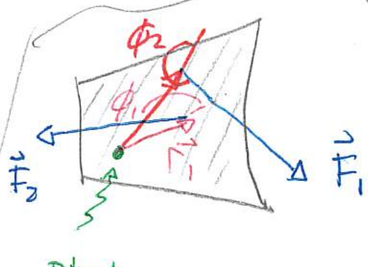
Lecture 36

Tues: Discussion /quiz

Ex: 380, 381, 383, 385, 387, 391, 392, 393

Rotational dynamics

The set-up for rotational dynamics is



specify forces and pivot

→ Determine torque produced by each force

$\tau_i = r_i F_i \sin\phi_i$

→ Determine net torque

$\tau_{net} = \tau_1 + \tau_2 + \ldots$

→ Determine moment of inertia about pivot

$$I = \sum_{\text{all masses}} m_i r_i^2 = \int r^2 dm$$

→ Rotational version of Newton's 2nd Law

$\tau_{net} = I\alpha$

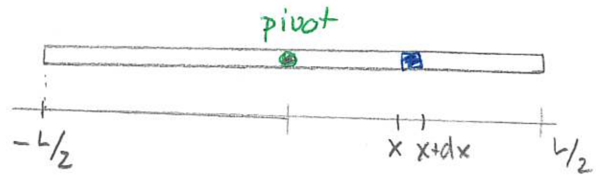
→ gives angular acceleration α

→ gives rate of change of angular velocity

$$\frac{d\omega}{dt} = \alpha$$

Moment of inertia for continuous distributions of mass

The original definition of moment of inertia involved point masses. This can be adapted to a continuous distribution of mass by breaking the mass into a collection of infinitesimally small pieces, and treating each as a point particle. As an example, consider a rod with negligible width and a uniform mass distribution. We aim to determine the moment of inertia about the center. Set up a co-ordinate axis as illustrated. Then



1) consider a segment from

$$x \to x+dx$$

2) this segment has mass

$$dm = \frac{M}{L}\,dx$$

3) this segment contributes

$$dI = r^2 dm \qquad = x^2\,dm$$

to the moment of inertia.

$$dI = \frac{M}{L} x^2 dx$$

4). The moment of inertia is the sum over all of these

$$I = \int dI = \int_{-L/2}^{L/2} \frac{M}{L} x^2 dx = \frac{M}{L} \frac{x^3}{3} \Big|_{-L/2}^{L/2}$$

$$\Rightarrow I = \frac{M}{L}\left[\frac{L^3}{24} - \frac{(-L)^3}{24}\right] = \frac{M}{L}\,\frac{L^3}{12} \Rightarrow I = \frac{1}{12} M L^2$$

We generically express this process as

$$\boxed{I = \int_{\substack{\text{entire}\\ \text{volume}}} r^2\, dm}$$

dm → sets up integral

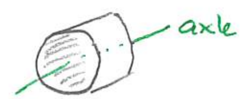
r^2 → varies throughout volume

~~Warm Up!~~

This can be done for two and three dimensional mass distributions.
Examples are listed in Table 12.2

1) Hoop mass M, radius R $\}$ $I = MR^2$

2) Solid cylinder mass M, radius R $\}$ $I = \frac{1}{2}MR^2$



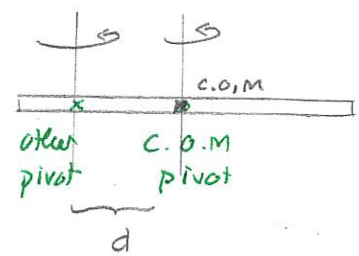
a

Warm up!

There are many possible axes of rotation. There will be a moment of inertia for each. They can be related.

Parallel Axis Theorem

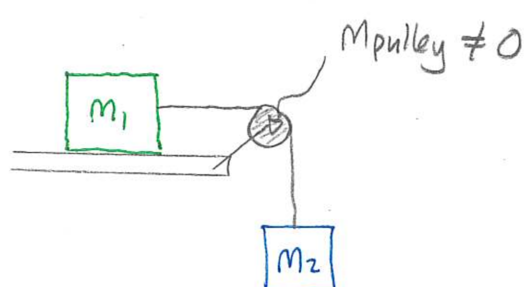
> Consider two axes of rotation such that:
>
> 1) one axis passes through the center-of-mass
> 2) the second axis is parallel to the first
>
> Then the moment of inertia for the second axis is
>
> $$I = I_{cm} + Md^2$$
>
> where d is the distance between axis



Quiz! 70% – 90% $\}$ 80%

Rotational Motion and Connected Objects

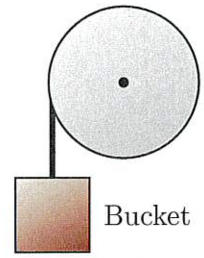
We will consider situations where there is rotational and linear motion. In the illustrated situation:



String does not slip, Blocks accelerate → Net torque on pulley ≠ 0 → Pulley accelerates

Warm up 2

395 Bucket suspended from a rotating pulley



A bucket with mass M is suspended from a massless string that is wrapped around a pulley. The pulley has with radius, R, and uniformly distributed mass, m and rotates about a frictionless axle through its center. The bucket is held at rest 1.5 m above the ground. The aim of this exercise is to find the time taken for the bucket to reach the ground. *(131F2024)*

a) Apply Newton's second law to the bucket and determine an expression for the acceleration of the bucket in terms of the tension in the string and other problem variables.

b) Apply the rotational version of Newton's second law to the pulley and use this to determine an expression for the angular acceleration of the pulley in terms of the tension in the string and other problem variables.

c) Relate the angular acceleration of the pulley to the acceleration of the bucket and use this and the previous expressions to find an expression for the acceleration of the bucket in terms of the masses, the pulley radius and g.

d) Determine the time taken for the bucket to reach the ground if its mass is 3.0 kg, the pulley's mass is 2.0 kg and the radius is 0.20 m.

Answer: a) The general scheme is.

Linear dynamics
↳ linear acceleration of bucket

→ Rotational dynamics
↳ angular acceleration of pulley

↳ No slip of string → relate linear and angular accel.

$$\sum F_{iy} = M a_y$$

$$\Rightarrow T - Mg = M a_y$$

Let $a =$ magnitude of acceleration of bucket

$$\Rightarrow a_y = -a \quad \Rightarrow \quad T - Mg = -Ma \quad \Rightarrow \boxed{Ma = Mg - T} \quad -(1)$$

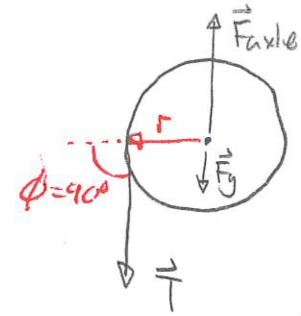
b) $\tau_{net} = I\alpha$

The moment of inertia of the disk is

$I = \frac{1}{2} M R^2$

The torques are obtained via

$\tau_{net} = \tau_{rope} + \tau_{grav} + \tau_{axle}$



Then

$\tau_{rope} = r F \sin\phi$

$= R T \sin 90° = RT$

$\tau_{axle} = \tau_{grav} = 0$ since $r = 0$.

Thus .

$\tau_{net} = RT \quad \Rightarrow \quad \boxed{T = \frac{1}{2} m R \alpha} \quad -(2)$

$RT = \frac{1}{2} M R^2 \alpha$

c) If the string does not slip then $\boxed{a = \alpha R} \quad -(3)$

Now combine (3) and (2) $\quad \alpha = a/R \quad \Rightarrow \quad T = \frac{1}{2} m R \frac{a}{R} = \frac{1}{2} m a$

Substitute into (1) gives

$Ma = Mg - \frac{1}{2} m a \quad \Rightarrow \quad \left[M + \frac{1}{2} m\right] a = Mg$

$\Rightarrow \quad a = \left[\frac{M}{M + \frac{1}{2} m}\right] g$

d) We can get the acceleration $\quad a = \left[\frac{3.0\,kg}{3.0\,kg + 1.0\,kg}\right] g = \frac{3}{4} g = 7.4\,m/s^2$

$\Rightarrow a_y = -7.4\,m/s^2$

Initial $\quad y_i = 1.5\,m \quad y_f = 0$

$V_{iy} = 0\,m/s \quad V_{fy} = ?$

Find $\quad a_y = -\frac{3}{4} g$

$\rightarrow \quad \cancel{y_f} = y_i + \cancel{v_{iy} \Delta t} + \frac{1}{2} a_y \Delta t^2$

$\Rightarrow \quad -\frac{2 y_i}{a_y} = \Delta t^2 \quad \Rightarrow \quad \Delta t^2 = \frac{2 y_i 4}{3 g}$

$\Delta t = \sqrt{\frac{8}{3} \frac{y_i}{g}} = \sqrt{\frac{8}{3} \frac{1.5\,m}{9.8\,m/s^2}} = 0.64\,s$