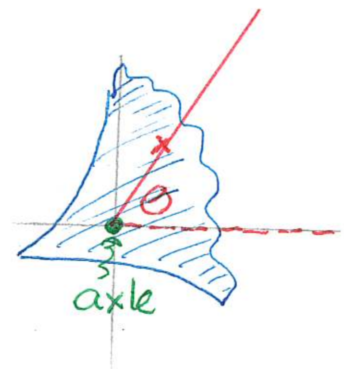
<u>Tues:</u> Discussion /quiz
EX 348, 350, 353, 357, 358, 363, 366, 367
<u>Weds:</u> Group exercise.

Rotational kinematics



We will describe rotational motion via adapting kinematics. Consider a rigid object that rotates about a fixed axle. We only need one co-ordinate to describe the object's configuration. We mark a point on the object and then use:

$$\boxed{\text{Angular position} = \text{angle measured (in radians) counterclockwise from a reference line.}}$$
θ

Then

$$\boxed{\text{Angular velocity} \equiv \omega = \frac{d\theta}{dt}}$$
rad/s

→ ω = slope of θ vs t

→ $\omega > 0 \Rightarrow$ rotates c.c.w
$\omega < 0 \Rightarrow$ rotates c.w

and

$$\boxed{\text{Angular acceleration} \equiv \alpha = \frac{d\omega}{dt}}$$
rad/s²

→ α = slope of ω vs t

<u>Quiz1</u> 60% - 80% & 40% - 60%

<u>Quiz2</u> 50% - 60% & 50% - 70%

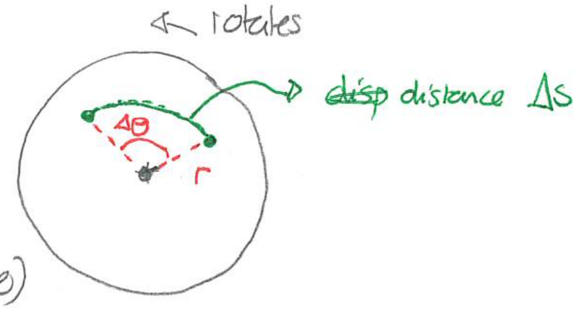
<u>DEMO</u>: PhET Ladybug Revolution → Rotation Tab.

* $\alpha = 0$ $\omega_i < 0$
* $\alpha > 0$ $\omega_i < 0$
* $\alpha < 0$ $\omega_i < 0$

Rotational and linear quantities

We can describe any single point on a rotating object using rotational or linear quantities. These are related u

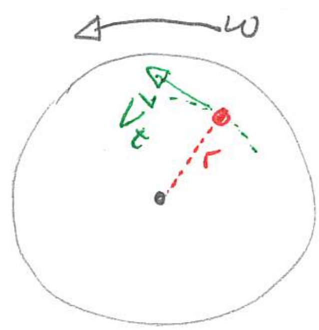
1) distance and displacement



For a point a distance r from the axle, the distance traveled when angle change by $\Delta\theta$ (angular displacement is $\Delta\theta$) is

$$\Delta s = r\,\Delta\theta$$

2) velocities

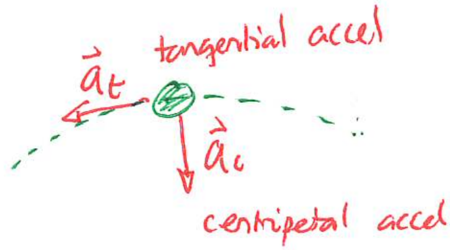


For a point distance r from the axle, the velocity is a vector tangential to the trajectory. This is denoted $\vec{v}_t$. The magnitude of the tangential velocity is the speed

$$v_t = \omega r$$

Warm Up 1

3) accelerations



The acceleration has two components:

a) centripetal acceleration radially inwards

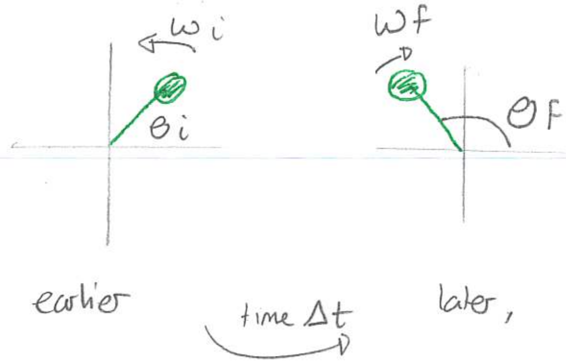
$$a_c = v_t^2/r = \omega^2 r$$

b) tangential acceleration $\quad a_t = \alpha r$

Warm Up 2.

Constant angular acceleration

If the angular acceleration is constant, then integration gives:



$$\boxed{\begin{aligned} \omega_f &= \omega_i + \alpha \Delta t \\ \theta_f &= \theta_i + \omega_i \Delta t + \tfrac{1}{2} \alpha \Delta t^2 \\ \omega_f^2 &= \omega_i^2 + 2\alpha(\theta_f - \theta_i) \end{aligned}}$$

Note that we often describe angular velocity in revolutions per minute (rpm). Then

$$1 \text{ rpm} = \frac{2\pi \text{ rad}}{60 s}$$

359 Accelerating turntable

A turntable (circular disk with an axle perpendicular to the disk through its center) initially rotates counterclockwise about the axle at 180 rpm (revolutions per minute) and subsequently speeds up at a constant rate, reaching 900 rpm 4.0 s later.

a) Convert the initial and final angular velocities into units of rad/s.

b) Determine the angular acceleration of the turntable in rad/s^2.

c) Determine angular displacement of any point on the turntable during this 4.0 s period.

a) $1\,\text{rpm} = \dfrac{2\pi\,\text{rad}}{60\text{s}} \Rightarrow 180\,\text{rpm} = 180 \times \dfrac{2\pi\,\text{rad}}{60\text{s}} = 6\pi\,\text{rad/s} = 18.8\,\text{rad/s}$

$\Rightarrow 900\,\text{rpm} = 900 \times \dfrac{2\pi\,\text{rad}}{60\text{s}} = 30\pi\,\text{rad/s} = 94.2\,\text{rad/s}$

b) $\omega_f = \omega_i + \alpha\Delta t \Rightarrow \dfrac{\omega_f - \omega_i}{\Delta t} = \alpha \Rightarrow \alpha = \dfrac{(30\pi - 6\pi)\,\text{rad/s}}{4.0\text{s}} = 6.0\pi\,\text{rad/s}^2$

$= 18.8\,\text{rad/s}^2$

c) $\theta_f = \theta_i + \omega_i\Delta t + \frac{1}{2}\alpha\Delta t^2$

$\Rightarrow \Delta\theta = \omega_i\Delta t + \frac{1}{2}\alpha\Delta t^2 \Rightarrow \Delta\theta = 6\pi\,\text{rad/s} \times 4.0\text{s} + \frac{1}{2}\, 6.0\,\pi\,\text{rad/s}^2\,(4.0\text{s})^2$

$= 24\pi\,\text{rad} + 48\,\pi\,\text{rad}$

$= \overset{72}{\cancel{80}}\,\pi\,\text{rad} = \cancel{157\,\text{rad}}.$

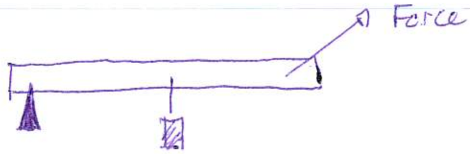
$50\pi\,\text{rad} = \dfrac{360^\circ}{2\pi\,\text{rad}}\,50\pi\,\text{rad} = 25 \times 360^\circ$

38 revolutions.

Rotational effects of forces

Forces can change the rotational state of motion of objects.

DEMO:

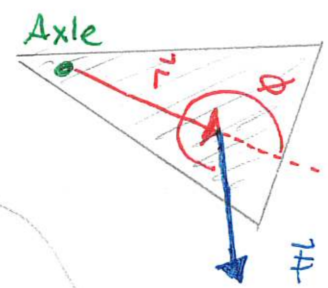


The demonstration shows that the rotational effects of forces depend on:

1) magnitude of the force
2) point at which the force acts
3) angle at which the force acts.

Those are described by torque.

Identify the force acting on the object and its location.



→ 1) Choose a reference point O (typically axle or pivot)

2) draw a vector $\vec{r}$ from O to where the force acts, and extend

3) Let ϕ be angle c.c.w. from extension to $\vec{F}$

4) The "torque produced by $\vec{F}$ about O" is

"tau" $\tau = rF\sin\phi$ Units N·m

↓

Torques will determine angular acceleration → Force tends to change angular velocity

Warm Up 3

Note: 1) In the definition r, F are never negative

2) If $0 \le \phi \le \pi$ then $\tau \geq 0$ → counterclockwise

$\pi \le \phi \le 2\pi$ then $\tau \leq 0$ → clockwise.

If multiple forces act on an object then the net torque is

$$\boxed{\tau_{net} = \underbrace{\tau_1}_{\text{Torque force 1}} + \underbrace{\tau_2}_{\text{Torque force 2}} + \tau_3 + \ldots = \sum \tau_i}$$

Quiz 3 50%-70%

60%-80%.