Tues: Discussion / quiz

Ex: 62, 65, 66, 69, 70, 73

Weds: –

Vector addition

Recall that a displacement vector is an arrow with a magnitude and direction. We now want to define algebraic operations on vectors. We first consider adding two vectors. The idea will be

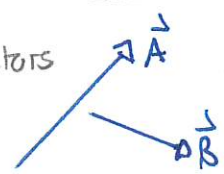
Given: Two vectors $\vec{A}, \vec{B}$ → **Sum**: The sum of $\vec{A}, \vec{B}$ is a single vector attained by successive displacements.

Demo: PhET Vector Addition
→ Explore 2D Tab → Select two vectors → Show sum

The procedure is:

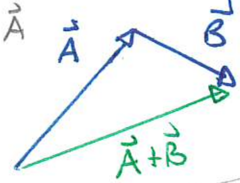
Given

Vectors



→ **Process**

Move $\vec{B}$ parallel to itself until tail of $\vec{B}$ touches tip of $\vec{A}$



→ **Sum**

The single displacement from tail of $\vec{A}$ to tip of $\vec{B}$ after rearrangement

Quiz 1 80% → 95 { 60% – 90%

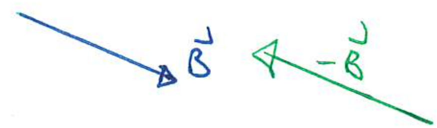
This illustrates that

$\boxed{\text{If } \vec{C} = \vec{A}+\vec{B} \text{ it is } \underline{\underline{\text{not}}} \text{ generally true that } \underbrace{C = A+B}_{\text{not usually true}}}$

Vector subtraction

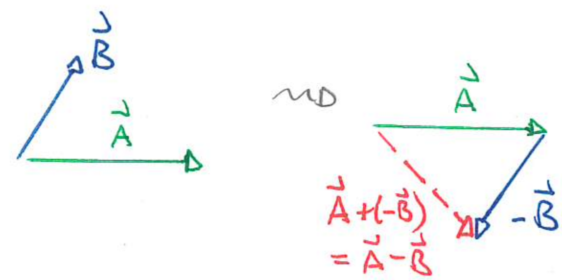
In algebra subtraction is an inverse operation to addition. This is related to negation. Here given a vector $\vec{B}$ the negative $-\vec{B}$ is another vector such that $\vec{B}+(-\vec{B}) = 0$. We can then see

> If $\vec{B}$ is any vector then
> $-\vec{B}$ is a vector with the same magnitude and opposite direction



Then

> For any vectors $\vec{A}, \vec{B}$,
> $$\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$$



Quiz 2 70% - 90% 80%

Warm Up 1

Scalar multiplication

We can repeatedly add the same vector e.g. $\vec{A}+\vec{A}+\vec{A} \stackrel{?}{=} 3\vec{A}$. This motivates scalar multiplication

> Let $\vec{A}$ be any vector and c any number. Then $c\vec{A}$ is a vector with
> 1) magnitude $|c|\vec{A}$
> 2) direction $= \begin{cases} \text{same as } \vec{A} & \text{if } c>0 \\ \text{opposite to } \vec{A} & \text{if } c<0 \end{cases}$

Vector components

We need to do vector algebra without using diagrams and eventually using numbers. The strategy to do this requires representing vectors in terms of components:

Given any vector $\vec{A}$ in two dimensions → Equivalent to 2 real numbers A_x, A_y

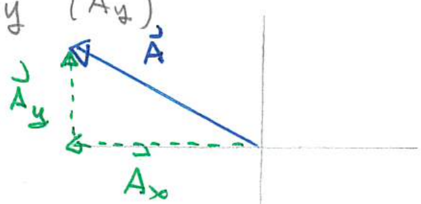
DEMO: PhET Vector addition – 2D Explore Tab.

→ Single vector → component vectors
→ component numbers.

The process to do this.

ESSENTIALLY ONE PAIR $\vec{A}_x, \vec{A}_y$ for any $\vec{A}$.

Given vector $\vec{A}$ decompose into two vectors, one along x $(\vec{A}_x)$ and the other along y $(\vec{A}_y)$



→ The components are two numbers:

Horizontal component A_x

$= \pm$ magnitude of A_x
+ if $\vec{A}_x$ right
− if $\vec{A}_x$ left

Vertical component A_y

$= \pm$ magnitude of A_y
+ if $\vec{A}_y$ up
− if $\vec{A}_y$ down

Warm Up 2

We can then use these for vector algebra via:

If $\vec{D} = \vec{A} + \vec{B} + \vec{C} + \ldots$ then

$D_x = A_x + B_x + C_x + \ldots$

$D_y = A_y + B_y + C_y + \ldots$

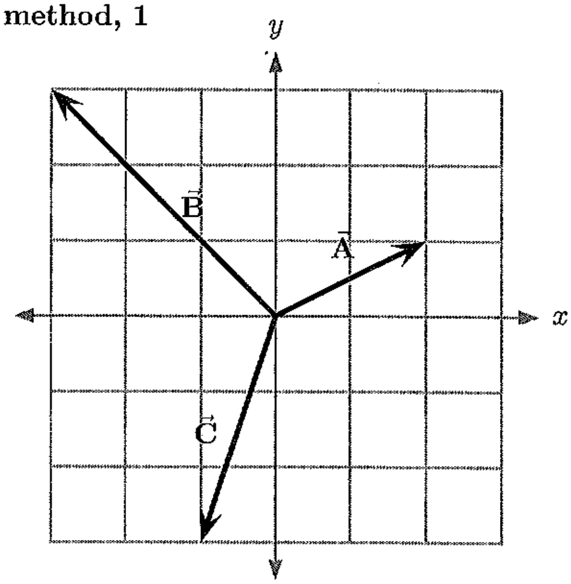
If $\vec{D} = c\vec{A}$ where c is a number then

$D_x = cA_x$

$D_y = cA_y$

82 Vector addition: graphical and algebraic method, 1

Displacement vectors, $\vec{\mathbf{A}}, \vec{\mathbf{B}}$, and $\vec{\mathbf{C}}$ are illustrated. Let $\vec{\mathbf{D}} = \vec{\mathbf{A}} + \vec{\mathbf{B}} + \vec{\mathbf{C}}$. *(131F2024)*



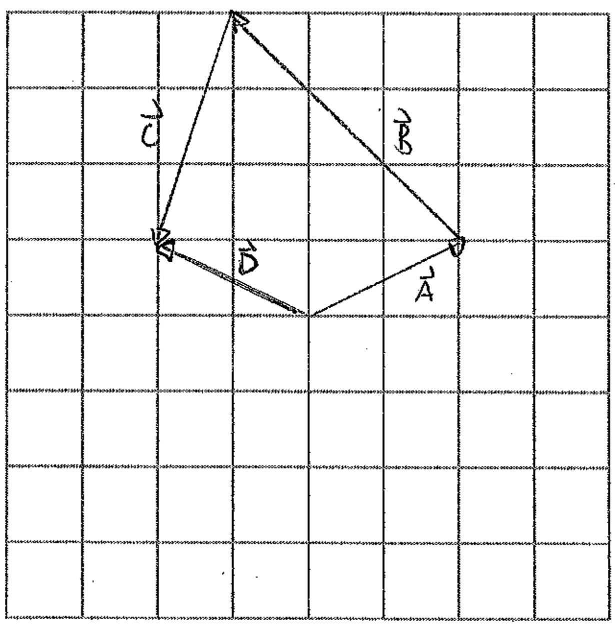
a) Using the graph sheet below, determine $\vec{\mathbf{D}}$ graphically via the head–to–tail method. Use the result to determine the magnitude of $\vec{\mathbf{D}}$.

b) List the horizontal and vertical components of each of $\vec{\mathbf{A}}, \vec{\mathbf{B}}$, and $\vec{\mathbf{C}}$ and use these components to determine the components of $\vec{\mathbf{D}}$. Use the result to determine the magnitude of $\vec{\mathbf{D}}$.

a)

$D = \sqrt{2^2+1^2}$

$= \sqrt{5} = 2.24$



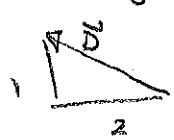
b) $A_x = 2$ $\quad A_y = 1$

$B_x = -3$ $\quad B_y = 3$

$C_x = -1$ $\quad C_y = -3$

$D_x = A_x + B_x + C_x = 2 - 3 - 1 = -2$

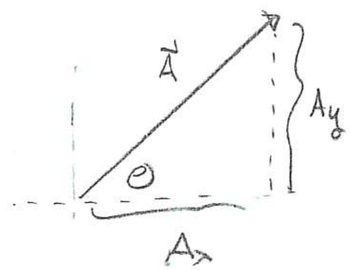
$D_y = A_y + B_y + C_y = 1 + 3 - 3 = 1$



$D = \sqrt{2^2+1^2} = \sqrt{5} = 2.24$

Calculating vector components

We can calculate vector components using trigonometry



Then A = magnitude of A (hypotenuse)

$\Rightarrow A_x = A\cos\theta$

$A_y = A\sin\theta$

Thus

Given $\vec{A}, \vec{B}$ want $\vec{D} = \alpha\vec{A} + \beta\vec{B}$

α, β real

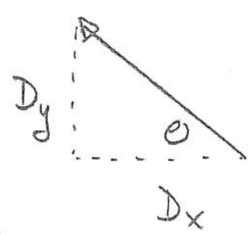
$\rightarrow$

$D_x = \alpha A_x + \beta B_x$

$D_y = \alpha A_y + \beta B_y$

gives components of $\vec{D}$

reconstruct $\vec{D}$ from components



$\rightarrow$

magnitude $D = \sqrt{D_x^2 + D_y^2}$

direction via angle, e.g

$\theta = \arctan D_y / D_x$

Quiz 3