Mon: Dec 2 Warm Up

Weds: Dec 4 Discussion/quiz

Rotational energy

We could use rotational energy to answer the question about the speed with which the tip of a rotating rod will fall. We find that the tip always falls with a greater speed than a freely falling object.

DEMO: MIT Hinged stick / falling ball video

DEMO: Falling chimney compilation

Vector description of rotational motion

The full description of a rotating object must account for the axis of rotation as well as the angular velocity and angular acceleration

DEMO: Show rotating disk about various axes

This is done using a vector description of angular velocity. The set-up is:

> The angular velocity $\vec{\omega}$ of an object is a vector with
>
> 1) magnitude $\omega = \left|\frac{d\theta}{dt}\right|$
>
> 2) direction - along the axis of rotation
>   - in a sense given by the right hand rule Fig 12.44

Quiz 1 80% - 90% & 60% - 90%

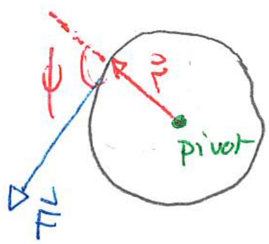
The angular acceleration is also a vector since

$$\boxed{\vec{\alpha} = \frac{d\vec{\omega}}{dt} = \lim_{\Delta t \to 0} \frac{\Delta \vec{\omega}}{\Delta t}.}$$

Then the rotational version of Newton's 2nd Law must be expressed in vector form:

$$\vec{\tau}_{net} = I\vec{\alpha}$$

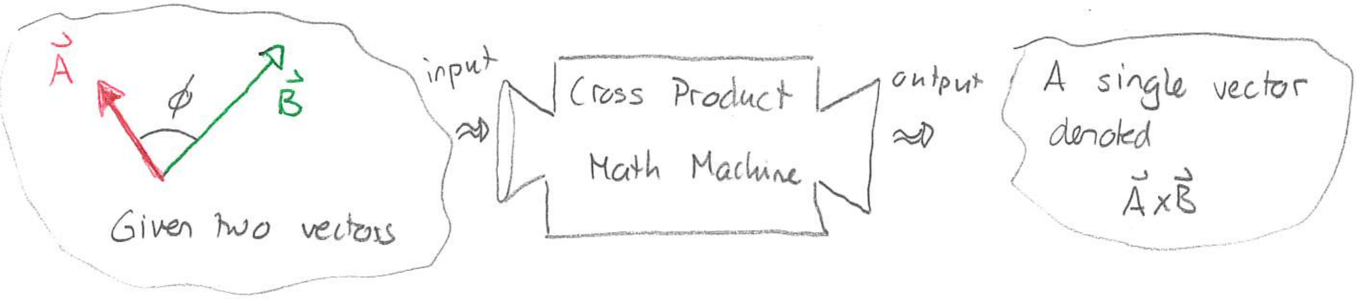
and this means that torque must be described in terms of vectors. Note that the torque produced by a force is constructed from two vectors:



$\rightsquigarrow$ need to multiply $\vec{r}, \vec{F}$ to produce a torque vector $\vec{\tau}$

Vector cross product

The cross product of two vectors produces a third vector.



The definition can be done in two ways

Method 1 The cross product of $\vec{A}$ with $\vec{B}$ denoted $\vec{A} \times \vec{B}$ in that order is a vector with: 1) magnitude $AB\sin\phi$ — A: magnitude of $\vec{A}$; B: magnitude of $\vec{B}$; ϕ: angle between $\vec{A}$ and $\vec{B}$ 2) direction — perpendicular to both $\vec{A}$ and $\vec{B}$ — sense given by r.h. rule — curl fingers from $\vec{A}$ to $\vec{B}$

Quiz2 90% $\geq$ 80% -

An alternative definition uses Cartesian basis vectors:

Method 2: The cross product of two vectors $\vec{A}, \vec{B}$ is 1) linear in the vectors 2) satisfies $\hat{i} \times \hat{i} = 0 \quad \hat{i} \times \hat{j} = \hat{k} \quad \hat{j} \times \hat{i} = -\hat{k}$ $\hat{j} \times \hat{j} = 0 \quad \hat{j} \times \hat{k} = \hat{i} \quad \hat{k} \times \hat{j} = -\hat{i}$ $\hat{k} \times \hat{k} = 0 \quad \hat{k} \times \hat{i} = \hat{j} \quad \hat{i} \times \hat{k} = -\hat{j}$

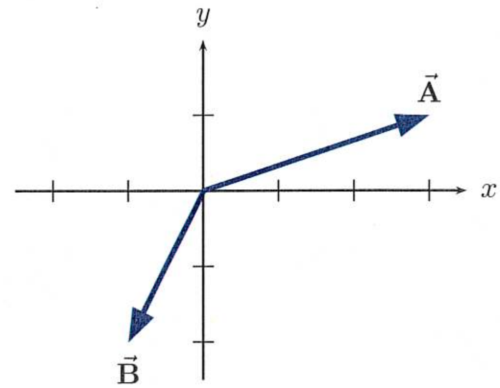
Quiz3 90%

One can show that:

1) $\vec{A} \times (\vec{B} + \vec{C}) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C}$ 2) $\hat{A} \times (\lambda \vec{B}) = \lambda (\vec{A} \times \vec{B})$ 3) $\vec{A} \times \vec{A} = 0$ 4) $\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$

415 Vector cross product

Two vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ are illustrated. Determine $\vec{\mathbf{A}} \times \vec{\mathbf{B}}$. *(131F2024)*



Answer

$$\vec{A} = 3\hat{i} + \hat{j}$$

$$\vec{B} = -\hat{i} - 2\hat{j}$$

$$\vec{A} \times \vec{B} = (3\hat{i}+\hat{j}) + (-\hat{i}-2\hat{j})$$

$$= 3\hat{i} \times (-\hat{i}) + 3\hat{i} \times (-2\hat{j}) + \hat{j} \times (-\hat{i}) + \hat{j} \times (-2\hat{j})$$

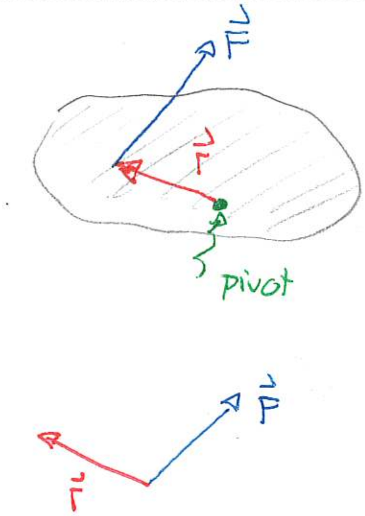
$$= -3 \underbrace{\hat{i} \times \hat{i}}_{0} - 6 \underbrace{\hat{i} \times \hat{j}}_{\hat{k}} - \underbrace{\hat{j} \times \hat{i}}_{-\hat{k}} - 2 \underbrace{\hat{j} \times \hat{j}}_{0}$$

$$= -6\hat{k} + \hat{k} \qquad = -5\hat{k}$$

$$\Rightarrow \quad \vec{A} \times \vec{B} = -5\hat{k}$$

Torque vector

The cross product allows for a vector definition of torque.



1) identify the force $\vec{F}$
2) construct $\vec{r}$ from the pivot to $\vec{F}$
3) rearrange $\vec{r}$ and $\vec{F}$ so that their tails coincide
4) the torque produced by $\vec{F}$ is

$$\vec{\tau} = \vec{r} \times \vec{F}$$

Quiz 4

DEMO: Bicycle wheel gyroscope

Angular momentum

In some circumstances it is useful to consider a rotational analogue of angular momentum.

The angular momentum of an object that rotates about a fixed axle is

$$\vec{L} = I\vec{\omega}$$

vector

units kgm^2/s

where I = moment of inertia about the axle
$\vec{\omega}$ = angular velocity about the axle

Then

$$\tau_{\substack{net \\ external}} = \frac{d\vec{L}}{dt}$$

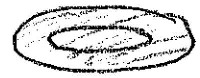
$\Rightarrow$ If the net external torque on a system is zero then the angular momentum of the system is constant

Conservation of Angular Momentum.

418 Rotational collision

A 0.150 kg solid disk with radius 0.050 m rotates at 180 revolutions per minute. A 0.400 kg ring with radius 0.030 m is held at rest and then gently dropped onto the disk so that its center coincides with the center of the disk. It sticks. Determine the angular velocity of the combination after the ring sticks to the disk. *(131F2024)*

Answer:



$\omega_{disk\,i} =$

$\omega_{ring\,i} = 0$

$\omega_{disk\,f} = \omega_{ring\,f} = \omega_f$

$$\omega_{disk\,i} = 180\,\frac{rev}{min} = 180\,\frac{rev}{min} \times \frac{2\pi\,rad}{1\,rev} \times \frac{1\,min}{60\,s} = 6\pi\ rad/s.$$

$$L_f = L_i$$

$$I_{disk}\,\omega_{disk\,f} + I_{ring}\,\omega_{ring\,f} = I_{disk}\,\omega_{disk\,i} + \cancel{I_{ring}\,\omega_{ring\,i}}$$

$$\Rightarrow (I_{disk} + I_{ring})\,\omega_f = I_{disk}\,\omega_{disk\,i}$$

$$\Rightarrow \omega_f = \frac{I_{disk}}{I_{disk} + I_{ring}}\,\omega_{disk\,i}$$

Then $I_{disk} = \frac{1}{2} M_{disk}\, r_{disk}^2 = \frac{1}{2} \times 0.150\,kg \times (0.050\,m)^2 = 0.000188\,kgm^2$

$I_{ring} = M_{ring}\, r_{ring}^2 = 0.400\,kg \times (0.030\,m)^2 = 0.00036\,kgm^2$

$$\Rightarrow \omega_f = \frac{0.000188\,kgm^2}{0.000188\,kgm^2 + 0.00036\,kgm^2}\, 6\pi\,rad/s = 2.1\pi\ rad/s.$$