or the same as the magnitude of the work done by gravity? Explain your answer.

267 Object sliding down a ramp

A 4.0 kg box slides down a ramp that makes an angle of 20° from the horizontal. The length of the ramp is 3.0 m. (131F2024)

- a) Determine the work done by gravity as the box slides all the way down the ramp from the top to the bottom.
- b) Determine the work done by the normal force as the box slides all the way down the ramp from the top to the bottom.

268 Cart sliding up and down a ramp

A cart approaches a ramp. The cart slides up the ramp, reverses direction and then slides down. (131F2024)

- a) Consider the segment of the motion where the cart slides up the ramp. Is the work done by gravity positive, negative or zero? Explain your answer.
- b) Consider the segment of the motion where the cart slides down the ramp. Is the work done by gravity positive, negative or zero? Explain your answer.

269 Barge in a canal

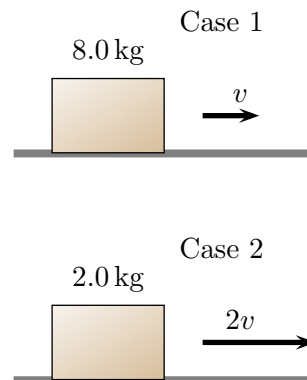
A barge can move along a straight canal. The barge is pulled by a donkey that walks along the bank of the canal. A rope connects the donkey to the barge. The donkey pulls the barge in a straight line for a distance of 750 m. (131F2024)

- a) Suppose that the rope makes an angle of 15° from the forwards direction of the barge's motion and the donkey exerts a constant force of 275 N on the barge. Determine the work done by the donkey.
- b) Now suppose that the rope length is changed and the result is that the rope makes an angle of 8.0° from the forwards direction of the barge's motion. Determine the force that must be exerted by the donkey so that it does the same work as it had done previously.

270 Kinetic energy

Two boxes move on a frictionless horizontal surface. The speed of the box in case 2 is twice the speed of the box in case 1. Let K_1 be the kinetic energy of the block in Case 1 and K_2 be the kinetic energy of the block in Case 2. Which of the following is true? Explain your answer. (131F2024)

- i) $K_1 = \frac{1}{4} K_2$
- ii) $K_1 = \frac{1}{2} K_2$
- iii) $K_1 = K_2$
- iv) $K_1 = 2K_2$
- v) $K_1 = 4K_2$



271 Pulling a box: energy and speed

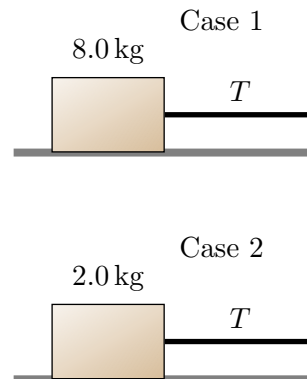
Two boxes on a frictionless horizontal surface are initially at rest. Both are pulled by a rope with the same tension for the same distance. (131F2024)

- a) Let K_1 be the kinetic energy of the block in Case 1 after it has been pulled through distance d . Let K_2 be the kinetic energy of the block in Case 2 after it has been pulled through distance d . Which of the following is true? Explain your answer.

- i) $K_1 = \frac{1}{4} K_2$
- ii) $K_1 = \frac{1}{2} K_2$
- iii) $K_1 = K_2$
- iv) $K_1 = 2K_2$
- v) $K_1 = 4K_2$

- b) Let v_1 be the speed of the block in Case 1 after it has been pulled through distance d . Let v_2 be the speed of the block in Case 2 after it has been pulled through distance d . Which of the following is true? Explain your answer.

- i) $v_1 = \frac{1}{4} v_2$
- ii) $v_1 = \frac{1}{2} v_2$
- iii) $v_1 = v_2$
- iv) $v_1 = 2v_2$
- v) $v_1 = 4v_2$



272 Molecular kinetic energy

The average kinetic energy of molecules in a gas only depends on the temperature of the gas (*side note: this is about translational kinetic energy only*). Consider a gas of helium molecules (each helium molecule consists of just one helium atom) and a gas of neon molecules (each neon molecule consists of one neon atom). Suppose that these gases are both at the same temperature. For these gases how does the typical speed of the helium molecules compare (same, smaller, larger, ...) to that of the neon molecules. (131F2024)

273 Energy, distance and speed

A rock is initially at rest on a horizontal sheet of ice. Alice pushes the rock with a constant horizontal force. The rock moves in a straight line, first passing Bob who is 5.0 m from the initial location and then Carol, who is 5.0 m from Bob. (131F2024)

- a) Let ΔK_1 be the change in the rock's kinetic energy from its initial location until it reaches Bob. Let ΔK_2 be the change in the rock's kinetic energy from the moment it passes Bob until it reaches Carol. Which of the following is true? Explain your answer.
- i) $\Delta K_2 < \Delta K_1$
 - ii) $\Delta K_2 = \Delta K_1$
 - iii) $\Delta K_2 > \Delta K_1$
- b) Let v_1 be the rock's speed as it reaches Bob. Let v_2 be the rock's speed as it reaches Carol. Which of the following is true? Explain your answer.
- i) $v_2 = v_1$
 - ii) $\sqrt{2}v_1 > v_2 > v_1$
 - iii) $v_2 = \sqrt{2}v_1$
 - iv) $v_2 = 2v_1$
 - v) $v_2 > 2v_1$

274 Work and motion

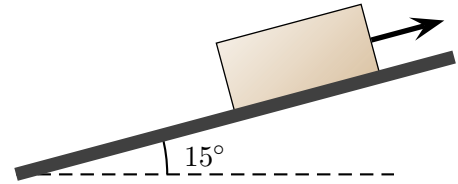
A 60 kg crate moves along a horizontal frictionless surface. Alice pulls up and left on the crate with a 300 N force as illustrated. Bob pulls right with a 400 N force. The crate is initially at rest and moves 5.0 m to the right while they pull on it. (131F2024)



- a) Determine the work done by Alice on the crate.
- b) Determine the work done by Bob on the crate.
- c) Determine the net work done on the crate.
- d) Determine the speed of the crate at moment that it reaches 5.0 m from its starting point.

275 Box lowered down a frictionless ramp

A 5.00 kg box is at rest at the top of a frictionless ramp that is inclined at 15° from the horizontal. The length of the ramp is 8.00 m. A rope pulls on the crate with a 9.00 N force parallel to and up the ramp. (131F2024)



- Determine the work done by gravity on the box from the top to bottom of the ramp.
- Determine the work done by the rope on the box from the top to bottom of the ramp.
- Determine the speed of the box at the bottom of the ramp.

276 Crate sliding down a frictionless ramp

A 6.00 kg crate is at rest at the top of a frictionless ramp that is inclined at 25° from the horizontal. The length of the ramp is 3.00 m. Use work and energy to answer the following questions. (131F2024)

- The crate is released. Determine the net work done on the crate as it moves from the top to the bottom of the ramp.
- Use the net work to determine the speed of the block as it reaches the bottom of the ramp.

277 Crate lowered down a frictionless ramp

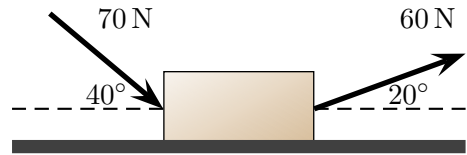
A 20.0 kg crate is at rest at the top of a frictionless ramp that is inclined at 30° from the horizontal. The length of the ramp is 5.00 m. A rope pulls on the crate with a 70.0 N force parallel to and up the ramp. Use work and energy to determine the speed of the crate when it reaches the bottom of the ramp. (131F2024)

278 Box pushed up a frictionless ramp

A 20.0 kg box is at rest at the bottom of a frictionless 3.50 m long ramp that is inclined at 15° from the horizontal. A person pushes horizontally on the box with a 60.0 N force. Determine the speed of the crate when it reaches the top of the ramp using energy. (131F2024)

279 Pushing a crate across a rough floor

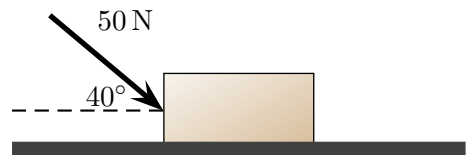
A 10 kg crate can move across a rough horizontal floor. One person pulls up on the crate from the right and another pushes down as illustrated. The crate is initially at rest and subsequently slides right for 5.0 m. The coefficient of kinetic friction between the crate and floor is 0.20. (131F2024)



- Determine the friction force on the crate.
- Determine the work done by each of the following forces: gravity, normal force, friction force and the forces exerted by the two people.
- Determine the speed of the crate at the moment that it has moved 5.0 m from its original location.

280 Pushing a crate at a constant speed along a rough floor

A worker pushes a crate a distance of 6.0 m along a rough horizontal surface. The crate moves with a constant speed. Use *work* to determine the frictional force acting on the cart. (131F2024)



281 Sliding hockey puck

A 0.150 kg hockey puck slides along a horizontal surface. Initially the surface is frictionless and the puck slides with speed 18 m/s. The puck then hits a rough section of floor which provides a constant friction force. The puck stops a distance of 2.5 m after hitting the rough section. (131F2024)

- Determine the net work done on the puck as it moves across the rough section. *Hint: consider kinetic energy.*
- Use the net work to determine the magnitude of the friction force acting on the puck.
- Determine the coefficient of kinetic friction between the puck and the floor.

282 Cart and ramp

A 20 kg cart is launched with speed 4.0 m/s at the base of a frictionless ramp. Determine the maximum vertical height that the cart ascends as it moves up the ramp. (131F2024)

283 Raising an elevator

An 2000 kg elevator is lifted by a cable attached to a motor. It moves at a constant speed through a vertical distance of 25 m. It takes 75 s to do this. (131F2024)

- a) Determine the work done by gravity as the elevator is lifted.
- b) Determine the work done by the cable as the elevator is lifted.
- c) Determine the power delivered by the motor as it raises the elevator.
- d) Suppose that the elevator were raised through the same distance and starting at the same speed but with the speed decreasing at a constant rate as it ascends. How would the magnitude of the work done by the cable compare (larger, smaller,...) to that done by gravity? Explain your answer using physics. What would this imply for the power delivered by the motor compared to the situation where the elevator is raised at a constant speed?

284 Cannonball energy

Two identical cannonballs are fired from the ground with the same speed. Cannonball Y is fired vertically upward while cannonball Z is fired at an angle of 60° above the horizontal. Consider the highest points reach by the cannonballs. (131F2024)

- a) Explain which cannonball has the largest kinetic energy at its highest point.
- b) Explain which cannonball has the largest gravitational potential energy at its highest point. Use this to determine which cannonball reaches a higher point.

285 Rocks launched from a bridge

Two rocks are launched from a bridge across a river. The point of launch is 4.0 m above the water in the river. In the following ignore air resistance and assume that there are no obstacles between the rocks and the river. (131F2024)

- a) Suppose that the two balls are identical. One is launched horizontally and the other is launched at an angle of 45° above the horizontal; their launch speeds are the same. Consider the speeds of each ball as they pass a point 3.0 m above the river. Which ball is traveling with a faster speed? Explain your answer.
- b) Consider the same situation but where the balls have different masses. The lighter ball is launched horizontally. Which ball is traveling with a faster speed? Explain your answer.

286 Energy and projectiles

A 15 kg cannonball is fired, leaving the ground with speed 30 m/s. We aim to determine the speed of the cannonball when it reaches a height of 25 m above the ground. (131F2024)

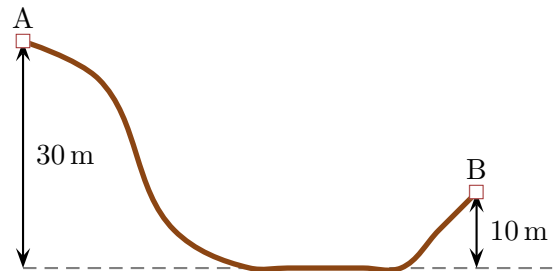
- Sketch the situation showing the trajectory of the cannonball. Indicate an “initial” and a “final” moment.
- Determine the total energy at the moment that the cannonball is launched.
- Determine the total energy when the cannonball is 25 m above the ground.
- Determine the kinetic energy of the cannonball when it is 25 m above the ground.
- Determine the speed of the cannonball when it is 25 m above the ground.

287 Cart and ramp

A 20 kg cart is launched with speed 4.0 m/s at the base of a frictionless ramp. Determine the maximum vertical height that the cart ascends as it moves up the ramp. (131F2024)

288 Ski jump

A 70 kg skier starts at rest at point A on the illustrated slope and then slides down, approaching launch point B. Ignore friction and air resistance. (131F2024)



- Determine the total energy of the skier at A.
- Determine the total energy of the skier at point B.
- Determine the speed of the skier at point B.
- Determine the maximum speed of the skier between points A and B.

289 Sledding

King Zog, with mass 160 kg, and Queen Geraldine, with mass 80 kg, sled down an icy hill. They start from rest at the same point above the bottom of the hill. Ignore friction and air resistance. Which of the following is true regarding their speeds at the bottom of the hill? Explain your answer. (131F2024)

- Same speeds.
- Geraldine’s speed is twice that of Zog.
- Geraldine’s speed is four times that of Zog.
- Zog’s speed is larger than Geraldine’s speed.

290 Loop-the-loop rollercoaster

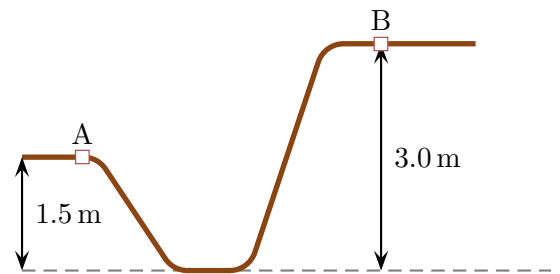
A 100 kg rollercoaster starts from rest at the top of the illustrated track. The rollercoaster completes the loop. Determine speed of the rollercoaster at the end of the track. Ignore friction and air resistance. (131F2024)



291 Skate park, 1

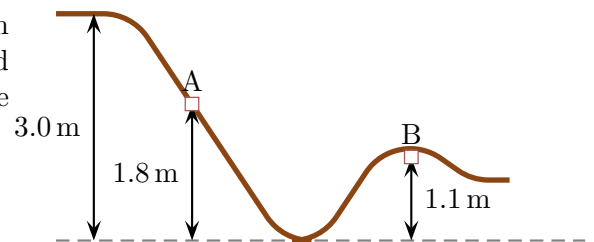
A skater approaches point A with speed 10 m/s. The skater slides along the illustrated track. Ignore friction and air resistance. (131F2024)

- Suppose that the mass of the skater is 60 kg. Determine the speed of the skater at point B.
- Determine the maximum speed of the 60 kg skater along the track.
- Determine the minimum speed that the 60 kg skater should have at A so that she reaches B.
- Would any of your results to the previous parts be different if the mass of the skater were different? Explain your answer.



292 Skate park, 2

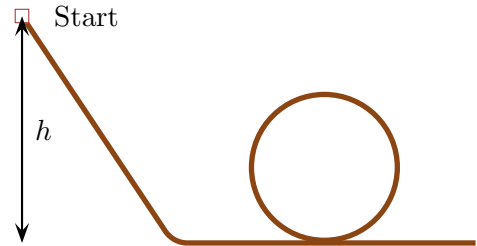
A 60 kg skater moves down a ramp at point A with speed 3.0 m/s. The skater slides along the illustrated track. Ignore friction and air resistance. Determine the speed of the skater at point B. (131F2024)



293 Completing the loop

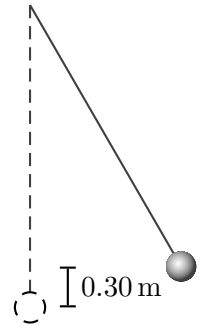
A cart slides down a frictionless track toward a circular loop with radius R . The cart is released from rest. (131F2024)

- Determine an expression for the minimum speed for the cart at the top of the loop such that it completes the loop.
- Determine an expression for the minimum height at which the cart should be released so that it completes the loop.



294 Pendulum

A pendulum consists of a ball that swings from a string. A 1.4 m long pendulum is raised so that it is released from rest 0.30 m above its lowest point. Determine the maximum speed of the pendulum. (131F2024)



295 Swinging monkey

A monkey grabs a 3.0 m rope and swings in a circular arc. The monkey reaches a maximum speed of 4.0 m/s. Determine the maximum angle (from the vertical) reached by the monkey as it swings. (131F2024)

296 Tarzan

Tarzan, with mass 80 kg, grabs a 5.0 m rope and swings in a circular arc. As Tarzan begins to swing the rope is at an angle of 35° from the vertical. Determine Tarzan's maximum speed. (131F2024)

297 Work done by a string

A ball swings in a circle at the end of a string. Which of the following is true? Explain your answer. (131F2024)

- i) The string does positive work on the ball whenever the ball speeds up and negative work on the ball whenever it slows down.
- ii) The string does positive work on the ball regardless of its speed.
- iii) The string does negative work on the ball regardless of its speed.
- iv) The string does zero work on the ball regardless of its speed.

298 Power delivered by engines

Two engines pull identical objects along horizontal surfaces. Engine A delivers 2000 W of power and engine B delivers 4000 W. Engine A pulls for 5 min and engine B for 2 min. Which of the following is true? Explain your answer. (131F2024)

- i) Engine A delivers more work than engine B.
- ii) Engine A delivers less work than engine B.
- iii) Engine A delivers the same work as engine B.
- iv) There is not enough information to decide.

299 Lifting bananas

A 150 kg bunch of bananas is lifted by a vertical rope through a distance of 7.5 m. It takes 16 s to do this and for all but the initial and final moments it moves with constant speed. Determine the power delivered by the tension in the rope. (131F2024)

300 Hoisting fish

King Zog and Queen Geraldine are fishing from a bridge and they catch identical twin fish, each with mass 5.0 kg. They hoist the fish at constant speeds to the bridge 8.0 m above the water. Zog takes 10 s to hoist his fish and Geraldine 7.5 s to hoist her fish. (131F2024)

- a) Which of the following is true? Explain your answer.
 - i) Zog and Geraldine do the same work.
 - ii) Zog does more work than Geraldine.
 - iii) Zog does less work than Geraldine.
- b) Which of the following is true? Explain your answer.
 - i) Zog and Geraldine expend the same power.
 - ii) Zog expends less power.
 - iii) Zog expends more power.

301 Solar energy

The Sun delivers energy in the form of electromagnetic waves to Earth. At the outer edge of Earth's atmosphere, the total power per square meter (*the solar irradiance*) is 1360 W/m^2 . This is diminished at Earth's surface by the atmosphere and depends on the angle of the Sun. At some location it is reduced to 1050 W/m^2 . (131F2024)

- a) A satellite uses solar panels to gather energy. If the satellite requires 10 kW of power, what would the necessary area of its solar panels be (assuming that the panels are 100% efficient)?
- b) Suppose that a solar plant on Earth is required to provide $5.0 \times 10^9 \text{ J}$ of energy per hour. Determine the area of the Earth's surface that the solar panels would have to cover if they are only 25% efficient.

302 Solar energy estimate

The aim of this exercise is to *estimate* the size of a solar array that could provide the electrical energy that Grand Junction uses. A detailed calculation will be very complicated but one can get a rough estimate using the following information.

The Sun delivers a total power per square meter (*the solar irradiance*) delivered to Earth's surface at Grand Junction of approximately 750 W/m^2 during winter and about 1000 W/m^2 during midsummer. A solar panel converts 25% of this to electrical energy. The US Energy Information Agency can provide data about typical electrical energy usage in the US.

Use this data to determine an approximate area that would have to be covered by solar panels to provide for all of Grand Junction's electrical energy. First describe *how* you would calculate this. Second, do the calculation, describing the data that you used, where you found it and the assumptions that you made. (131F2024)

303 Spring forces

A 5.0 kg bag is suspended by a spring from the ceiling of an elevator. The spring constant is 4000 N/m. (131F2024)

- a) The elevator and spring move up with a constant speed of 0.50 m/s. Determine the amount by which the spring is stretched.
- b) The elevator and bag accelerate upwards with an acceleration of 3.0 m/s^2 . Determine the amount by which the spring is stretched.

304 Monkey on a vertical spring

A monkey hangs from a spring which is attached to the ceiling of a building. The spring hangs vertically and the monkey bounces up and down without touching the floor. (131F2024)

- a) As the monkey ascends toward and nears its highest point, the spring is compressed. Which of the following is true while this happens?
 - i) The spring does positive work, gravity does positive work.
 - ii) The spring does positive work, gravity does negative work.
 - iii) The spring does negative work, gravity does positive work.
 - iv) The spring does negative work, gravity does negative work.
- b) As the monkey begins to descend away its highest point, the spring is still compressed. Which of the following is true while this happens?
 - i) The spring does positive work, gravity does positive work.
 - ii) The spring does positive work, gravity does negative work.
 - iii) The spring does negative work, gravity does positive work.
 - iv) The spring does negative work, gravity does negative work.

305 Oscillating block

A 6.0 kg block can slide along a frictionless table. It is attached to a spring with spring constant 30 N/m. The block is pulled 0.20 m from the spring's equilibrium position. The block oscillates. (131F2024)

- a) Describe whether you can use constant acceleration kinematics to predict the block's speed as it passes the equilibrium point.
- b) Determine the speed of the box as it passes the equilibrium point.

306 Oscillating cart

A 0.60 kg cart can slide along a frictionless track. It is attached to a spring with spring constant 3.5 N/m. The cart is pulled 0.40 m from the spring's equilibrium position and released from rest. (131F2024)

- a) Describe whether you can use constant acceleration kinematics to predict the cart's speed at any moment after it has been released.
- b) Determine the maximum speed of the cart.

307 Work done by a spring

A 20 kg box on a rough horizontal floor is pushed against a spring with constant 800 N/m. The spring is compressed by 0.10 m. The coefficient of kinetic friction between the box and floor is 0.15. The box departs from the spring when the spring reaches its equilibrium state. (131F2024)

- a) Determine the work done by the friction force on the box from the point of release until the box departs.
- b) Determine the work done by spring on the box from the point of release until the box departs.
- c) Determine the speed of the box at the point of departure.
- d) Is the acceleration of the box constant during this process? Could you use constant acceleration kinematics and Newton's second law to determine the speed of the box at the point of departure.

308 Spring bumper

Two walrus (named X and Y), with the same masses, slide along horizontal sheets of ice. Each collides with a horizontal spring mounted to a wall; the springs are identical. Prior to hitting the spring, walrus X moved with speed twice that of walrus Y. The springs compress, bringing each walrus to a stop. Which of the following is true regarding the distances by which the springs compress? Explain your answer. (131F2024)

- i) Springs compress by the same distance.
- ii) X compresses spring by twice as much Y.
- iii) X compresses spring by four times as much Y.
- iv) X compresses spring by half as much Y.
- v) X compresses spring by a quarter of what Y compresses.

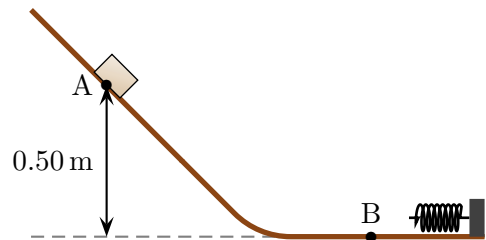
309 Bungee jumper

A 100 kg person is attached to a bungee cord and, starting at rest, jumps off a bridge that is 120 m above a river. The bungee cord behaves like a spring and the length of the cord when it is unstretched is 100 m. The spring constant of the cord needs to be such that person stops just above the river. (131F2024)

- Determine the total energy of the system at the moment that the person jumps.
- Determine the total energy of the system at the moment that the person stops just above the river and use the result to determine the spring constant of the bungee cord.
- Determine the maximum force that the bungee cord exerts on this person.
- Now suppose that a person with mass 70 kg jumps from the same bridge using the same cord. Determine the maximum stretch in the spring, the height above the river at which the person reverses direction and the maximum force exerted on the person.

310 Box, ramp and spring

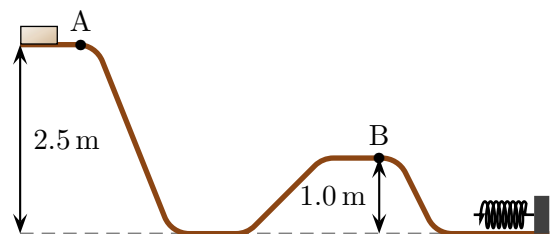
A 6.0 kg box is released from rest at on the illustrated frictionless track. The box slides down the ramp, passes B and collides with a spring (constant 6000 N/m). While it moves friction and air resistance can be ignored. (131F2024)



- Determine the speed of the box at point B.
- Determine the maximum distance by which the spring is compressed.

311 Sled and spring bumper

A 100 kg sled moves along the illustrated track. At point A it moves right with speed 5.0 m/s. While it moves friction and air resistance can be ignored. (131F2024)



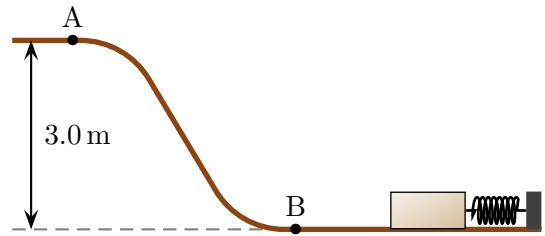
- Determine the energy of the sled at point A.
- Determine the kinetic energy of the sled when it is at point B.
- Determine the speed of the sled at point B.

The sled approaches a spring with spring constant 5000 N/m.

- Determine the speed of the sled just before it hits the spring.
- Determine the maximum distance by which the spring is compressed.
- Determine the maximum force exerted by the spring as the sled compresses it.

312 Spring-launched sled

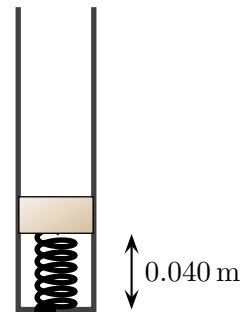
A 10.0 kg sled is pushed against a spring with spring constant 4000 N/m. The sled is held at rest while the spring is compressed by 0.45 m. It is released and then travels along the illustrated frictionless surface. (131F2024)



- By the time that the sled passes point B it has left the spring. Determine the speed of the sled at point B.
- Determine the speed of the sled at point A.

313 Spring gun

A spring gun consists of a vertical pipe with frictionless walls. A 30 g block is pushed against a spring, which is attached to the base of the pipe. In equilibrium the top of the spring is 0.050 m above the base of the pipe. The block is pushed against the spring and released from the indicated location while the pipe points vertically. (131F2024)



- Determine the spring constant so that the maximum height above the base of the pipe that the block reaches is 12.00 m.
- Determine the maximum velocity of the block.

314 Crate and ramp

A 10 kg crate is launched with speed 16 m/s at the base of a frictionless ramp that is inclined at 25° from the horizontal. Use work and energy to answer the following questions. (131F2024)

- Ignoring air resistance, determine the distance along the ramp that the crate travels before stopping (briefly at its highest point).
- Ignoring air resistance, determine the vertical height gained by the crate at the highest point that it reaches.
- Suppose that the crate only travels 20 m because of air resistance that results in a force opposite to the crate's motion. Assuming that the air resistance is constant, determine the magnitude of the air resistance force.

315 Spring, cart and hand

A 5.0 kg cart can slide along a frictionless horizontal surface. A spring, with spring constant 400 N/m connects the cart to a wall. The spring is initially compressed by 0.25 m and the cart is held at rest. It is released and subsequently a hand pushes with a constant 30 N force against the spring as the spring relaxes. Determine the speed of the cart when the spring reaches its equilibrium position. (131F2024)



316 Spring-launched cart

A 40 kg cart can move along a horizontal surface. The cart is pushed against a spring, with spring constant 3000 N/m, compressing it by 0.35 m. It is released and begins to move right across a frictionless surface, eventually leaving the spring. After it has left the spring it enters section where there is friction. The coefficient of friction between the cart and the surface is 0.20. The cart eventually stops in this section. (131F2024)

- Determine the speed of the cart at the moment that it leaves the spring.
- Determine the distance which the cart travels from the moment that it encounters the friction until it stops. *Hint: use work and energy.*

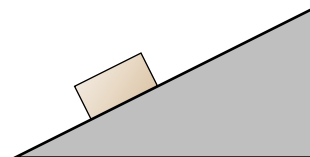
317 Bumper cart

A 200 kg bumper cart can move on a rough horizontal surface. The cart is pushed against a spring, with spring constant 6000 N/m, compressing it by 0.25 m. It is released and begins to move right. The coefficient of friction between the cart and the surface is 0.20. (131F2024)

- Determine the speed of the cart at the moment that it leaves the spring.
- Suppose that while the cart moves right it is also pulled with a rope that pulls with constant tension 600 N to the right. Determine the speed of the cart when it leaves the spring.

318 Box on a rough ramp

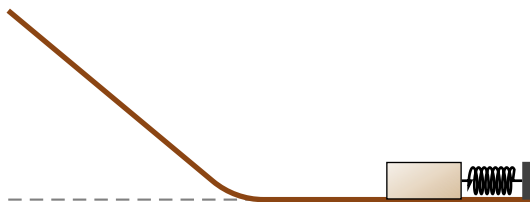
A 5.0 kg box approaches a rough ramp, inclined at an angle of 20° above the horizontal. The box's speed at the base of the ramp is 16 m/s. The coefficient of kinetic friction between the box and ramp is 0.28. (131F2024)



- Use *work and energy* to determine the distance along the ramp which the box slides before returning back down the ramp.
- Use *work and energy* to determine the speed of the box when it reaches the bottom of the ramp again.
- (*Challenging*) Suppose that the angle between the ramp and the horizontal is θ . Determine an expression for the minimum coefficient of friction so that the box does not reach the bottom of the ramp.

319 Spring-launched crate

An 8.0 kg crate is held at rest against a spring with spring constant 1600 N/m compressing it by 0.25 m. It is released and then travels along the illustrated surface. The lower 0.50 m long horizontal portion of the surface is rough with coefficient of friction 0.35. The rest of the surface is frictionless. Ignoring air resistance determine the maximum vertical height reached by the crate. (131F2024)



320 Work along various trajectories

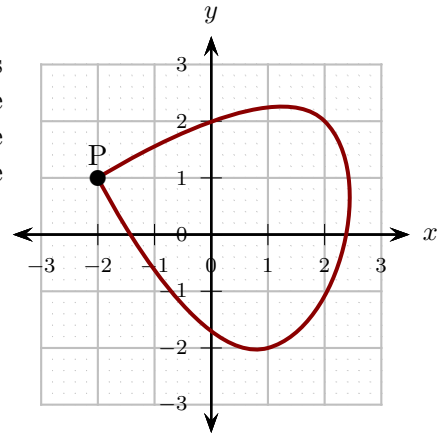
A force $\vec{F} = 40\text{N}\hat{i} - 30\text{N}\hat{j}$ acts on a particle. The particle could move along one of the following three trajectories given in Cartesian coordinates (x, y) :

- Path A: $(0, 0) \rightarrow (2, 0) \rightarrow (2, 3)$
- Path B: $(0, 0) \rightarrow (0, 3) \rightarrow (2, 3)$
- Path C: $(0, 0) \rightarrow (2, 3)$

- Determine the work done along each trajectory. (131F2024)
- Could the force be conservative?

321 Work in a loop

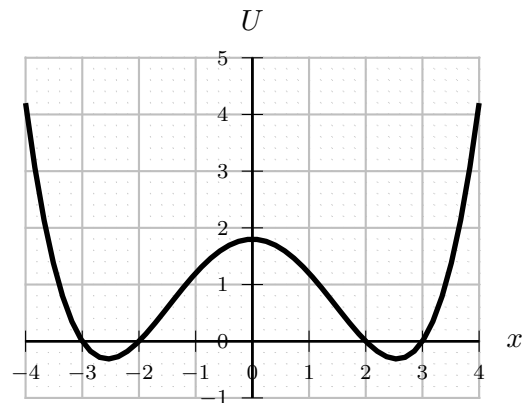
A coin is pressed against a flat vertical board. Various forces including gravity and friction act on the coin while it moves. The coin is initially at point P and follows the loop clockwise, finally ending at point P. In each of the following, explain your answers. (131F2024)



- a) Which of the following is true about the work done by gravity for the motion around the loop?
 - i) $W_{\text{grav}} = 0$.
 - ii) $W_{\text{grav}} < 0$.
 - iii) $W_{\text{grav}} > 0$.
- b) Which of the following is true about the work done by friction for the motion around the loop?
 - i) $W_f = 0$.
 - ii) $W_f < 0$.
 - iii) $W_f > 0$.

322 Potential and motion, 1

A particle that can move along the x axis is subjected to the potential whose graph is as illustrated (vertical axis units are J and horizontal axis units are m). (131F2024)

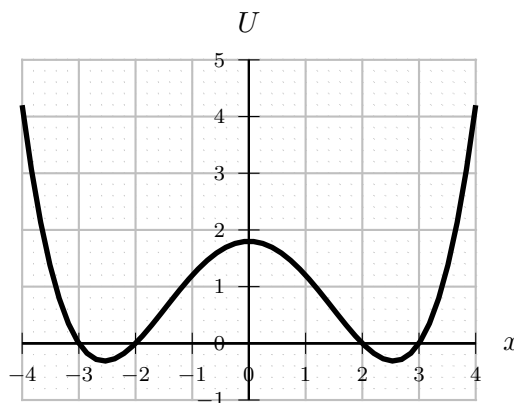


- a) The particle is released from rest at $x = 1.0$ m. Describe the direction of the force acting on the particle at this moment. Describe the subsequent motion of the particle (left, right, slowing, speeding up, ...). Approximately what is the furthest right that this particle reaches? Explain your answer.
- b) The particle is released from rest at $x = 4.0$ m. Describe the direction of the force acting on the particle at this moment. Describe the subsequent motion of the particle (left, right, slowing, speeding up, ...). Approximately what is the furthest left that this particle reaches? Explain your answer.

323 Potential and motion, 2

A particle that can move along the x axis is subjected to the potential whose graph is as illustrated. (131F2024)

- Suppose that the potential energy of the particle is zero. Use the plot to explain whether the force on the particle is also zero in this situation.
- Suppose that the force on the particle is zero. Use the plot to explain whether the potential energy of the particle is also zero in this situation.

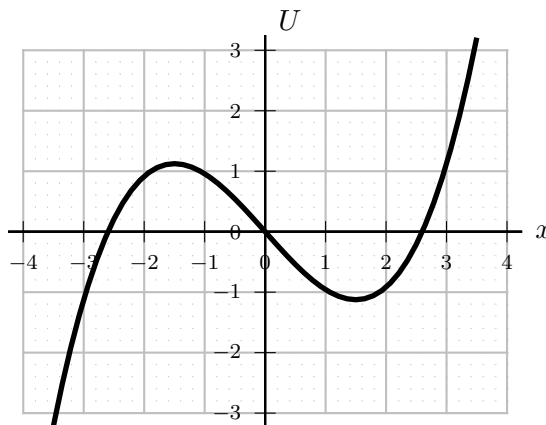


324 Motion under a complicated potential

A particle that can move along the x axis is subjected to the potential

$$U(x) = (x^3 - 6.75x)/5.$$

- Determine an expression for the horizontal component of the force on the object.
- Determine locations where the object is in equilibrium and for each describe whether the equilibrium is stable or not. (131F2024)



325 Particle in a quadratic potential

A particle moves subject to an interaction described by the potential

$$U(x) = \frac{1}{2}kx^2 - bx$$

where $k = 140 \text{ N/m}$ and $b = 35.0 \text{ N}$. (131F2024)

- Determine and expression for the force associated with the potential.
- Determine any locations where the force on the particle is zero. Is $U = 0 \text{ J}$ at these locations?
- Suppose that the particle is held at rest at $x = 0.0 \text{ m}$. In which direction will it begin to move? Explain your answer.

326 Force for a quadratic potential

A particle moves subject to an interaction described by the potential

$$U(x) = ax^2 + bx + c$$

where $a = -4.0 \text{ J/m}^2$, $b = 10.0 \text{ J/m}$ and $c = 5.0 \text{ J}$ are constants. (131F2024)

- a) Determine any locations where the force on the particle is zero.
- b) Suppose that the particle is held at rest at $x = 0.0 \text{ m}$. In which direction will it begin to move? Explain your answer.

327 Force for the Lennard-Jones potential

The Lennard-Jones potential for two air molecules is

$$U(x) = \epsilon \left[\left(\frac{a}{x} \right)^{12} - \left(\frac{a}{x} \right)^6 \right]$$

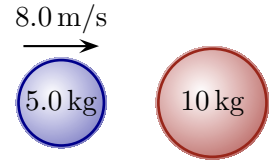
where a is a constant with units of meters and ϵ is a constant with units of energy. (131F2024)

- a) Determine an expression for the force associated with this potential. This is the force that one molecule exerts on the other.
- b) Determine the force when $x = a$. For air $a = 3.42 \times 10^{-10} \text{ m}$ and $\epsilon = 1.65 \times 10^{-21} \text{ J}$.
- c) Determine the force when $x = 2a$. For air $a = 3.42 \times 10^{-10} \text{ m}$ and $\epsilon = 1.65 \times 10^{-21} \text{ J}$.

Momentum

328 Colliding balls, 1

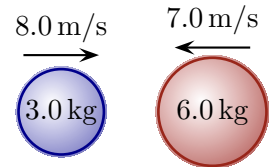
Two balls, isolated from all other objects, are initially as illustrated. The 5.0 kg ball moves directly toward the 10 kg ball. They collide and subsequently the 10 kg ball moves right with speed 3.0 m/s. The aim of this exercise will be to determine the speed and direction of motion of the smaller ball after the collision. (131F2024)



- Determine the momentum of the smaller ball before the collision.
- Determine the momentum of the larger ball before the collision.
- Determine the momentum of the system before the collision.
- Determine the momentum of the larger ball after the collision.
- Determine the momentum of the smaller ball after the collision.
- Determine the velocity of the smaller ball after the collision.
- Suppose that the construction of the balls was different and that after the collision the larger ball moves right with speed 5.0 m/s. Determine the velocity of the smaller ball after the collision.

329 Colliding balls, 2

Two balls, isolated from all other objects, initially move toward each other as illustrated. The 3.0 kg ball moves directly toward the 6.0 kg ball. The balls collide and subsequently the 3.0 kg ball moves left with speed 4.0 m/s. Determine the velocity of the larger ball after the collision. (131F2024)



330 Space collision

A 100 kg astronaut is at rest in space. A 0.0050 kg fleck of paint moves toward the astronaut with speed 15×10^3 m/s. It collides with and sticks to the astronaut. (131F2024)

- Write an expression for the total momentum of the system before the collision in terms of the masses and speeds of the astronaut and paint. Determine the total momentum of the system before the collision (assume that the paint fleck initially moves along the positive x axis).
- Write an expression for the total momentum of the system after the collision in terms of the masses and speeds (after collision) of the astronaut and paint.
- Use momentum conservation to determine the speed of the astronaut after the collision.
- Now suppose that the paint fleck bounced off the astronaut and reverses direction with speed 8.0×10^3 m/s. Determine the speed of the astronaut after the collision.

331 Railroad car carrying sand

An empty 3000 kg rail car has an open top and moves right with speed 5.0 m/s (it slides along the tracks without a locomotive pulling it) It passes beneath a chute that pours sand into the car. The sand hits the car vertically and after it has passed the car moves with speed 3.0m/s. Determine the mass of the sand delivered to the car. (131F2024)

332 Sledder catching a ball

A person sits at rest on a sled on a sheet of ice; the combined mass of the person and sled is 95.0 kg. A 5.0 kg bowling ball is launched horizontally rightwards toward the person with speed 20 m/s. The person catches the ball and the person, sled and ball slide together at the same speed. (131F2024)

- a) Determine their velocity after the ball has been caught.
- b) Is kinetic energy conserved in this process?

Suppose that the person and sled had been moving left with speed 2.0 m/s prior to catching the ball.

- c) Determine their velocity after the ball has been caught.

333 Bumper carts on ice

A parent slides in a cart (mass of both 100 kg) to the right with speed 6.0 m/s. A child on another cart (mass of both 60 kg) also slides to the right, initially ahead of the parent with speed 4.0 m/s. When they collide the parent holds onto the other cart. Both slide together. Determine their velocity after the collision. (131F2024)

334 Bouncing ball

A ball is thrown towards a vertical wall and is traveling horizontally at the moment that it hits the wall. It bounces back horizontally with approximately the same speed. (131F2024)

- a) Someone correctly observes that the momentum of the ball is not conserved during this process. Explain why this is true.
- b) Is the momentum of the system consisting of the ball and wall conserved during this process? Explain your answer.

335 Dropping a rock on a planet

An astronaut stands on a planet, holding a rock at rest above the surface of the planet. The planet, astronaut and rock are initially at rest. The astronaut releases the rock and it falls toward the planet. (131F2024)

- a) While the rock falls is the total momentum of the planet/astronaut/rock system zero or not? Explain your answer.

b) While the rock falls does the planet remain at rest or not? Explain your answer.

336 Rebounding subatomic particles

A subatomic particle, labeled 1, with mass m_1 travels with speed v_1 directly toward another subatomic particle, labeled 2, with mass m_2 and initially at rest. The particle with mass m_1 rebounds, traveling left with speed $0.80v_1$ and the other particle travels right with speed v_2 . Determine an expression for m_2 in terms of v_1, v_2 and m_1 . (131F2024)

337 Jumping crowd

A crowd of 100000 people, each with mass 80 kg are at rest on Earth's surface. They all jump up simultaneously, leaving Earth's surface with speed 3.0 m/s (relative to the background). Determine Earth's recoil speed. (131F2024)

338 Mollusc propulsion

Many molluscs (e.g. octopus, squid, scallop) can move by propelling a jet of water away from their bodies. These creatures ingest a small quantity of water which they subsequently expel. Assume that the creature and ingested water is initially at rest. (131F2024)

- a) A particular type of squid has mass 100g and moves with speed 0.70 m/s immediately after expelling a jet of water. The jet moves with speed 1.3 m/s. Determine the mass of the expelled water in the jet.
- b) A particular scallop has mass 41 g. Assume that this expels 5.0 g of water with speed 2.5 m/s. Determine the speed with which the scallop moves afterward.

339 Separating carts

Two carts are at rest on a frictionless horizontal track. The mass of cart A is exactly four times the mass of cart B. A spring between the carts is released, the carts separate and cart A moves left with speed v . (131F2024)

- a) Which of the following is true for cart B after they separate? Explain your answer.
 - i) B moves left with speed v .
 - ii) B moves left with speed $2v$.
 - iii) B moves left with speed $4v$.
 - iv) B moves right with speed v .
 - v) B moves right with speed $2v$.
 - vi) B moves right with speed $4v$.
- b) Which cart gains the most kinetic energy? Explain your answer.

340 Ball launched from a moving cart

A 400 g cart slides along a laboratory bench with velocity 10.0 m/s to the right. A 100 g ball is initially on the cart and at rest relative to the cart. It is subsequently fired to the left and, after this, the velocity of the ball relative to the laboratory is 10.0 m/s to the left. Determine the velocity of the cart after the ball has been fired. (131F2024)

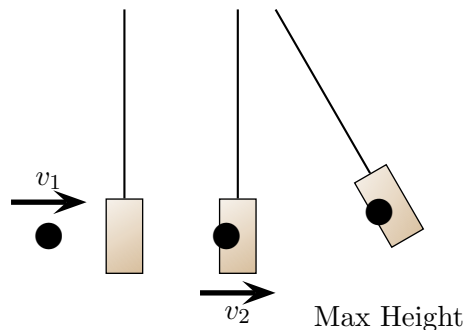
341 Brick thrown back and forth

Two people, each mass 70 kg, stand at opposite ends of a small 100 kg cart. The person on the left holds a 10 kg brick. They are all at rest initially. (131F2024)

- The person on the left throws the brick to the person on the right horizontally with speed 8.0 m/s. Determine the speed of the cart after the brick has been launched.
- The person on the right catches the brick and eventually holds it at rest relative to the cart. Determine the speed of the cart after this.

342 Ballistic pendulum

A ballistic pendulum consists of a 1.5 kg wooden block suspended at rest from a cord. A 0.0080 kg bullet is fired horizontally into the block. It embeds itself causing the block to swing upward, reaching a point 0.20 m above its lowest point. The idea is to use the maximum height reached by the pendulum to determine the speed of the bullet immediately before hitting the block. The process is illustrated at three instants. Let instant 1 be the moment just before the bullet hits, instant 2, the moment just after it embeds and instant 3, the moment at which it reaches its highest point. (131F2024)



- Is momentum conserved from instant 1 to instant 2? Is mechanical energy, $E = K + U_{\text{grav}}$, conserved from instant 1 to instant 2?
- Is momentum conserved from instant 2 to instant 3? Is mechanical energy, $E = K + U_{\text{grav}}$, conserved from instant 2 to instant 3?
- Consider the period from instant 2 to instant 3 and use this to determine the speed of the bullet and block just after it embeds.
- Consider the period from instant 1 to instant 2 and use this to determine the speed of the bullet and block just before it embeds.

343 Ballistic pendulum

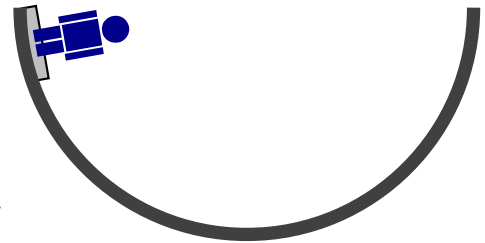
A 0.010 kg bullet is fired horizontally into a 2.0 kg block that is suspended at rest from a string just above the ground. The bullet collides with the block and sticks. The two then swing in a circle and the block reaches a height of 0.30 m above the ground. (131F2024)

- Determine the speed of the bullet and block just after the bullet has stuck to the block.
- Determine the speed of the bullet just before the bullet hits the block.

344 Half pipe

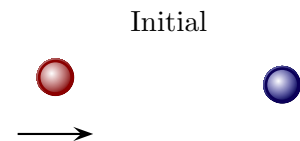
A 60 kg person rides a 5.0 kg skateboard. Both are at rest at the top of a half-pipe of radius 10 m. Ignore any friction and the rotation of the wheels. (131F2024)

- Determine the speed of the skateboarder at the bottom of the half pipe.
- Suppose that there was another skateboarder, of mass 80 kg on a 5.0 kg skateboard at rest at the bottom of the pipe. The two skateboarders collide, hold each other and move together. Determine their speed moments after they collide.
- Determine how high up the pipe the combined pair of skateboarders move.



345 Pool ball collision

A ball rolls right along a horizontal surface toward another ball, initially at rest. The diagram illustrates the approximate configuration of the balls. The balls collide and this collision is not necessarily head-on. Explain whether each of the following is a possible trajectory afterward. (131F2024)

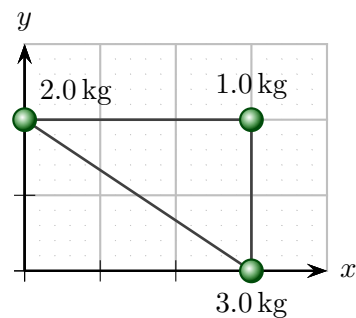


- Both balls are at rest.
- One ball moves \uparrow and the other moves \rightarrow .
- One ball moves \uparrow and the other moves \downarrow .
- One ball moves \nearrow and the other moves \rightarrow .
- One ball moves \nearrow and the other moves \searrow .

Rotational Motion

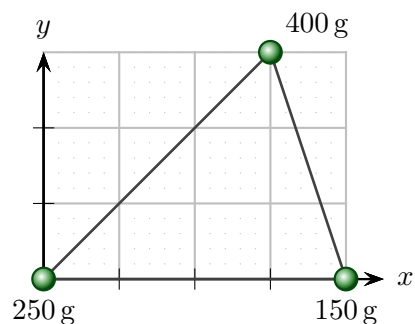
346 Center of mass of three balls, 1

Three small balls are connected via massless rods in the illustrated configuration. Each grid block is 2.0 cm long. Determine the center of mass of the system. (131F2024)



347 Center of mass of three balls, 2

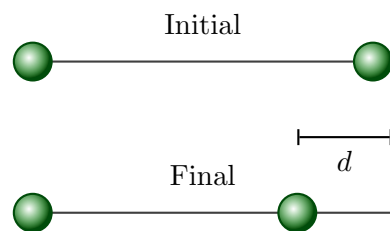
Three small balls are connected via massless rods in the illustrated configuration. Each grid block is 5.0 cm long. Determine the center of mass of the system. (131F2024)



348 Center of mass

Two identical balls are attached to a rigid rod with negligible mass. The ball on the right is shifted left by distance d . Which of the following is correct about the center of mass (ignore the rod) after the ball has been shifted? (131F2024)

- The c.o.m. has remained where it was before the mass was shifted.
- The c.o.m. moved less than $d/2$ to the left.
- The c.o.m. moved exactly $d/2$ to the left.
- The c.o.m. moved between $d/2$ and d to the left.
- The c.o.m. moved exactly d to the left.



349 Jumping crowd and Earth displacement

A crowd of 100000 people each with mass 80 kg is initially at rest on Earth's surface. They all jump upward simultaneously and their maximum height from their starting point is 0.50 m. Determine the maximum displacement of Earth from its starting point. (131F2024)

350 Rotating bar bell

A rigid barbell rotates about the axle at O . The distance from the end labeled B to O is three times the distance from the end labeled A to O . (131F2024)



- a) Which of the following is true? Explain your answer.
- i) The linear speed of B is nine times that of A.
 - ii) The linear speed of B is three times that of A.
 - iii) The linear speed of B is the same as that of A.
 - iv) The linear speed of B is one third that of A.
 - v) The linear speed of B is one ninth that of A.
- b) Explain as precisely as possible how the centripetal acceleration of A is related (e.g. same, four times, ...) to that of B.

351 Bugs on a rotating disk

The PhET animation

<http://phet.colorado.edu/en/simulation/rotation>

illustrates bugs on a rotating disk. Use the link to run the animation. Once the animation has opened, use the Intro tab (this is the default tab). The two bugs can be moved to various parts of the disk and the disk can be made to rotate. Place one bug a quarter of the way from the center to the edge. Place the other half way from the center to the edge. (131F2024)

- a) Predict how many times larger (or smaller) the velocity of the bug that is closer to the center is than that of the bug further from the center. Explain your prediction.
- b) Predict how many times larger (or smaller) the centripetal acceleration of the bug that is closer to the center is than that of the bug further from the center. Explain your prediction.
- c) Run the animation by entering various angular velocities. It should display the velocity and acceleration vectors. Do your results for the previous parts agree with the observed acceleration vectors? Do your predictions from the previous parts depend on the angular velocity?

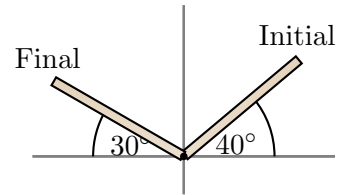
352 Rotating wheel RPM

A wheel rotates at 300 rpm (revolutions per minute). Determine the angular velocity of the wheel in rad/s. (131F2024)

353 Rotating beam

A narrow metal beam rotates at a constant rate about the axle through one end. Its positions are indicated at two instants 0.60 s apart. (131F2024)

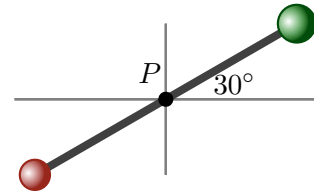
- Determine the angular velocity of the beam in rad/s.
- Determine the angular velocity of the beam in rpm.



354 Rotating bar bell angular position

A rigid barbell rotates about its midpoint P . At an initial moment the beam makes an angle of 30° from the horizontal. It subsequently rotates with a constant angular velocity of 1.5 rad/s . (131F2024)

- Determine the angle between the bar and the horizontal at an instant 1.0 s after the initial instant.
- Determine the angle between the bar and the horizontal at an instant 2.0 s after the initial instant.



355 Accelerating disk

A disk initially rotates with angular velocity 30 rad/s . It speeds up at a constant rate, reaching 100 rad/s at an instant 14 unitss later. Determine the angular acceleration of the wheel in rad/s^2 . (131F2024)

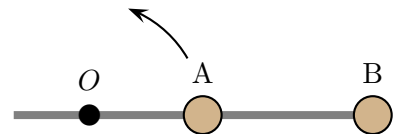
356 Accelerating wheel

A wheel initially rotates clockwise at 300 rpm . It speeds up at a constant rate, reaching 700 rpm at an instant 8.0 s later. Determine the angular acceleration of the wheel in rad/s^2 . (131F2024)

357 Rotating masses on a rod

Two masses are fixed as illustrated on a 0.50 m long rigid rod that rotates about the axis at O . The distance from O to the left end of the rod is 0.10 m , the distance from A to the left end of the rod is 0.25 m and B is at the right end of the rod. (131F2024)

- Suppose that the rod rotates at a constant rate of 720 rpm . Determine the speed of each mass.
- Suppose that the rod rotates in such a way that the angle from the horizontal is $\theta = (10 \text{ rad/s}^2) t^2$. Determine an expression for the angular velocity and angular acceleration of each ball. Determine the total distance traveled by each mass from $t = 0.0 \text{ s}$ to $t = 60.0 \text{ s}$.



358 Hard drive rotation

A hard drive in a computer contains a disk that rotates at a rate of 7200 rpm. The diameter of the disk is 6.25 cm. (131F2024)

- a) Determine the speed of a point at the edge of the disk while it is rotating at 7200 rpm.
- b) Assume that the disk is initially at rest and accelerates at a constant rate reaching 7200 rpm in 200 ms. Determine the magnitude of the angular acceleration of the disk.
- c) Determine the tangential component of the acceleration of a point on the edge of the disk and express it as a multiple of g .

359 Accelerating turntable

A turntable (circular disk with an axle perpendicular to the disk through its center) initially rotates counterclockwise about the axle at 180 rpm (revolutions per minute) and subsequently speeds up at a constant rate, reaching 900 rpm 4.0 s later.

- a) Convert the initial and final angular velocities into units of rad/s.
- b) Determine the angular acceleration of the turntable in rad/s^2 .
- c) Determine angular displacement of any point on the turntable during this 4.0 s period.

360 Slowing turntable

A turntable (circular disk with an axle perpendicular to the disk through its center) initially rotates counterclockwise about the axle at 4800 rpm (revolutions per minute) and subsequently slows at a constant rate. It stops after exactly 30 revolutions. (131F2024)

- a) Determine the angular acceleration of the turntable.
- b) Determine the speed of a point 0.15 m from the turntable axle at the instant when it has completed 15 revolutions.

361 Rotating bicycle wheels

A bicycle travels with speed 6.5 m/s (about 15 mph). The diameter of the bicycle wheel is 0.700 m. Determine the angular velocity of the wheel (about the axle). (131F2024)

362 Rotating Frisbee

A 30cm diameter Frisbee rotates about an axis through its center with a constant angular acceleration of $20\pi \text{ rad/s}^2$ starting from rest. (131F2024)

- a) Determine the angular velocity of a point on the rim of the Frisbee 3.0s after it starts rotating.
- b) Determine the angular velocity of a point halfway from the axis to the rim of the Frisbee 3.0s after it starts rotating.
- c) Determine the total distance traveled by a point on the rim of the Frisbee in the first 3.0s after it starts rotating.
- d) Determine the total distance traveled by a point halfway from the axis to the rim of the Frisbee in the first 3.0s after it starts rotating.

363 Snail on a turntable

A turntable (circular disk with an axle perpendicular to the disk through its center) is mounted horizontally. A 10 g snail sticks to a turntable at a distance of r from the turntable axle. The turntable is initially at rest and subsequently rotates with constant angular acceleration α . (131F2024)

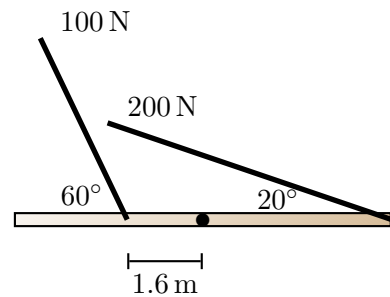
- a) Determine an expression for the ratio of the snail's centripetal acceleration to the tangential acceleration as a function of α , r and the time after the turntable starts to rotate.
- b) Determine how long it takes before the centripetal acceleration dominates.
- c) Describe how the direction of the net force on the snail evolves as time passes.
- d) Suppose that the acceleration is such that the disk rotates at a rate of 60 rpm after 5.0 s. If the snail is 0.30 m from the axle, determine the time that it would take for the snail's acceleration to reach $2g$.

364 Rotation on Earth

Two people stand on Earth's surface. One is in Houston (latitude 29.8° N) and the other in Seattle (latitude 47.6° N). Assuming that Earth is a perfect sphere and rotates at a constant angular rate, determine the speed, the angular velocity and the acceleration of each person. (131F2024)

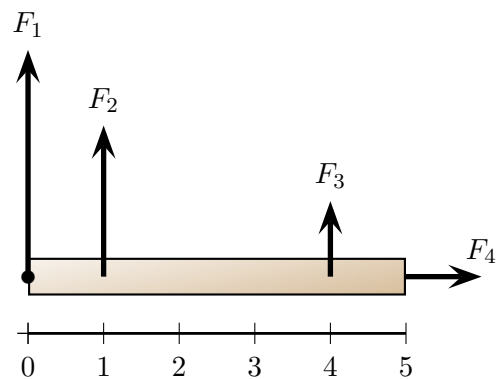
365 Multiple torques on a rod

A 3.0 kg beam with length 8.0 m can pivot about its midpoint. Two ropes pull as illustrated. The thickness of the rod is negligible. Determine the net torque on the rod. (131F2024)



366 Torques on a beam

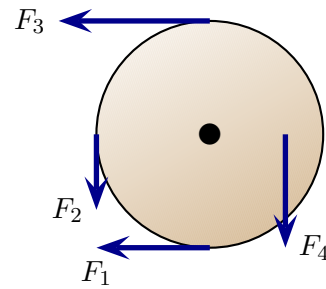
A beam can pivot about its left end. Various forces act on the beam as illustrated. An axis is provided for scale. The magnitudes of the forces are related via $F_1 = 3F_3$, $F_2 = 2F_3$, and $F_4 = F_3$. Rank the torques produced by the forces about the left end of the beam in order of increasing torque. Explain your answer. (131F2024)



367 Torques on a disk

A disk, with diameter 60 cm can rotate about an axle through its center and perpendicular to the disk. The illustrated forces act on the disk. The magnitudes of the forces are $F_1 = 15\text{ N}$, $F_2 = 10\text{ N}$, $F_3 = 20\text{ N}$ and $F_4 = 15\text{ N}$. The force F_4 acts at a point $2/3$ of the distance from the center of the disk to the rim. (131F2024)

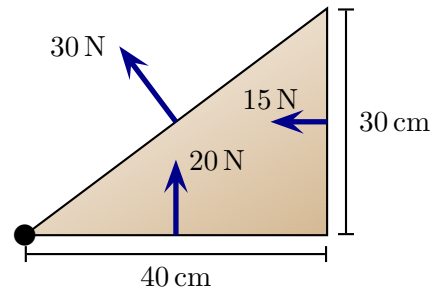
- Determine the torque exerted by each force about the axle.
- Determine the net torque on the disk.



368 Torques on a plywood triangle

Three forces act on a triangular piece of plywood as illustrated. Each acts midway along the edge. The plywood can pivot about the axle at the bottom left corner. (131F2024)

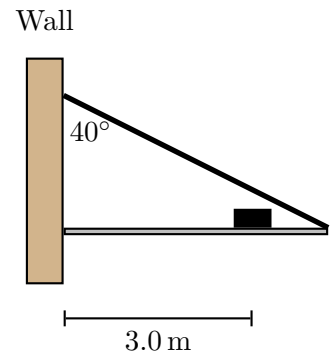
- Determine the torque exerted by each force about the axle.
- Determine the net torque on the plywood.



369 Beam perpendicular to a wall

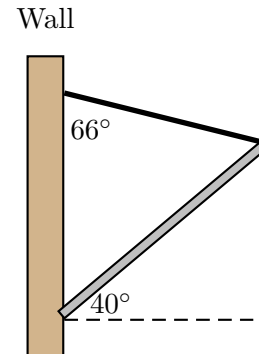
A 4.0 m long, 80 kg beam is anchored to a wall and held at rest horizontally by a rope at the illustrated angle. A 10 kg box rests on the beam at the illustrated point. The aim of this exercise is to determine the tension in the rope. This would enable one to decide on the breaking strength of the rope. The thickness of the beam is negligible. (131F2024)

- State the conditions for equilibrium.
- Draw all the force vectors on the beam.
- Identify a pivot point (there are many correct possibilities – one is much more useful than the others) and determine expressions for the torque exerted by each force about the pivot.
- Substitute the individual torques into one of the conditions for equilibrium and obtain an expression for the tension in the rope.



370 Raised drawbridge

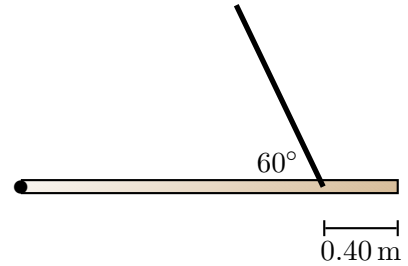
A 5.0 m long, 600 kg drawbridge is anchored to a castle wall. A rope holds the drawbridge at rest at the illustrated angle. While this happens a 120 kg knight in armor dangles vertically downward from the right end of the drawbridge. Determine the tension in the rope. (131F2024)



371 Beam in equilibrium, 1

A 10 kg beam with length 2.0 m can pivot about its left end. A rope is attached as illustrated. The thickness of the beam is negligible. (131F2024)

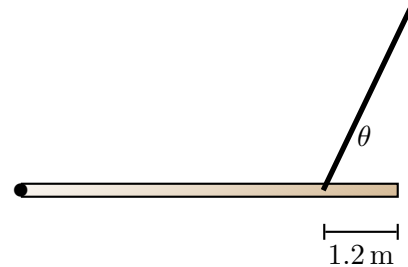
- State the conditions for equilibrium.
- Draw all the force vectors on the beam.
- Identify a pivot point (there are many correct possibilities – one is much more useful than the others) and determine expressions for the torque exerted by each force about the pivot.
- Substitute the individual torques into one of the conditions for equilibrium and determine the tension in the rope.



372 Beam in equilibrium, 2

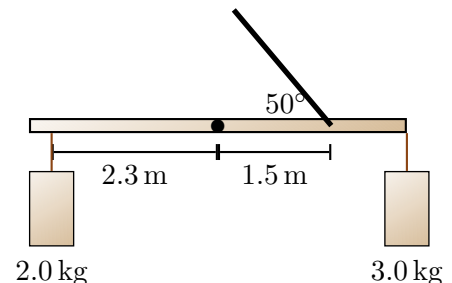
A 50 kg beam with length 6.0 m can pivot about its left end. A rope is attached as illustrated. The thickness of the beam is negligible. (131F2024)

- Determine an expression for the tension so that the beam stays at rest horizontally.
- Does the tension needed stay constant, increase or decrease as the angle $0^\circ \leq \theta \leq 90^\circ$ increases? Explain your answer.



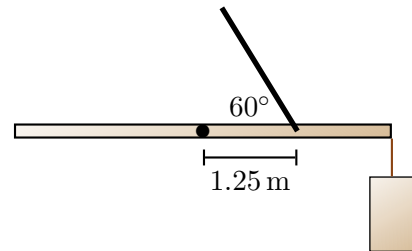
373 Balanced rod, unknown tension

A 5.0 m long rod can pivot about an axle through its midpoint. Masses are suspended at the illustrated points. A rope is attached 1.5 m from the midpoint and pulls at the illustrated angle. The thickness of the rod is negligible. Determine the tension in the rope that will keep the rod at rest horizontally. (131F2024)



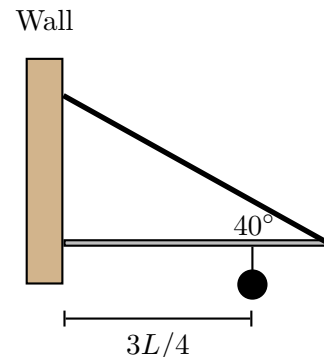
374 Balanced rod, unknown mass

A 5.0 m long rod can pivot about an axle through its midpoint. A block is suspended at the illustrated point. A rope is attached 1.25 m from the midpoint and pulls with tension 80 N at the illustrated angle. The thickness of the rod is negligible. Determine the mass of the suspended block that will keep the rod at rest horizontally. (131F2024)



375 Beam supporting a ball

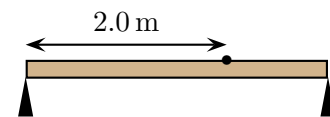
A beam with mass M and length L is anchored to a wall and held at rest horizontally by a rope as illustrated. A ball with mass m is suspended from the beam at the illustrated point. The aim of this exercise is to determine the tension in the rope. This would enable one to decide on the breaking strength of the rope. The thickness of the beam is negligible. (131F2024)



- State the conditions for equilibrium.
- Draw all the force vectors on the beam.
- Identify a pivot point (there are many correct possibilities – one is much more useful than the others) and determine expressions for the torque exerted by each force about the pivot.
- Substitute the individual torques into one of the conditions for equilibrium and obtain an expression for the tension in the rope.
- What must be the minimum required breaking tension for the rope if it is to support a 40 kg beam with length 3.0 m while there is a 8.0 kg ball suspended as illustrated?

376 Balance beam, 1

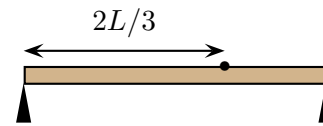
A 60 kg person stands on a 20 kg horizontal beam, with length 3.0 m, at the point illustrated by the dot. The beam is supported by stands at either end and is in equilibrium. Each support exerts an upward force. The thickness of the beam is negligible. (131F2024)



- Determine the force provided by the support on the right. *Hint: Consider torques about the left support and don't forget the mass of the beam.*
- Determine the force provided by the support on the left. *Hint: Consider torques about the right support and don't forget the mass of the beam.*

377 Balance beam, 2

A person with mass m_p stands on a horizontal beam, with mass m_b and length L , at the point illustrated by the dot. The beam is supported by stands at either end and is in equilibrium. Each support exerts an upward force. The thickness of the beam is negligible. (131F2024)

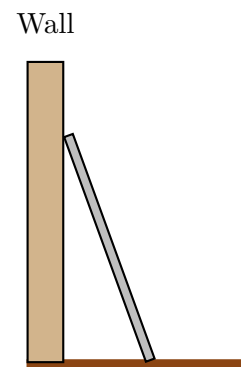


- Determine an expression for the force that each support provides. Does this depend on the length of the beam?
- Determine the force that each support provides if the beam has mass 40 kg and the person 90 kg.

378 Ladder against a wall

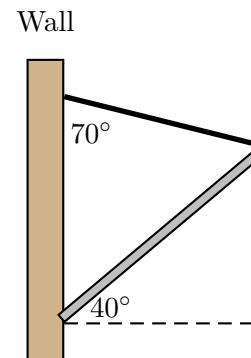
A ladder with mass M and length L rests against a wall as illustrated. The angle between the ladder and the ground is 70° . There is no friction between the ladder and the wall. (131F2024)

- Determine an expression for the force exerted by the wall on the ladder.
- Determine an expression for the magnitude of the force exerted by the ground on the ladder.



379 Raising a drawbridge

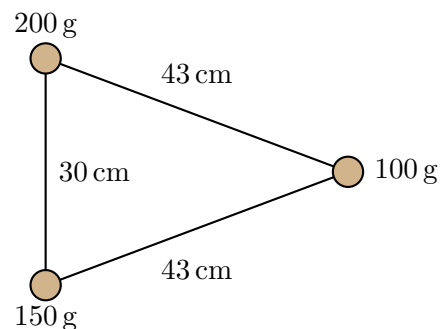
A 5.0 m long, 400 kg drawbridge is anchored to a castle wall. A rope with tension 2000 N raises the drawbridge at the illustrated angle. Determine the net torque on the drawbridge about the point where the drawbridge is attached to the wall. (131F2024)



380 Moment of inertia for point masses

Three point masses are connected by massless rods. Determine the moment of inertia about an axis perpendicular to the page and that passes through:

- the 150 g mass,
- the 100 g mass, and
- the 200 g mass. (131F2024)



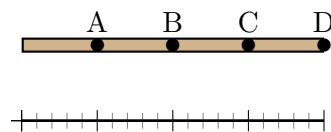
381 Moments of inertia

Two circular disks are made of the same material. Disk A has a radius three times that of disk B; their thicknesses are the same. Let I_A be the moment of inertia of A and I_B be the moment of inertia of B (each about an axis perpendicular to the page). Which of the following is true? Explain your answer. (131F2024)

- i) $I_A = I_B$
- ii) $I_A = \frac{9}{2} I_B$
- iii) $I_A = 9 I_B$
- iv) $I_A = \frac{81}{2} I_B$
- v) $I_A = 81 I_B$

382 Moment of inertia of a rod about different axes

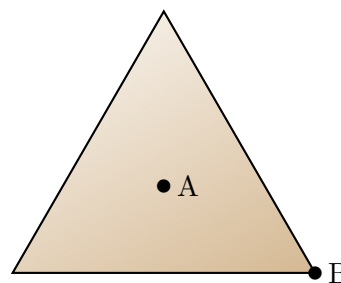
A thin rod can rotate about any of the illustrated four axes. The mass is uniformly distributed along the rod. Let I_A be the moment of inertia about point A, I_B be the moment of inertia about point B, etc. Rank the moments of inertia from smallest to largest, indicating any equality. Explain your answer. (131F2024)



383 Moment of inertia of a triangle about different axes

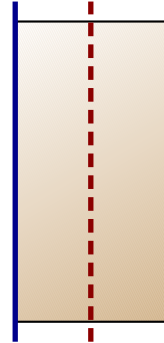
A flat equilateral triangular plate can rotate about one of two axes. Point A is at the center and point B is at one of the corners. Let I_A be the moment of inertia about A and I_B be the moment of inertia about B (each about an axis perpendicular to the page). Which of the following is true? Explain your answer. (131F2024)

- i) $I_A = I_B$
- ii) $I_A > I_B$
- iii) $I_A < I_B$



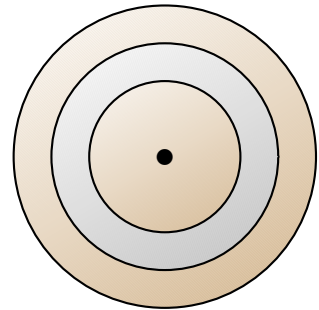
384 Moment of inertia of a door

A door could rotate about one of two axes. One axis (solid dark blue in the illustration) passes along the long edge of the door. The other axis (dashed dark red in the illustration) passes parallel to the long edge and through the center of the door. Estimate the mass and dimensions of a door in your room, state what your estimates are and use these to calculate the moment of inertia about each axis. *Note: The moment of inertia of a rectangular object about an axis parallel to the edge is the same as that for a rod rotating about the same axis.* (131F2024)



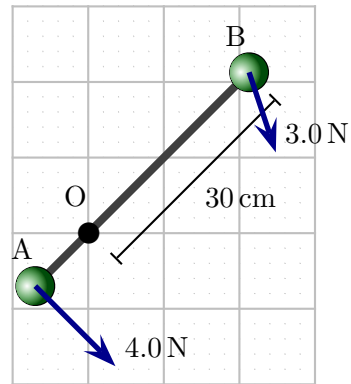
385 Moment of inertia of rotating objects

A turntable with mass M and radius R can rotate about an axis through its center and perpendicular to the disk. A ring with mass m , inner radius $R/4$ and outer radius $R/2$ is fixed to the disk concentrically. Determine an expression for the total moment of inertia of the system. (131F2024)



386 Rotational dynamics of a barbell

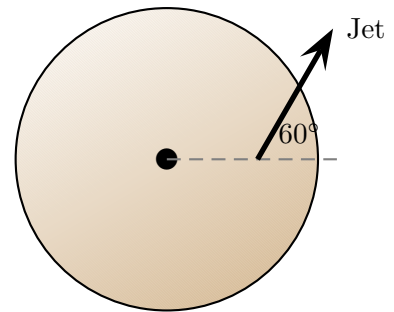
A rigid barbell consists of two heavy balls mounted at the ends of a light rigid 40 cm long rod. The barbell can rotate about an axle (pointing perpendicular to the board/page) at O. The mass of A is 600 g, the mass of B is 300 g and the mass of the rod is negligible. One force acts on each ball and the force on ball A is perpendicular to the rod. The angle between the force on B and the rod is 63° . The set-up is such that gravitational forces are irrelevant. At an indicated moment the rod makes a 45° angle with respect to the usual x axis. (131F2024)



- Determine the net torque on the barbell (about O).
- Determine the moment of inertia of the barbell (about O).
- Determine the angular acceleration of the barbell (about O).

387 Jet-propelled disk

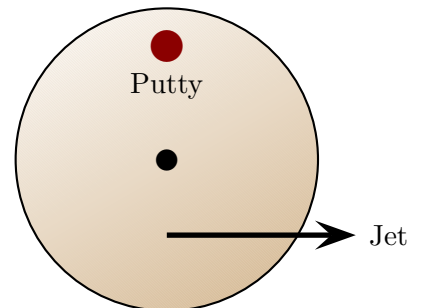
A 3.0 kg solid, uniform disk has radius 0.25 m and can rotate horizontally about a frictionless axle through its center. A small jet, which is attached 0.15 m from the center of the disk, exerts a 4.0 N force as illustrated. The aim of this exercise is to determine the angular acceleration of the disk. (131F2024)



- Write the rotational version of Newton's second law.
- Determine the moment of inertia of the disk.
- Determine the net torque acting on the disk.
- Determine the angular acceleration of the disk.

388 Rotating turntable and putty

A 2.0 kg turntable (disk) has radius 0.10 m and can rotate horizontally about a frictionless axle through its center. A 1.2 kg blob of putty is stuck to a point on the disk three quarters of the distance from the center to the edge. A jet attached halfway from the center to the edge of the disk exerts a 4.0 N tangential force as illustrated. The aim of this exercise is to determine the angular acceleration of the disk. (131F2024)



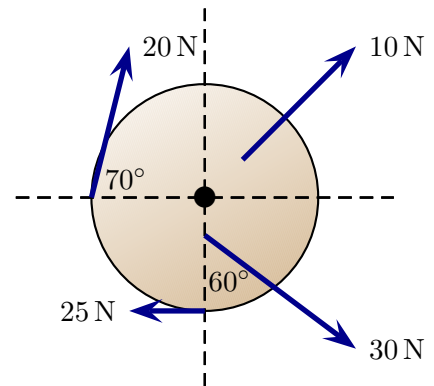
- Write the rotational version of Newton's second law.
- Determine the moment of inertia of the disk plus putty.
- Determine the net torque acting on the disk.

- d) Determine the angular acceleration of the disk.
- e) Suppose that a brake pad presses on the rim of the disk, producing a frictional force with magnitude 1.5 N while the jet is operating. If the wheel is rotating counterclockwise, determine the angular acceleration of the disk in this situation.

389 Rotating disk

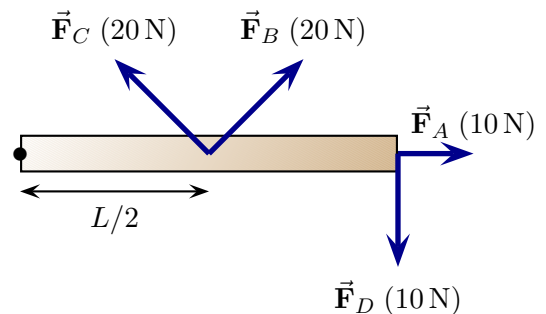
An 8.0 kg disk with radius 0.15 m can rotate about an axle through its center and perpendicular to the disk. The illustrated forces act on the disk. The 30 N force acts at a point 0.050 m from the axle. The 10 N force acts at a point 0.08 m from the axle. (131F2024)

- a) Determine the torque about the axle produced by each force on the disk.
- b) Determine net torque on the disk.
- c) Determine the angular acceleration of the disk.



390 Rotating rod

A rod with length L lies on a frictionless horizontal surface and can rotate about an axle at its left end. In separate instances one force is applied to the rod as illustrated. The angles between forces B and C and the rod are the same. Rank the magnitude of the angular accelerations in order from largest to smallest. Explain your answer. (131F2024)



391 Rod rotating in a vertical plane

A thin long rod with uniformly distributed mass M and length L can rotate in a vertical plane (i.e. up and down) about an axis through one end. A hand pushes vertically upward at the other end of the rod with force F . Consider the instant at which the rod is horizontal. For each of the following situations explain whether the rod is speeding up, slowing down, maintaining a constant velocity or the situation is impossible. (131F2024)

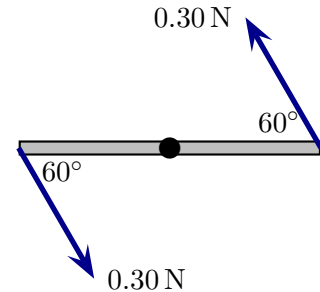


- a) The rod rotates counterclockwise and $F = Mg$.
- b) The rod rotates counterclockwise and $F = Mg/2$.

- c) The rod rotates counterclockwise and $F = Mg/4$.
- d) The rod rotates clockwise and $F = Mg$.
- e) The rod rotates clockwise and $F = Mg/2$.
- f) The rod rotates clockwise and $F = Mg/4$.

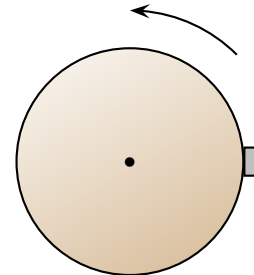
392 Spinning rod

A thin 0.25 kg, 0.50 m long rod can rotate about an axis through its center. Two jets are attached to the end of the rod and each exerts the same force at the same angle relative to the rod as illustrated. Suppose that the rod was initially at rest. Determine the time taken for the rod to rotate at 2000 rpm. (131F2024)



393 Braking wheel

A wheel with mass M and radius R can rotate about an axle through its center and perpendicular to the wheel. The wheel is a solid disk with uniform mass distribution. The wheel initially rotates with angular velocity ω_i . A brake pushes against perpendicularly against the edge of the wheel, exerting a (normal) force F . The coefficient of kinetic friction between the rim and brake is μ_k . (131F2024)



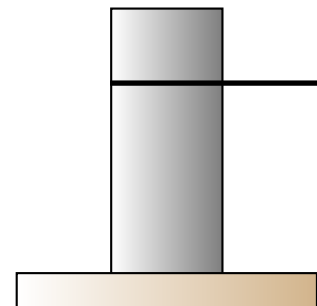
- a) Show that the time taken to stop the wheel is

$$T = \frac{MR\omega_i}{2\mu_k F}.$$

- b) Determine the time taken to stop a 2.5 kg wheel with radius 0.40 m that is initially rotating at 100 rpm. The force acting on the wheel is 35 N and the coefficient of kinetic friction is 0.40.

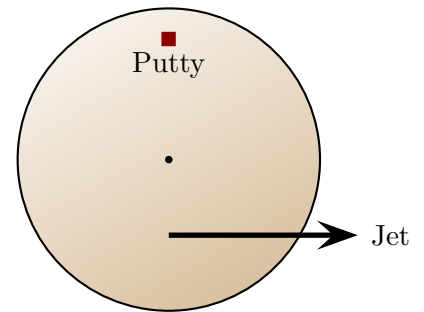
394 Flywheel and axle

A particular flywheel is a solid disk with mass 0.150 kg and radius 0.075 m. This is mounted to an 0.400 kg axle which is a hollow cylinder with radius 0.020 m. The entire arrangement is initially at rest and is subsequently pulled with constant tension by a string that is wound around the axle. It reaches an angular velocity of 40 rad/s in 5.0 s. A side view is illustrated. Determine the tension in the string. (131F2024)



395 Rotating turntable and putty

A turntable with radius R and uniformly distributed mass M can rotate horizontally about a frictionless axle through its center. A small blob of putty with mass m is stuck to a point on the disk $4/5$ of the distance from the center to the edge. A jet attached halfway from the center to the edge of the disk exerts a tangential force, with magnitude F_{jet} , as illustrated. (131F2024)



- Determine an expression for the angular acceleration of the disk in terms of R , M , m , F_{jet} and numbers.
- Assuming that the disk is initially at rest, determine an expression for the time it takes to complete one rotation.

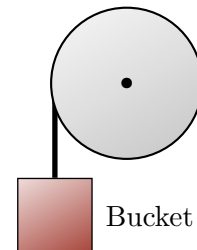
396 Rotating solid sphere

A solid sphere has radius 0.30 m and mass 6.0 kg. A thin cylindrical axle protrudes from the north pole and south pole of the sphere. This has radius 0.0080 m and its mass (both ends) is 0.250 kg. The sphere is initially at rest. A string is wound around the axle and is pulled with constant tension of 9.0 N. You aim to determine the time taken for the sphere to reach a certain speed. (131F2024)

- Given the parameters, does it make sense to consider the mass of the axle in the problem? Explain your answer.
- Determine the time taken for the sphere to reach 60 rpm.

397 Bucket suspended from a rotating pulley

A bucket with mass M is suspended from a massless string that is wrapped around a pulley. The pulley has with radius, R , and uniformly distributed mass, m and rotates about a frictionless axle through its center. The bucket is held at rest 1.5 m above the ground. The aim of this exercise is to find the time taken for the bucket to reach the ground. (131F2024)

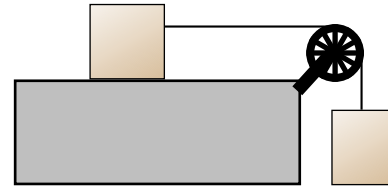


- Apply Newton's second law to the bucket and determine an expression for the acceleration of the bucket in terms of the tension in the string and other problem variables.
- Apply the rotational version of Newton's second law to the pulley and use this to determine an expression for the angular acceleration of the pulley in terms of the tension in the string and other problem variables.
- Relate the angular acceleration of the pulley to the acceleration of the bucket and use this and the previous expressions to find an expression for the acceleration of the bucket in terms of the masses, the pulley radius and g .

- d) Determine the time taken for the bucket to reach the ground if its mass is 3.0 kg, the pulley's mass is 2.0 kg and the radius is 0.20 m.

398 Dynamics of connected objects: level/suspended blocks with a massive pulley

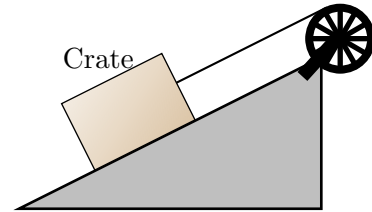
A box, with mass m_b , can slide along a horizontal frictionless surface. It is connected to a suspended object, with mass m_s , by a massless string. The string runs horizontally over a pulley, which is a solid disk with uniformly distributed mass m_p and radius R . These objects are held at rest and are then released.
(131F2024)



- Is the angular acceleration of the pulley after the objects are released zero or not? Explain your answer.
- How does the tension in the vertical part of string compare (smaller, larger, same) to that in the horizontal part of the string as the objects move after they are released? Explain your answer.
- Determine an expression for the acceleration of the objects after they are released.

399 Crate sliding down a ramp while connected to a heavy pulley

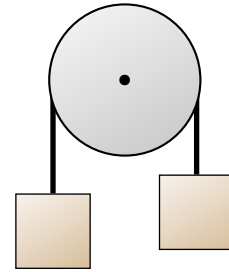
A crate on a rough ramp is connected to a string which is wrapped around a pulley. The string runs parallel to the ramp. At one instant the crate slides down the ramp.
(131F2024)



- Determine the acceleration of the crate if the ramp is inclined at an angle of 35° above the horizontal, the mass of the crate is 4.0 kg, the mass of the pulley is 2.0 kg and the coefficient of kinetic friction is 0.40. Does the crate speed up or slow down?
- Determine a general expression for the acceleration of the crate sliding down the ramp if the ramp is inclined at an angle of θ above the horizontal and the coefficient of kinetic friction is μ_k . Check that your expression is correct in the limit as the mass of the pulley approaches zero or the limit as the ramp becomes horizontal or vertical.

400 Blocks suspended from a rotating pulley, 1

A pulley, with moment of inertia I , can rotate about a frictionless axle through its center. Two blocks are suspended from a string, which runs, without slipping, over the pulley. The block on the left has mass m_1 and that on the right has mass $m_2 < m_1$. The system is held at rest and then released. (131F2024)



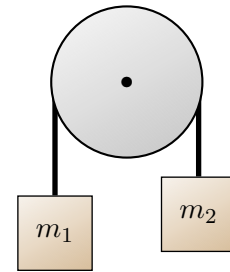
- Write the rotational version of Newton's second law for the pulley.
- Determine the net torque acting on the pulley, assuming that the tension in the string on the left might be different from that on the right.
- Determine an expression for the angular acceleration of the pulley in terms of the tensions in the strings.
- If the moment of inertia of the pulley is non-zero and the pulley is accelerating, can the two tensions be equal?
- Apply Newton's second law to each of the suspended masses, in each case relating the magnitude of the acceleration to the tension in the string.
- Combine the results from the previous parts to show that the magnitude of the acceleration of the blocks is

$$a = \frac{m_1 - m_2}{m_1 + m_2 + I/R^2} g$$

where R is the radius of the pulley.

401 Blocks suspended from a rotating pulley, 2

A pulley, with moment of inertia I , can rotate about a frictionless axle through its center. Two blocks are suspended from a string, which runs, without slipping, over the pulley. The block on the left has mass m_1 and that on the right has mass $m_2 < m_1$. The system is held at rest and then released. (131F2024)



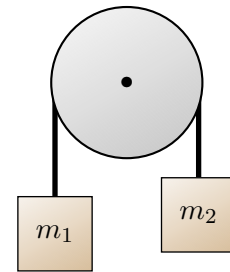
- a) If the moment of inertia of the pulley is non-zero and the pulley is accelerating, can the tension in the string on the left equal that on the right? Explain your answer.
- b) Show that the magnitude of the acceleration of the blocks is

$$a = \frac{m_1 - m_2}{m_1 + m_2 + I/R^2} g$$

where R is the radius of the pulley.

402 Blocks suspended from a massive rotating pulley and tension

A pulley, with mass m_p and radius R , can rotate about a frictionless axle through its center. Two blocks are suspended from a string, which runs, without slipping, over the pulley. The block on the left has mass m_1 and that on the right has mass $m_2 < m_1$. Let T_l be the tension in the left string and T_r be the tension in the right string. (131F2024)



- a) The blocks are initially held at rest and are then released. Which of the following is true *after* they are released? Explain your answer.
 - i) $T_l < T_r$
 - ii) $T_l = T_r$
 - iii) $T_l > T_r$
- b) The block on the right is given a brief sharp downward tug and, after this it begins to descend. Which of the following is true while it descends but after the tug? Explain your answer.
 - i) $T_l < T_r$
 - ii) $T_l = T_r$
 - iii) $T_l > T_r$