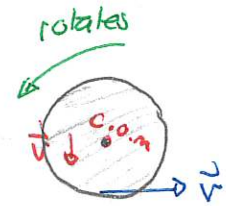
Thurs: Seminar 12:30
Wubben 203

Fri: HW 5pm

Ex 390, 394, 395, 396, 398, 399, 401, 402

Rotational Energy



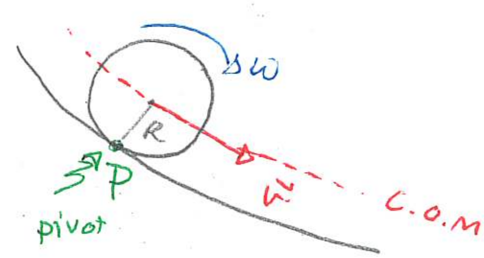
Consider a solid rotating object such as a pulley. Even though the center-of-mass is fixed, there must be some kinetic energy present as a result of the rotating mass. We cannot just express the kinetic energy as $\frac{1}{2}mv^2$ since there is a continuous range of velocities in the object. A general rule is:

> Consider an object that rotates about some axis. The object has rotational kinetic energy
>
> $$K_{rot} = \frac{1}{2} I \omega^2$$
>
> where I = moment of inertia about the rotation axis
> ω = angular velocity about the rotation axis.

For a proof see section 12.3.

Rotational and translational energies.

We now consider objects the simultaneously undergo translational and rotational motion. Let



* $\vec{V}$ = velocity of the center-of-mass
* ω = angular velocity about the pivot P

Then one can show that

If the net work done by non-conservative forces is zero, then the total energy

$$E = K_{trans} + K_{rot} + U_{grav} + U_{spring} + \ldots$$

stays constant. Here

$K_{rot} = \frac{1}{2} I \omega^2$ where I = moment of inertia about c.o.m.

$K_{trans} = \frac{1}{2} m v^2$

<u>Rolling without slipping</u>

A special case of this motion is where the object rolls without slipping. Then the speed of the center of mass satisfies

$$v = \omega R$$

and this can be used to eliminate ω from the energy expression. So

$$K_{trans} + K_{rot} = \frac{1}{2} m v^2 + \frac{1}{2} I \omega^2$$

$$\omega = v/R$$

$$\rightarrow K_{trans} + K_{rot} = \frac{1}{2} m v^2 + \frac{1}{2} I \frac{v^2}{R^2}$$

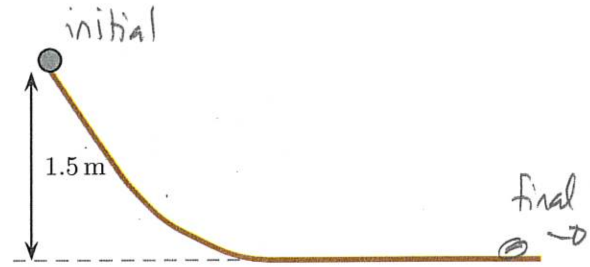
Quiz 1 30% - 60% ≶ 30% ~

The energy conservation rule emerges by applying Newton's 2nd and 3rd Laws to the constituents of the rotating objects and finally translating into rotational motion terms.

~~Quiz~~

410 Rolling cylinder

A cylinder with mass M and radius R is released from rest at the top of a track with height 1.5 m. It rolls without slipping. Determine the speed of the cylinder at the bottom of the track. *(131F2024)*



$E_f = E_i$

$y_i = 1.5\,\mathrm{m}$	$y_f = 0\,\mathrm{m}$
$v_i = 0\,\mathrm{m/s}$	$v_f = ?$
$\omega_i = 0\,\mathrm{rad/s}$	$\omega_f = ?$

$K_{trans\,f} + K_{rot\,f} + U_{g\,f} = K_{trans\,i} + K_{rot\,i} + U_{g\,i}$

$\frac{1}{2}Mv_f^2 + \frac{1}{2}I\omega_f^2 + \cancel{Mgy_f} = \cancel{\frac{1}{2}Mv_i^2} + \cancel{\frac{1}{2}I\omega_i^2} + Mgy_i$

Now $I = \frac{1}{2}MR^2$

$\Rightarrow \frac{1}{2}\cancel{M}v_f^2 + \frac{1}{4}\cancel{M}R^2\omega_f^2 = \cancel{M}gy_i$

No-slip $\Rightarrow$ $\omega_f = v_f/R$ and thus

$\frac{1}{2}v_f^2 + \frac{1}{4}v_f^2 = gy_i \Rightarrow \frac{3}{4}v_f^2 = gy_i$

$\Rightarrow v_f = \sqrt{\frac{4gy_i}{3}} = \sqrt{\frac{4 \times 9.8\,\mathrm{m/s^2} \times 1.5\,\mathrm{m}}{3}}$

$= 4.4\,\mathrm{m/s}$

Note that if the ball slid without rolling

$\frac{1}{2}v_f^2 = gy_i \Rightarrow v_f = \sqrt{2gy_i}$

which is larger than the rolling case

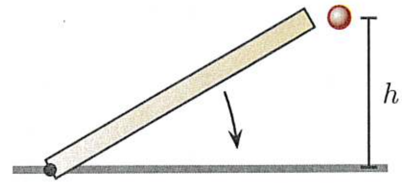
Quiz 2 very few.

~~Quiz~~

Quiz 3

405 Toppling rod versus freely falling ball, 1

A rigid rod with uniformly distributed mass M and length L can pivot about a frictionless axle in a horizontal surface. The rod is held at rest with one end height h above the surface. A ball with mass m is also held at rest alongside the tip of the rod. Both are released at the same time. *(131F2024)*



a) Determine the speed of the ball just before it hits the horizontal surface.

b) Determine the speed of the tip of the rod just before it hits the horizontal surface.

c) Which hits first?

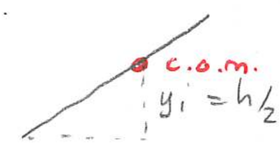
a) $E_f = E_i \Rightarrow K_f + U_{gf} = K_i + U_{gi}$

$$\Rightarrow \frac{1}{2}\cancel{m} v_f^2 + \cancel{mgy_f} = \cancel{\tfrac{1}{2}mv_i^2} + \cancel{m}gy_i \Rightarrow v_f = \sqrt{2gh}$$

b) $E_f = E_i$

$$K_{rot\,f} + \cancel{K_{trans\,f}} + \cancel{U_{gf}} = \cancel{K_{rot\,i}} + \cancel{K_{trans\,i}} + U_{gi}$$

The rotational kinetic energy is about the pivot, translational is at the pivot $\Rightarrow K_{trans} = 0$



$$\frac{1}{2} \underset{\text{left end}}{I_{rod}} \omega_f^2 = Mgh/2$$

$$\Rightarrow \cancel{\tfrac{1}{2}}\frac{1}{3}\cancel{M}L^2\omega_f^2 = \cancel{\tfrac{1}{2}}\cancel{M}gh$$

$$\Rightarrow \omega_f^2 L^2 = 3gh. \qquad \text{But} \qquad \omega_f L = v_f \equiv \text{speed of tip}$$

$$\Rightarrow v_f = \sqrt{3gh}$$

DEMO: U Iowa Falling Chimney

DEMO: YouTube video