Phys 131 2024

Lecture 27

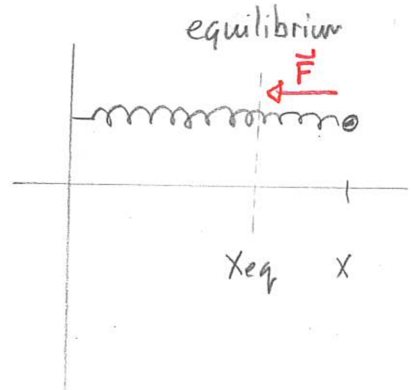
Fri: HW 5pm

Mon: Warm Up 11 D2L

Tues: Discussion /quiz

Spring forces

The force law for springs is determined by



1) the spring has an equilibrium configuration where it exerts no force.

2) when the spring is disturbed from equilibrium it exerts a restoring force back to equilibrium

3) set up the x-axis along the spring axis. Then the x-component of the spring force is

$$\boxed{F_x = -k\Delta x}$$ Hooke's Law

where $\Delta x = x - x_{eq}$ and

k = spring constant (measured in N/m)

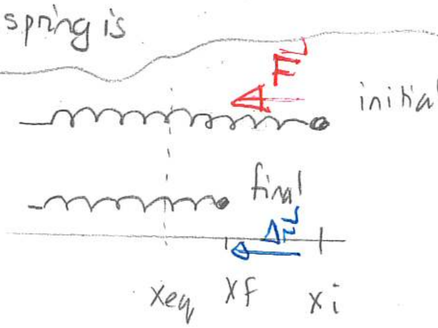
Work done by a spring

We can show that the work done by a spring is

$$W_{spring} = -\frac{1}{2}k(\Delta x_f)^2 + \frac{1}{2}k(\Delta x_i)^2$$

where $\Delta x_f = x_f - x_{eq}$

$\Delta x_i = x_i - x_{eq}$



Proof: $$W_{spring} = \int_{x_i}^{x_f} F_x \, dx = \int_{x_i}^{x_f} -k(x - x_{eq}) \, dx$$

$$= -\frac{1}{2}k(x - x_{eq})^2 \Big|_{x_i}^{x_f} = -\frac{1}{2}k(x_f - x_{eq})^2 + \frac{1}{2}k(x_i - x_{eq})^2 \quad \blacksquare$$

Quiz! 70% $\{$ 50% – 70%

Energy conservation and spring forces

Suppose that the only force that does non-zero work is a spring force and that the spring exerts a force along the x-axis. Then between an initial and final point, Then

$$\Delta K = W_{net} \Rightarrow K_f - K_i = W_{spring}$$

$$\Rightarrow K_f - K_i = -\frac{1}{2}k(\Delta x_f)^2 + \frac{1}{2}(\Delta x_i)^2$$

$$\Rightarrow K_f + \frac{1}{2}k(\Delta x_f)^2 = K_i + \frac{1}{2}k(\Delta x_i)^2$$

We therefore define

> The elastic potential energy of a spring is
>
> $$U_{sp} = \frac{1}{2}k(\Delta s)^2$$
>
> where Δs is the magnitude of the displacement from equilibrium

It follows that

> If the only force that does non-zero work is a spring force then the total energy
>
> $$E = K + U_{sp}$$
>
> is constant.

Fundamental Mechanics: Group Exercise 6

25 October 2024

Names: __________________ __________________ __________________

__________________ __________________

1 Oscillating cart

A 0.60 kg cart can slide along a frictionless track. It is attached to a spring with spring constant 3.5 N/m. The cart is pulled 0.40 m from the spring's equilibrium position and released from rest. *(131F2024)*

a) Describe whether you can use constant acceleration kinematics to predict the cart's speed at any moment after it has been released.

b) Determine the maximum speed of the cart.

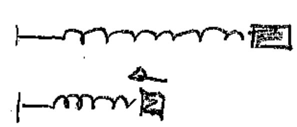
Answer: a) No, the force varies as position changes and thus the acceleration will change

b) $E_f = E_i$

initial

final

(at equilib)



$K_f + U_{spf} = K_i + U_{spi}$

$\Delta s_i = 0.40\,m$, $V_i = 0\,m/s$; $\Delta s_f =$, $V_f =$

$\frac{1}{2} m v_f^2 + \frac{1}{2} k (\Delta s_f)^2 = \cancel{\frac{1}{2} m v_i^2} + \frac{1}{2} k (\Delta s_i)^2$

this will be largest when Δs_f^2 is smallest $\Rightarrow \Delta s_f = 0$

$\Rightarrow \frac{1}{2} m v_f^2 = \frac{1}{2} k (\Delta s_i)^2 \Rightarrow v_f^2 = \frac{k}{m} (\Delta s_i)^2$

$\Rightarrow v_f = \sqrt{\frac{k}{m}}\, \Delta s_i$

$= \sqrt{\frac{3.5\,N/m}{0.60\,kg}} \times 0.40\,m$

$= 0.97\,m/s$

We can extend energy conservation to include gravity and springs

$$\boxed{\begin{array}{l}\text{If the only force that do non-zero work are gravity and springs} \\ \text{then the total energy} \\ \qquad E = K + U_{grav} + U_{spring} \\ \text{stays constant}\end{array}}$$

This is another example of the conservation of energy

308 Bungee jumper

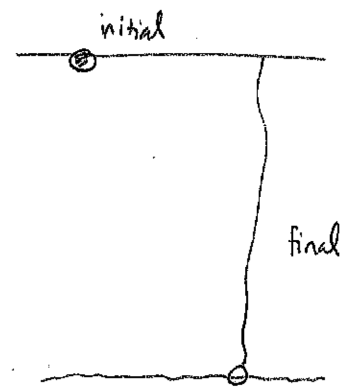
A 100 kg person is attached to a bungee cord and, starting at rest, jumps off a bridge that is 120 m above a river. The bungee cord behaves like a spring and the length of the cord when it is unstretched is 100 m. The spring constant of the cord needs to be such that person stops just above the river. *(131F2024)*

a) Determine the total energy of the system at the moment that the person jumps.

b) Determine the total energy of the system at the moment that the person stops just above the river and use the result to determine the spring constant of the bungee cord.

c) Determine the maximum force that the bungee cord exerts on this person.

d) Now suppose that a person with mass 70 kg jumps from the same bridge using the same cord. Determine the maximum stretch in the spring, the height above the river at which the person reverses direction and the maximum force exerted on the person.



$y_i = 120m$ $\quad y_f = 0m$

$v_i = 0m/s$ $\quad v_f = 0m/s$

$\Delta s_i = 0m$ $\quad \Delta s_f = 20m$

Answer: a) $E_i = K_i + U_{grav,i} + U_{sp,i}$

$= \cancel{\frac{1}{2}mv_i^2}^{\,0} + mgy_i + \cancel{\frac{1}{2}k(\Delta s_i)^2}$

$= 100kg \times 9.8m/s^2 \times 120m = 1.18\times10^5 J$

b) $E_f = K_f + U_{grav f} + U_{sp f}$

$1.18\times10^5 J = \cancel{\frac{1}{2}mv_f^2} + \cancel{mgy_f} + \frac{1}{2}k(\Delta s_f)^2$

$1.18\times10^5 J = \frac{1}{2}k(20m)^2$

$= 200m^2\, k$

$\Rightarrow k = 588\ N/m$

c) $|F_{sp}| = k\Delta s$ $\qquad$ max $\Delta s = 20m$

$= 588N/m \times 20m = 11.8\ kN$