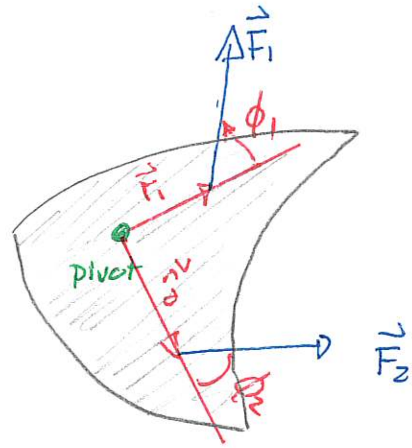
Mon: Warm Up 14

Tues: Discussion /quiz

Payscale data

Torques and dynamics



We now consider how forces affect the rotational state of motion of an object. The system will involve

Determine the net torque on the object

↳ The net torque is proportional to the angular acceleration $\boxed{\tau_{net} = I\alpha}$ where I is a constant that depends on the mass arrangement in a way to be determines.

Note that the individual torques are computed using

$$\boxed{\tau_i = r_i F_i \sin\phi_i}$$

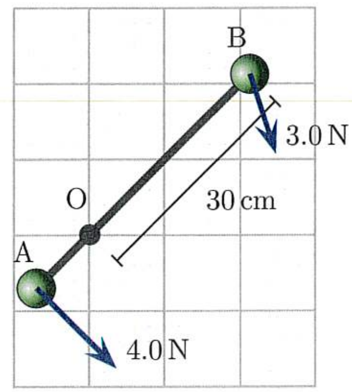
and

$$\boxed{\tau_{net} = \tau_1 + \tau_2 + \cdots}$$

Quiz 1 80% ≷ 90%

386 Rotational dynamics of a barbell

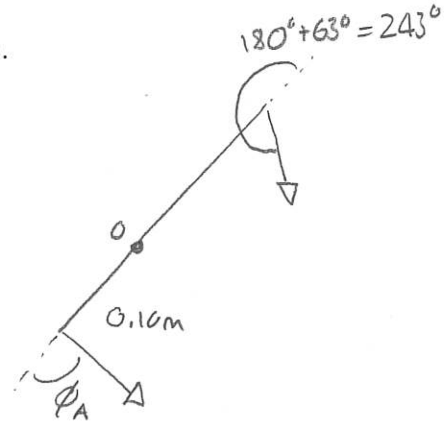
A rigid barbell consists of two heavy balls mounted at the ends of a light rigid 40 cm long rod. The barbell can rotate about an axle (pointing perpendicular to the board/page) at O. The mass of A is 600 g, the mass of B is 300 g and the mass of the rod is negligible. One force acts on each ball and the force on ball A is perpendicular to the rod. The angle between the force on B and the rod is 63°. The set-up is such that gravitational forces are irrelevant. At an indicated moment the rod makes a 45° angle with respect to the usual x axis. *(131Sp2023)*



a) Determine the net torque on the barbell (about O).

b) Determine the moment of inertia of the barbell (about O).

c) Determine the angular acceleration of the barbell (about O).



Answer: a) $\tau_{net} = \tau_{axle} + \tau_A + \tau_B$

Force A: $\tau_A = r_A F_A \sin\phi_A$

$= 0.10\text{m} \times 4.0\text{N} \sin 90°$

$= 0.40\text{Nm}$

Force B: $\tau_B = r_B F_B \sin\phi_B$

$= 0.30\text{m} \times 3.0\text{N} \sin 243°$

$= -0.80\text{Nm}$

Axle: $\tau_{Axle} = \overset{0}{\cancel{r_{Axle}}} F_{Axle} \sin\phi_{Axle} = 0\text{N.m.}$

$\tau_{net} = 0.40\text{NM} - 0.80\text{NM} = -0.40\text{NM}.$

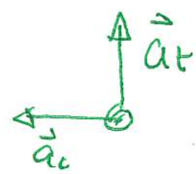
The actual acceleration will depend on the mass arrangement.

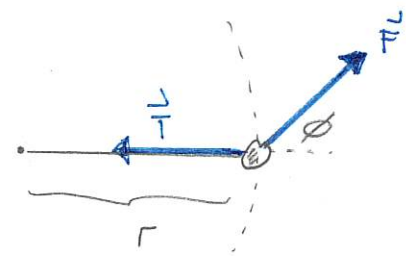
DEMO: RMS set up

To get an idea of how to account for this consider a single point particle swinging in a circle on a horizontal frictionless surface.

Then

$$\vec{F}_{net} = m\vec{a}$$





$$\Rightarrow \vec{T} + \vec{F} = m\vec{a}$$

But $\vec{a} = -a_c\hat{\imath} + a_t\hat{\jmath}$

$$\Rightarrow -T\hat{\imath} + [F\cos\phi\,\hat{\imath} + F\sin\phi\,\hat{\jmath}] = -ma_c\hat{\imath} + ma_t\hat{\jmath}$$

The $\hat{\jmath}$ components give

$$F\sin\phi = ma_t = mr\alpha \quad \Rightarrow \quad rF\sin\phi = mr^2\alpha$$

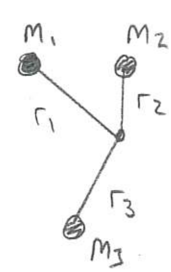
Then the torque produced by $\vec{F}$ is $rF\sin\phi$. Since $\vec{T}$ produces zero torque, in this case

$$\tau = (mr^2)\alpha$$

Thus we define:

The moment of inertia of a system of point particles about a pivot is

$$I = m_1r_1^2 + m_2r_2^2 + \ldots = \sum_{\text{all particles}} m_ir_i^2$$



where m_i = mass of object i

r_i = distance from object i to pivot

Units: $kg\,m^2$

Quiz 2 40% – 80% $\gtrless$ 40% ~ 90%

Quiz 3 30% – 80% $\gtrless$ 30% ~ 80%

This extends to objects with a continuous mass distribution. In this case the moment of inertia requires decomposition and integration. It is still true that

$$\boxed{\tau_{net} = I\alpha}$$

Quiz 4

Exercise 386 continued.

b) $I = m_1 r_1^2 + m_2 r_2^2 = 0.600kg \times 0.10m^2 + 0.300kg \times (0.30m)^2$

$= 0.033 kg\, m^2$

c) $\tau_{net} = I\alpha \Rightarrow -0.40Nm = 0.033 kg m^2 \alpha$

$\Rightarrow \alpha = \dfrac{-0.40Nm}{0.033 kg m^2}$

$\Rightarrow \alpha = -12.1 rad/s^2$.

Thus the velocity changes by -12rad/s every second.